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Legendre Wavelet for Solving Linear System of Fredholm And Volterra Integral Equations

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Abstract
In this work, we employ Legendre wavelet method to find numerical solution of system of linear Fredholm and Volterra integral equations, which uses zeros of Legendre wavelets for collocation points is introduced and used to reduce this type of system of integral equations to a system of algebraic equations.
Keywords: system of linear Fredholm integral equations, system of linear Volterra integral equations, Legendre wavelets, linear algebraic equations.

I. Introduction

Consider the system of linear Fredholm and Volterra integral equations

$$U(x)-\int_0^1 K(x,t)U(x,t)dt=F(x),\quad x\in[0,1], \tag{1}$$

and

$$U(x)-\int_0^x K(x,t)U(x,t)dt=F(x),\quad x\in[0,1]. \tag{2}$$

Where

$$U(x)=[u_1(x),u_2(x),\dots,u_n(x)]^T,$$
$$F(x)=[F_1(x),F_2(x),\dots,F_n(x)]^T,$$
$$K(x,t)=\left[k_{ij}(x,t)\right],\ \ i,j=1,2,\dots,n. \tag{3}$$

In system (1), (2) the known kernel K(x, t) is continuous, the function F (x) is given, and U(x) is the solution to be determined.

The system (1) and (2) have been solved by several methods. For instance, homotopy analysis method (HAM) [1], the Adomian decomposition method (AMD) [2], Taylor-series expansion method [3], Modified homotopy perturbation method (MHPM) [4], delta functions (DFs) [5], hat basis functions [6].

In recent years, wavelets have found their way into many different fields of science and engineering. The uses of wavelets method for solving integral equations has considered by many authors: Legendre and Chebyshev wavelets method [7, 8], Wavelet Galerkin method [9].

In this study, we want to solve system (1) by using Legendre wavelets. The method consist of reducing the solution of (1) to a system of linear algebraic equations and also a collocation method is used for solving system (2).The method consist of expanding the solution by Legendre wavelets with unknown coefficients. For a suitable collocation points we choose zeros of Legendre wavelets. The properties of Legendre wavelets together with zeros of Legendre wavelets are then utilized to evaluate the unknown coefficients and to find an approximate solution to system (2).

II. Properties of Legendre wavelets

2.1 Wavelets and Legendre wavelets

Wavelets constitute a family of functions constructed from dilation and translation of a single function called the mother wavelet. When the dilation parameter a and the translation parameter b vary continuously we have the following family of continuous wavelets [10]

$$\psi_{a,b}(t)=|a|^{-\frac{1}{2}}\psi\left(\frac{t-b}{a}\right),\quad a,b\in\ \mathbb{R}\ \ ,a\neq 0. \tag{4}$$

If we restrict the parameters a and b to discrete values as $a={a_0}^{-k}$, $b=nb_0{a_0}^{-k}$, $a_0>1, b_0>0$ and $n\ and\ k\ \epsilon\ \mathbb{N}$ we have the following family of discrete wavelets:

$$\psi_{k,n}(t)=|a|^{\frac{k}{2}}\psi({a_0}^k t-nb_0). \tag{5}$$

Where $\psi_{k,n}(t)$ form a wavelet basis for $L^2(R)$. In particular, when $a_0 = 2$ and $b_0 = 1$ then $\psi_{k,n}(t)$ forms an orthonormal basis [10]. Legendre wavelets $\psi_{nm}(t) = \psi(k, \hat{n}, m, t)$have four arguments; ;$\hat{n} = 2n - 1, n = 0, 1, 2, \ldots, 2^{k-1}, k$ can assume any positive integer, m is the order for Legendre polynomials and t is the normalized time. They are defined on the interval [0, 1):

$$\psi_{nm}(t) = \begin{cases} \sqrt{2m+1}2^{\frac{k}{2}}P_m(2^k t - n), & for\ \frac{n}{2^k} \le t \le \frac{n+1}{2^k} \\ 0. & otherwise \end{cases}, \tag{6}$$

where $m = 0, 1, \ldots, M - 1\ and\ n = 0, 1, 2, \ldots, 2^{k-1}$. The coefficient $\sqrt{m + \frac{1}{2}}$ is for orthonormality,the dilation parameter is $a = 2^{-k}$ and translation parameter is $b = \hat{n}2^{-k}$.Here, L_m (t) are the well-known Legendre polynomials of order m, which are orthogonal with respect to the weight function w (t) = 1 and satisfy the following recursive formula:

$$L_0(t) = 1, \qquad L_1(t) = t,$$

$$L_{m+1}(t) = \left(\frac{2m+1}{m+1}\right) t\, L_m(t) - \left(\frac{m}{m+1}\right) L_{m-1}(t), m = 1,2,3, \ldots. \tag{7}$$

