

## **Modeling and Optimization of Integrated Production Inventory Distribution Network for a Tyre Retread Industry**

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**Abstract-** Industries try to be on the winning side by maximising their profits or minimizing their costs. An integrated production inventory distribution model is formulated as a mixed integer programming problem with an aim to minimize the cost of an industry. The industry selected is tyre retread manufacturing company General Rubbers. This operations research problem is coded using computer software Lingo/Lindo and solved. Data of three months from the industry is given as input. Branch and bound method is used by Lingo to solve the model. The optimum solution is obtained as objective value, which is the minimum cost. The solution obtained when interpreted gives the required actions to be taken to attain that minimum cost by considering production, inventory and distribution factors of the selected industry. A large amount of money would be saved when the output recommendations are practised in terms of cost of production, cost of inventory and cost of distribution. The output is distribution schedules. This work has taken into account integrated production, inventory and distribution factors of the industry which forms a complex model which is solved and optimised so as to reduce total cost.

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### **I. INTRODUCTION**

Companies are looking for new ways to design, manufacture and distribute products in an efficient manner in today's competitive environment. After years of focussing on reduction in production and operating costs companies are looking towards reduction in distribution costs. In this work an integrated model of production, inventory and distribution for reduction in cost is proposed. The goal is to coordinate interrelated decisions related to production schedules, inventory policy, truckload allocation etc. The model formulated is an Integer linear programming model. The integrated integer programming model is solved by using branch and bound algorithm method with the help of the operations research software Lingo. The algorithm is applied on real distribution scenario of Midas Mileage (a tyre retread manufacturing company). The distribution network consisted of 3 plants, 3 distribution centres, 3 customer zones, 3 products, 3 inbound and outbound shipment carriers over a period of 3 months.

### **II. LITERATURE REVIEW**

Erden Eskigun [1] designed outbound supply chain network considering lead times, location of distribution facilities and choice of transportation mode. A Lagrangian heuristic is provided which gives solution to this model. Scenario analyses are conducted on industrial data using this algorithm.

Young Hae Lee [2] developed analytic models to solve integrated production distribution models in supply chain management. Operation time in the analytic model is considered as a dynamic factor and adjusted by the results from independently developed simulation models which include general production distribution characteristics.

M.A.Hoque [3] presented vendor buyer integrated production inventory model with normal distribution of lead time in a two stage supply chain. The vendor produces a product at a finite rate and delivers the lot to the buyer with a number of equal sized batches. An optimal solution technique is derived to the model to obtain minimum joint total cost that follows development of solution algorithm. Extensive comparative studies on the results of some numerical problems are carried out to highlight the potential significance of the present method. Sensitivity analysis to the solutions with variations of some parameter values are also carried out.

A. Bellabdaoui [4] presented a mixed integer programming model for producing steel making continuous casting production. The mixed integer programming formulation is solved using standard software packages.

Annilie. I. Pettersson[5] analyzes supply chain cost and measurements of supply chain cost in industry. She prescribes a model for measuring supply chain cost. The study shows that general thorough cost and supply chain analyses in many companies can be improved and further developed.

Henrick Andersson and Arild Hoff[6] describes industrial aspects of combined inventory management and routing in maritime and road based transportation and gives a classification and comprehensive literature review of advanced state of research.

Kejia Chen and Ping Ji [7] present a MIP model which gives system integration of the production planning and shopfloor scheduling problems. The proposed APS model explicitly considers capacity constraints, operation sequences, lead times and due dates in a multi order environment. The objective of the model is to seek the minimum cost of both production idle time and tardiness. The output of the model is operation schedules with order starting time and finish time.