2.2 Function approximation

A function $f(t)$ defined over [0, 1) may be expanded as

$$f(x,t) = \sum_{n=1}^{\infty}\sum_{m=0}^{\infty} c_{n,m}\psi_{n,m}(x). \tag{8}$$

Where $c_{n,m} = (f(t), \psi_{n,m}(x))$, in which (.,.) denotes the inner product. If the infinite series in Eq.(8) is truncated, then Eq.(8) can be written as In the infinite series in eq.(4) is truncated , then eq.(4) can be written as :

$$f(t) \cong \sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1} c_{n,m}\psi_{n,m}(t) = \mathrm{C}^T(x)\Psi(t). \tag{9}$$

Where C and $\Psi(t)$ are $2^{k-1}M \times 1$ matrices given by

$$\mathrm{C} = \left[c_{10}, c_{11}, \ldots, c_{1M-1}, \ldots, c_{20}, c_{21}, \ldots, c_{2M-1}, \ldots, c_{2^{k-1}0}, c_{2^{k-1}1}, \ldots, c_{2^{k-1}M-1}\right]^T, \tag{10}$$

$$\Psi(t) = \left[\psi_{10}, \psi_{11}, \ldots, \psi_{1M-1}, \ldots, \psi_{20}, \psi_{21}, \ldots, \psi_{2M-1}, \ldots, \psi_{2^{k-1}0}, \psi_{2^{k-1}1}, \ldots, \psi_{2^{k-1}M-1}\right]^T. \tag{11}$$

Similarly, a function $k(x,t) \in L^2([0,1] \times [0,1])$ may be approximated as:

$$k(x,t) \cong \Psi^T(x)\, K\, \Psi(t), \tag{12}$$

where $K\ is\ 2^{k-1}\, M \times 2^{k-1}\, M$ matrix, with

$$K_{ij} = (\psi_i(x), k(x,t)\psi_j(x)), \tag{13}$$

The integration of the product of two Legendre wavelets vector function is obtained as:

$$I = \int_0^1 \Psi(t)\Psi^T(t)dt, \tag{14}$$

where I is an identity matrix.

III. System of linear integral equations

In this section, we use Legendre wavelets method for solving system of linear Fredholm and Volterra integral equations.

3.1 Legendre wavelets method for solving System of linear Fredholm integral equations:

Consider the system of linear Fredholm integral equations as follows:

$$U(x) - \int_0^1 K(x,t)U(x,t)dt = F(x), \quad x \in [0,1]. \tag{15}$$

Where

$$U(x) = [u_1(x), u_2(x), \ldots, u_n(x)]^T,$$

$$F(x) = [F_1(x), F_2(x), \ldots, F_n(x)]^T,$$

$$K(x,t) = \left[k_{ij}(x,t)\right], \ i,j = 1,2, \ldots, n. \tag{16}$$

In Eq. (15), the functions K and F are given, and U is the vector function of the solution of system (15) that will be determined. Consider the *ith* equation of (15)

$$u_i(x) = f_i(x) + \int_0^1 \sum_{j=1}^{n} k_{ij}(x,t)\, u_j(t)dt\, , i = 1,2, \ldots, n. \tag{17}$$

Where $f_i \epsilon L^2[0,1)$, $k_{ij} \epsilon L^2[0,1) \times L^2[0,1)$, and $u_i$is an unknown function.We approximate f_i, u_i *and* k_{ij} by (6)-(9) as follows:

$$f_i(x) \cong F_i^T \Psi(t),\quad u_i(x) \cong C_i^T \Psi(t)\,,\qquad k_{ij}(x,t) \cong \Psi^T(x) K_{ij} \Psi(t). \tag{18}$$

By substituting the above relation in (17), we have:

$$\Psi^T(x) C_i = \Psi^T(x) F_i + \int_0^1 \sum_{j=1}^{n} \Psi^T(x) K_{ij} \Psi(t) \Psi^T(t) C_j\, dt,$$

$$= \Psi^T(x) F_i + \Psi^T(x) \sum_{j=1}^{n} K_{ij} \left(\int_0^1 \Psi(t) \Psi^T(t)\, dt \right) C_j \ = \Psi^T(x) F_i + \Psi^T(x) \sum_{j=1}^{n} K_{ij}\, C_j. \tag{19}$$

Then we have the following linear system:

$$C_i = F_i + \sum_{j=1}^{n} K_{ij}\, C_j. \tag{20}$$

3.2 Legendre wavelets method for solving System of linear Volterra integral equations:

For solving the system of linear Volterra integral equations (2), we consider the ith equation of (17):

$$u_i(x) = f_i(x) + \int_0^x \sum_{j=1}^{n} k_{ij}(x,t)\, u_j(t) dt\,, i = 1,2,\dots,n, \tag{22}$$

where $f_i \epsilon L^2[0,1)$, $k_{ij} \epsilon L^2[0,1) \times L^2[0,1)$, and $u_i$is an unknown function.In order to use Legendre wavelets, we first approximate $u_i\ (x)$ as

$$u_i(x) = C_i^T \Psi(t)\,, \tag{23}$$

where C and $\Psi(t)$ are defined similarly to Eq.(10) and (11). In view of Eqs. (22) and (23) we have

$$C_i^T \Psi(x) = f_i(x) + \int_0^x \sum_{j=1}^{n} K_{ij}(\text{x}, \text{t}) \Psi(x)\, C_j^T\, dt, \qquad i = 1,2,\dots,n. \tag{24}$$

Now we collocate Eq.(24) at 2^{k-1}M points.

$$C_i^T \Psi(x_s) = f_i(x_s) + \int_0^{x_s} \sum_{j=1}^{n} K_{ij}(x_s, \text{t}) \Psi(x_s)\, C_j^T\, dt, \qquad i = 1,2,\dots,n. \tag{25}$$

Suitable collocation points are zeros of Chebyshev polynomial [11]

$$x_s = \cos\left(\frac{(2s+1)\pi}{2^k M},\quad s = 1,2,\dots,2^{k-1}M.\right.$$

However, in this method we choose zeros of Legendre wavelets for collocation points. For this purpose, we put k = 1 in Eq. (6), so the Legendre wavelet of order m is computed as follows

$$\psi_{1,m}(t) = \sqrt{m + \frac{1}{2}}\, 2^{\frac{1}{2}} L_m(2t-1), \tag{26}$$

where $\psi_{1,m}(t)$ are Legendre wavelet defined on the interval [0, 1) and $L_m(t)$ are the well-known Legendre polynomials. Eq. (25) gives ($n2^{k-1}$ M) linear equations which can be solved for the vector $C_i, i = 1, 2, \dots, n$ in Eq.(23).

IV. Illustrative examples

To demonstrate the effectiveness of the method, here we consider some system of linear Fredholm and Volterra integral equations. The Legendre wavelets are defined only for t [0, 1], we take a = 0, b = 1. The computations associated with the examples were performed using Mathematica and Maple.

Example 4.1[3]: Consider the following system of Fredholm integral equations

$$\begin{cases} u(x) - \int_0^1 (x-t)^3 u(t) dt - \int_0^1 (x-t)^2 v(t) dt = \frac{1}{20} - \frac{11}{30} x + \frac{5}{3} x^2 - \frac{1}{3} x^3, \\ v(x) - \int_0^1 (x-t)^4 u(t) dt - \int_0^1 (x-t)^3 v(t) dt = -\frac{1}{30} - \frac{41}{60} x + \frac{3}{20} x^2 + \frac{23}{12} x^3 - \frac{1}{3} x^4. \end{cases} \tag{27}$$

Taylor-series expansion method has been applied to solve the system (27) by K. Maleknejad, N. Aghazadeh and M. Rabbani en [3]. The better result accomplished with m=10 and the maximum absolute errors for u(x) is0.01656 and for v(x) is 0.1105 (see table 1 in [3]). We apply the presented method in this section and solve Eq. (27) with k = 1 and M = 5. Using Eq. (6), (7) and (13) we obtain

$$\psi_{1,0} = 1,\ \ \psi_{1,1} = \sqrt{3}(-1+2x),\ \ \psi_{1,2} = \frac{\sqrt{5}}{2}(-1+3(-1+2x)^2),$$

$\psi_{1,3}=\sqrt{7}(-1+12x-30x^2+20x^3),\quad \psi_{1,4}=3-60x+270x^2-420x^3+210,$ (28)

where

$\Psi=(\psi_{1,0}\quad \psi_{1,1}\quad \psi_{1,2}\quad \psi_{1,3}\quad \psi_{1,4})^T,\quad C_1=(c_{1,0}\quad c_{1,1}\quad c_{1,2}\quad c_{1,3}\quad c_{1,4})^T,$

$C_2=\left(c'_{1,0}\quad c'_{1,1}\quad c'_{1,2}\quad c'_{1,3}\quad c'_{1,4}\right)^T,\quad F_1=(f_0\quad f_1\quad f_2\quad f_3\quad f_4)^T,$

$F_2=(f'_0\quad f'_1\quad f'_2\quad f'_3\quad f'_4)^T.$ (29)

Using Eq.(20) and Eq.(27) we have

$C_1+k_{11}C_1+k_{12}C_2-F_1=0,\qquad C_2+k_{21}C_1+k_{22}C_2-F_2=0,$ (30)

where

$$K_{11}=\begin{pmatrix}0&-\frac{1}{5\sqrt{3}}&0&-\frac{1}{20\sqrt{7}}&0\\ \frac{1}{5\sqrt{3}}&0&\frac{1}{4\sqrt{15}}&0&0\\ 0&-\frac{1}{4\sqrt{15}}&0&0&0\\ \frac{1}{20\sqrt{7}}&0&0&0&0\\ 0&0&0&0&0\end{pmatrix};K_{12}=\begin{pmatrix}\frac{1}{6}&0&\frac{1}{6\sqrt{5}}&0&0\\ 0&-\frac{1}{6}&0&0&0\\ \frac{1}{6\sqrt{5}}&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\end{pmatrix};$$

$$K_{21}=\begin{pmatrix}\frac{1}{15}&0&\frac{\sqrt{5}}{42}&0&\frac{1}{210}\\ 0&-\frac{1}{10}&0&-\frac{1}{10\sqrt{21}}&0\\ \frac{\sqrt{5}}{42}&0&\frac{1}{30}&0&0\\ 0&-\frac{1}{10\sqrt{21}}&0&0&0\\ \frac{1}{210}&0&0&0&0\end{pmatrix};K_{22}=\begin{pmatrix}0&-\frac{1}{5\sqrt{3}}&0&-\frac{1}{20\sqrt{7}}&0\\ \frac{1}{5\sqrt{3}}&0&\frac{1}{4\sqrt{15}}&0&0\\ 0&-\frac{1}{4\sqrt{15}}&0&0&0\\ \frac{1}{20\sqrt{7}}&0&0&0&0\\ 0&0&0&0&0\end{pmatrix}.$$

$$F_1=\begin{pmatrix}\frac{61}{180}&\frac{1}{2\sqrt{3}}&\frac{7}{36\sqrt{5}_2}&-\frac{1}{60\sqrt{7}}&0\end{pmatrix}^T,F_2=\begin{pmatrix}\frac{7}{80}&\frac{37}{80\sqrt{3}}&\frac{229}{560\sqrt{5}}&\frac{1}{16\sqrt{7}}&-\frac{1}{630}\end{pmatrix}^T. \quad (31)$$

Substituting from (29)-(31) into (20) and solving the resulting system of algibric equations we have

$$C_1=\begin{pmatrix}\frac{1}{3}&\frac{1}{2\sqrt{3}}&\frac{1}{6\sqrt{5}}&0&0\end{pmatrix}^T,C_2=\begin{pmatrix}\frac{1}{12}&\frac{3\sqrt{3}}{20}&\frac{\sqrt{5}}{12}&\frac{1}{20\sqrt{7}}&0\end{pmatrix}^T. \quad (32)$$

From Eq. (9) we have

$$\begin{cases}u(x)=\left(\frac{1}{3}\right)\psi_{10}(x)+\left(\frac{1}{2\sqrt{3}}\right)\psi_{11}(x)+\left(\frac{1}{6\sqrt{5}}\right)\psi_{12}(x)+(0)\psi_{13}(x)+(0)\psi_{14}(x)=x^2,\\ v(x)=\left(\frac{1}{12}\right)\psi_{10}(x)+\left(\frac{3\sqrt{3}}{20}\right)\psi_{11}(x)+\left(\frac{\sqrt{5}}{12}\right)\psi_{12}(x)+\left(\frac{1}{20\sqrt{7}}\right)\psi_{13}(x)+(0)\psi_{14}(x)=-x+x^2+x^3.\end{cases} \quad (33)$$

Which is exact solution and provide that our method is more accurate than the methods in [3]

Example 4.2([4]): Consider the following system of Fredholm integral equations

$$\begin{cases}u(x)=-\sin(5x)-x\left(-\frac{23\cos(5)}{125}+\frac{2\mathrm{Sin}(5)}{25}-\frac{2}{125}\right)-x\left(-\frac{4e^{-3}}{9}+\frac{1}{9}\right)-\int_0^1[(x^2tu(t)+xtv(t)]dt,\\ v(x)=e^{-3}-x\left(\frac{2}{5}\cos(5)-\frac{1}{25}\sin(5)-\frac{1}{5}\right)-x^2\left(-\frac{4}{9}e^{-3}+\frac{1}{9}\right)+\int_0^1[x(t+1)u(t)-xt^2v(t)]dt.\end{cases} \quad (34)$$

With exact solution $u(x)=\sin(-5x)$, $v(x)=e^{-3x}$. M. Javidi en [4] solved the system (34) by modified homotopy perturbation method when N=5 the corresponding absolute error for u(x) and v(x) are 6.297×10^{-4} and 8.710×10^{-4} respectively, and when N=10 the corresponding absolute error for u(x) and v(x) are 8.776×10^{-6} and 8.776×10^{-5} respectively (see table1 in [4]). Now, we apply the technique described in example 4.1 with k = 1, M = 10 to obtain the approximate solution of (34) as

$$u(x)=A^T\Psi(x)=\sum_{i=0}^{10}a_i\psi_i(x)=-0.00000111232664946545-4.999856176152356x$$

$$-0.004553951674927781x^2 + 20.894969567459103x^3 - 0.44178699167780167x^4$$
$$-24.184300473987907x^5 - 4.7870258289191625x^6 + 23.0325403822005x^7$$
$$-6.643571146357272x^8 - 3.3123978445104973x^9 + 1.40490916190625x^{10},$$

$$v(x) = A^T\Psi(x) = \sum_{i=0}^{10} b_i\psi_i(x) = 0.999999998347311 - 2.9999997820320528x$$
$$+4.4999928797388495x^2 - 4.499899204789171x^3 + 3.374230463705209x^4$$
$$-2.021460931847776x^5 + 1.0020851183688309x^6 - 0.4136992789471499x^7$$
$$+0.1365293313484685x^8 - 0.03179825055419336x^9 + 0.003806726432140963x^{10}.$$

Fig.1 shows the solution of $u(x)$, $v(x)$obtained by the present method and the exact solution and the error computed in table 1, which is more accurate, again than the solution obtained in [4].

Fig.1 exact and Legendre wavelet solution of u(x) and v(x),$x \in [0,1]$.

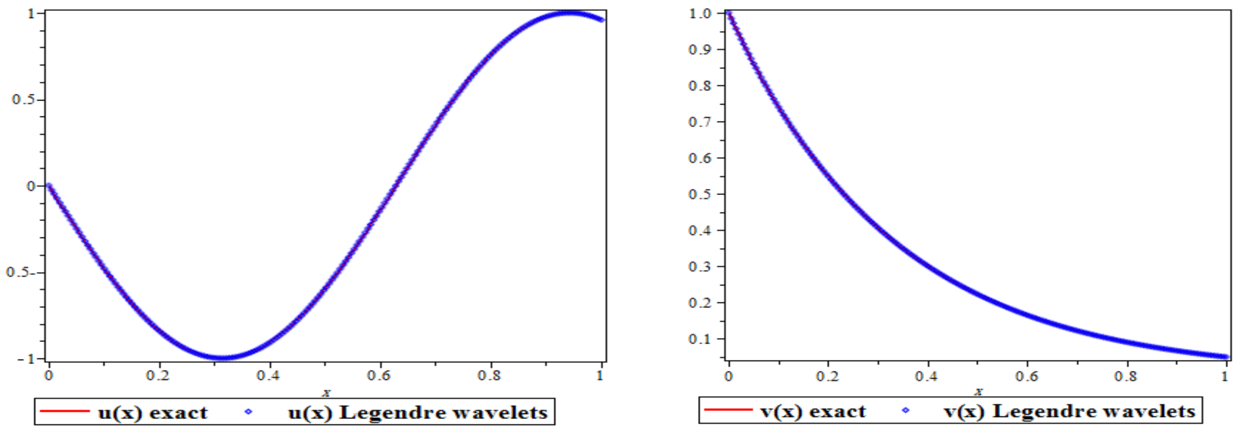


Table 1. The error of u(x) and v(x) of example 4.2

x	Error u	Error v
0	1.11233×10^{-6}	1.65269×10^{-9}
0.2	2.87621×10^{-7}	3.62850×10^{-10}
0.4	1.96009×10^{-7}	2.55138×10^{-10}
0.6	2.33588×10^{-7}	2.28141×10^{-10}
0.8	2.97986×10^{-7}	3.37538×10^{-10}
1.0	1.31130×10^{-6}	1.40260×10^{-9}

Example 4.3 ([12]): Consider the following system of Fredholm integral equations

$$\begin{cases} u(x) + \int_0^1 t\cos(x)\,u(t)dt + \int_0^1 x\sin(t)v(t)dt = x + \dfrac{\cos x}{3} + \dfrac{x\ \sin 1^2}{2}, \\ v(x) + \int_0^1 e^{xt^2}\,u(t)dt + \int_0^1 (x+\ t)v(t)dt = \cos x + \dfrac{e^x - 1}{2x} + (x+1)\sin 1 + cos1 - 1. \end{cases} \quad (30)$$

With exact solution $u(x) = x$ *and* $v(x) = cos(x)$. K. Maleknejad and F. Mirzaee [12] solved (2.72) with k= 16 and 32. The best results of (2.72) obtained at k=32 and the maximum absolute error evaluated in [12] of u(x), v(x) are 0.01566 and 0.01321.We apply the technique described in example 4.1and solved Eq.(30).In addition, Fig.2 show the solution of$u(x)$, $v(x)$obtained by the present method with k = 1, M = 5 and the exact solution and the error computed in table 2.Through the results, obtained using our method (see table 2) prove it more accurate than the results that have been obtained in [12].

Table 2. The error of u(x) and v(x) of example 4.3

x	Error u	Error v
0	1.10145×10^{-11}	$.43682\times10^{-5}$
0.2	1.64643×10^{-12}	2.63044×10^{-6}
0.4	8.10324×10^{-12}	4.94216×10^{-6}
0.6	1.82401×10^{-11}	4.7315×10^{-6}
0.8	2.87543×10^{-11}	2.18088×10^{-6}
1.0	3.96211×10^{-11}	1.69812×10^{-5}

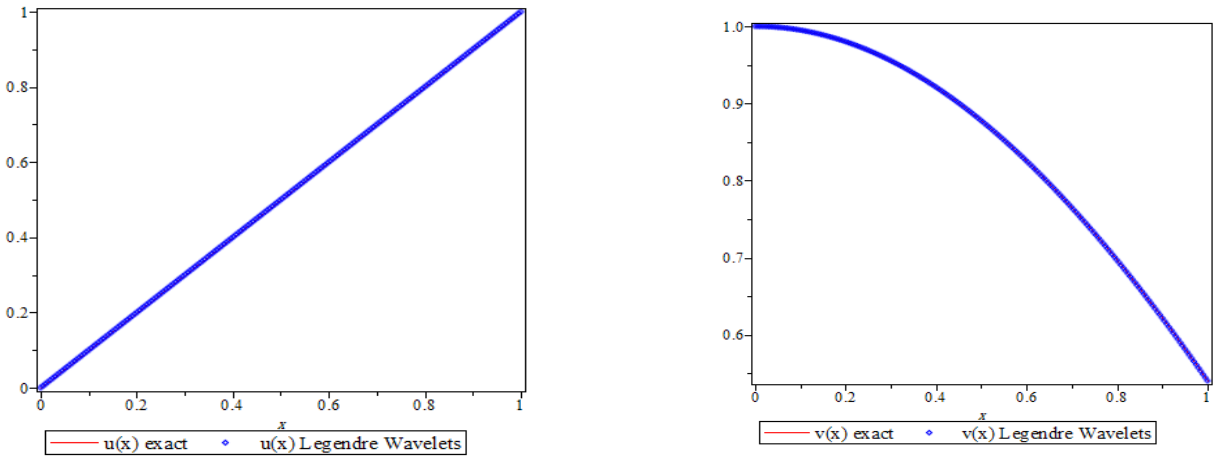


Fig.2 exact and Legendre wavelet solution of u(x) and v(x),$x \in [0,1]$.

Example 4.4 ([15, 16]): Consider the following system of Volterra integral equations

$$\begin{cases} u(x) + \int_0^x e^{x-t}u(t)dt + \int_0^x \cos(t-x)\, v(t)dt = \cosh(x) + x\sin(x), \\ v(x) + \int_0^x e^{x+t}u(t)dt + \int_0^x x\cos(t)v(t)dt = 2\sin(x) + x(\sin(x)^2 + e^x). \end{cases} \quad (31)$$

With exact solution $u(x) = e^{-x}$, $v(x) = 2\sin x$. The system (31) solved by M.I Berenguer and D. Gámez [15] by Biorthogonal systems and also the solution of (31) founded by J. Biazar and H. Ghazvini with homotopy perturbation method [16]. The results obtained by M.I Berenguer and D. Gámez [15] with j= 9, 17, 33 and the best results is calculating when j=33 which the absolute maximum error of u(x) and v(x) are 2.81E-2 and 6.71E-2 respectively (see table 3 in [15]). In addition, the maximum error obtained by J. Biazar and H. Ghazvini [16] with 5 terms for u (x), v (x) are 1.0E-3 and 1.4E-3 respectively (see table 1 in [16]). We apply the Legendre wavelets approach (22) - (25) and solved Eq.(31). Fig.3 show the solution of$u(x)$, $v(x)$obtained by the present method with k = 1, M = 5 and the exact solution and the error computed in table 3.

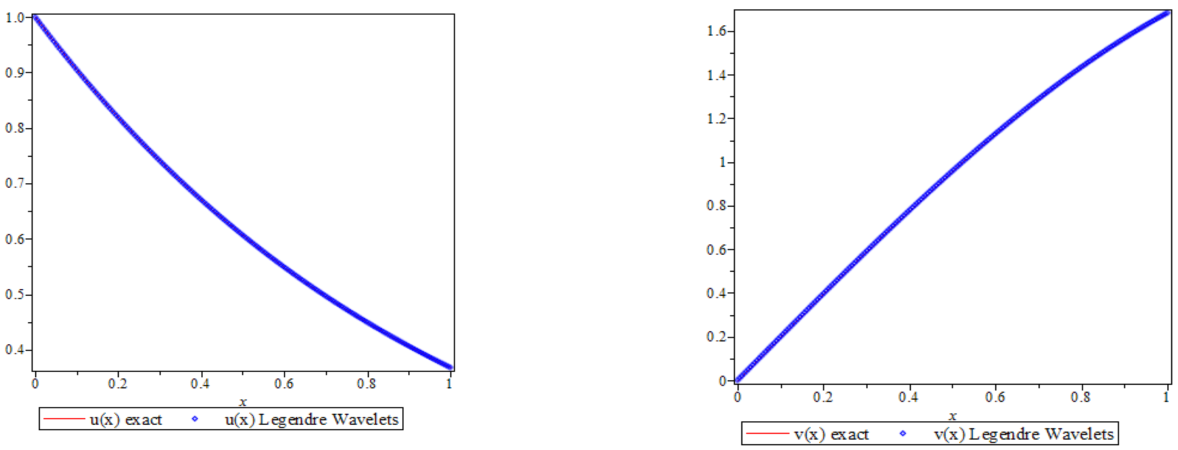


Fig.3 exact and Legendre wavelet solution of u(x) and v(x), $x \in [0,1]$.

Table 3. The error of u(x) and v(x) of example 4.4.

x	u exact – u Lw	v exact – v Lw
0.2	1.2906×10^{-5}	1.2448×10^{-5}
0.4	8.8772×10^{-6}	1.4751×10^{-5}
0.6	2.6478×10^{-7}	4.0975×10^{-6}
0.8	4.0576×10^{-6}	1.0665×10^{-5}
1	3.7849×10^{-5}	8.2414×10^{-5}

Example 4.5 ([6]): Consider the following system of Volterra integral equations

$$\begin{cases} u(x) - \int_0^x \sin(x-t)\ u(x)dt + \int_0^x \cos(x+t)\, v(x)dt = -3\cos x + 3\,, \\ v(x) + \int_0^x u(x)dt + \int_0^x e^{x-t} v(x)dt = e^x - \frac{1}{3}x^3 - 1. \end{cases} \quad (32)$$

With exact solution $u(x) = x^2$ and $v(x) = x$. E. Babolian M. Mordad [6] used hat basis functions for the approximate solution of Eq. (2.73) with n=64 and they obtain the best results E. Babolian M. Mordad [6] by doing large steps to obtain this solution. The maximum absolute error obtained is 0.00539 for u(x) and 0.00717 for v(x) (see table 5 in [6]).

We apply the Legendre wavelets approach (22) – (25) and solved Eq.(32), and Fig.4 show the solution of $u(x)$, $v(x)$ obtained by the present method with k = 1, M = 5 and the exact solution and the error computed in table 4 .

Table 4. The error of u(x) and v(x) of example 4.5

x	u exact – u Lw	v exact – v Lw
0	3.10862×10^{-15}	1.93352×10^{-15}
0.2	1.31839×10^{-16}	4.99600×10^{-16}
0.4	1.11022×10^{-16}	3.33067×10^{-16}
0.6	1.11022×10^{-16}	2.22045×10^{-16}
0.8	3.33067×10^{-16}	0.
1	3.33067×10^{-16}	2.22045×10^{-15}

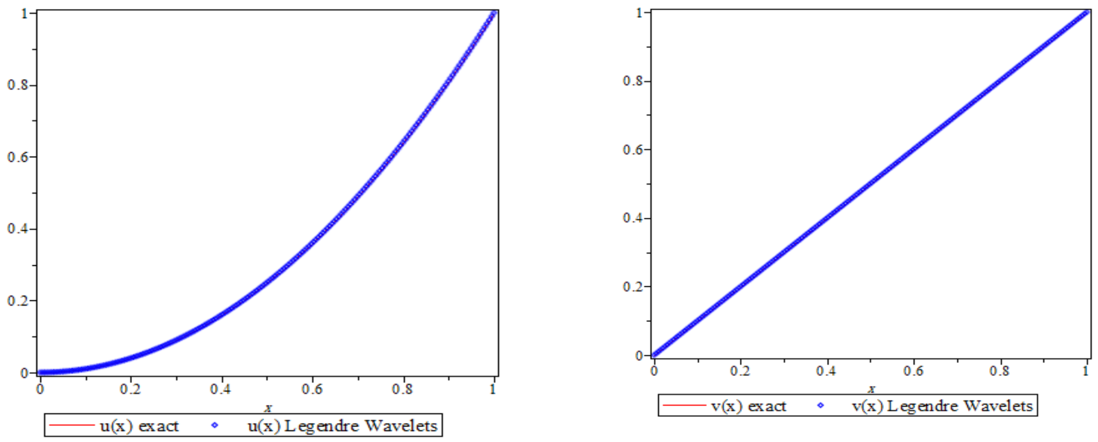


Fig.4 exact and Legendre wavelet solution of u(x) and v(x), $x \in [0,1]$

Example 4.6([15, 17, 18]): Consider the following system of Volterra integral equations

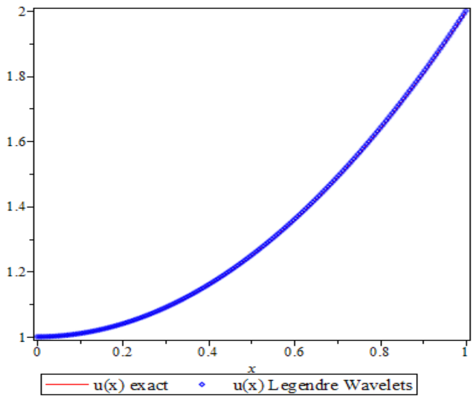
$$\begin{cases} u(x) - \int_0^x (x-t)^3 u(x)dt - \int_0^x (x-t)^2 v(x)dt = -\frac{1}{3}(x^3+x^4)+x^2+1, \\ v(x) - \int_0^x (x-t)^4 u(x)dt - \int_0^x (x-t)^3 v(x)dt = -\frac{1}{420}x^7 - \frac{1}{4}(x^4+x^5) - x^3 + x + 1. \end{cases} \quad (33)$$

With exact solution $u(x) = x^2 + 1$, $v(x) = x - x^3 + 1$.

The system (33) solved by three method, first by block-by-block method by R. Kataniand and S. Shahmorad [18], second, an expansion method by M. Rabbania and K. Maleknejad [17], third, Biorthogonal systems M.I. Berenguer and D. Gámez [15]. The best results is obtained with h=0.05, m=1 and j=33 by these three methods respectively. The maximum absolute errors are 1.469E-6, 3.84E-2, 3.48E-2 for u(x) and 1.128E-6, 3.32E-2, 5.01E-4 for v(x) by using method 1(see table 5 in [18]), method 2 (see table 1in [17]) and method 3 (see table 1in [15]) respectively. We apply the Legendre wavelets approach (22) – (25) and solved Eq.(32). Fig.5 show the solution of$u(x)$, $v(x)$obtained by the present method with k = 1, M = 5 and the exact solution and the error computed in table 5. Our method more accurate (see table 5) then these methods and get this best results with a few steps.

Table 5. The error of u(x) and v(x) of example 4.6

x	**u exact – u Lw**	**v exact – v Lw**
0	2.88658×10^{-15}	8.21565×10^{-15}
0.2	2.22045×10^{-16}	6.66134×10^{-16}
0.4	2.22045×10^{-16}	0.
0.6	6.66134×10^{-16}	4.44089×10^{-16}
0.8	0.	2.22045×10^{-16}
1	8.88178×10^{-16}	3.55271×10^{-15}



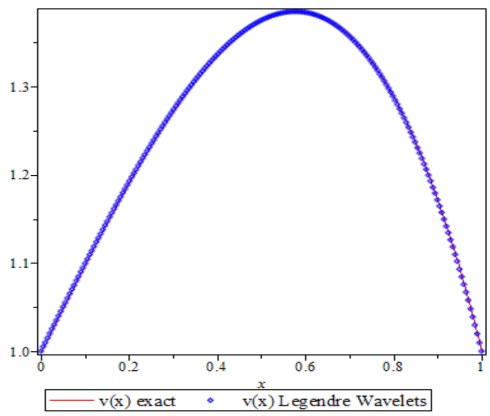


Fig.5 exact and Legendre wavelet solution of u(x) and v(x),$x \in [0,1]$

V. Conclusions

In this paper, Legendre wavelets method was aplied for solving system of linear integral equations. The present method reduced the system of integral equations to a system of linear algebraic equations. We have provided some illustrative examples to illustrate the feasibility of and validity of this method.The results showed that the method is very accurate and simple.

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