### III. PROBLEM DESCRIPTION

The problem is that of configuring an integrated production inventory distribution system where a set of plants produces multiple items. The distribution centres act as intermediate centres between plants and customers and facilitate distribution of products between the two echelons. A mathematical model is developed to assist decision making in the integrated production inventory distribution system. The problem formulated attempts to minimize total costs by considering various factors like production schedule, level of inventory, distribution batch size etc. The industry chosen is rubber retread manufacturer Midas Mileage. There are three plants and three distribution centres. They also have 3 major customer zones. Mainly three products are produced. Precured tread, Camel back and Orbi tread (OTR) and Bonding gum. Inbound and outbound shipment carriers are used (Trucks of capacity varying from 10 to 30 tonnes). Input data is given for a period of three months. The problem under study involves the minimization of total cost

### IV. MATHEMATICAL MODELING

#### 4.1 Notations

$i$  represents plants,  $i = 1,2,3$ .

$j$  represents DC's,  $j = 1,2,3$ .

$q$  represents customer zones,  $q = 1,2,3$ .

$l$  represents products,  $l = 1,2,3$ .

$m$  represents inbound shipment carriers,  $m = 1,2,3$ .

$n$  represents outbound shipment carriers,  $n = 1,2,3$ .

$t$  represents time period,  $t = 1,2,3$ (months)

#### 4.2 Input parameters

$A_{ilt}$  = Fixed production cost for a product  $l$  at plant  $i$  in period  $t$ .

$B_{ilt}$  = Variable cost for producing a unit of product  $l$  at plant  $i$  in period  $t$ .

$C_{ilt}$  = Inventory cost for a unit of product  $l$  at plant  $i$  in period  $t$ .

$D_{qit}$  = Demand for product  $l$  by customer zone  $q$  in period  $t$ .

$E_{jlt}$  = Inventory cost a unit of product  $l$  in DC  $j$  in period  $t$ .

$F_{ijlmt}$  = Transportation cost for shipping a unit of product  $l$  from plant  $i$  to DC  $j$  when using carrier  $m$  in period  $t$ .

$G_{jqint}$  = Transportation cost for shipping a unit of product  $l$  from DC  $j$  to customer  $q$  when using carrier  $n$  in period  $t$ .

$H_{ilt}$  = Production capacity for product  $l$  at plant  $i$  in period  $t$ .

$I_{ilt}$  = Inventory capacity for product  $l$  at plant  $i$  in period  $t$ .

$J_{jlt}$  = Inventory capacity for product  $l$  in DC  $j$  in period  $t$ .

$K_{jt}$  = Upper bound on throughput capacity in DC  $j$  in period  $t$ .

$L_{jt}$  = Lower bound on throughput capacity in DC  $j$  in period  $t$ .

$M_{mt}$  = Truckload capacity of inbound shipments carrier  $m$  in period  $t$ .

$N_{nt}$  = Truckload capacity of outbound shipments carrier  $n$  in period  $t$ .

$O_{mt}$  = Driver capacity of inbound shipments carrier  $m$  in period  $t$ .

$Q_{nt}$  = Driver capacity of outbound shipments carrier  $n$  in period  $t$ .

$R_{lmt}$  = Average truckload for a standard vehicle shipping product  $l$  for inbound shipments carrier  $m$  in period  $t$ .

$S_{lnt}$  = Average truckload for a standard vehicle shipping product  $l$  for outbound shipments carrier  $n$  in period  $t$ .

$T_{lmt}$  = Average trips a driver of inbound shipments carrier  $m$  can make for product  $l$  in period  $t$ .

$U_{lnt}$  = Average trips a driver of outbound shipments carrier  $n$  can make for product  $l$  in period  $t$ .

$V_{ilo}$  = Starting inventory level for product  $l$  at plant  $i$ .

$W_{jlo}$  = Starting inventory level for product  $l$  at DC  $j$ .

$\beta_{qit}$  = Shipping requirement(the degree of consolidation or break bulk) of customer  $q$  for product  $l$  in period  $t$ .

**4.3 Output parameters**

$X_{ijlmt}$  = Amount of product l shipped from plant i to DC j when using inbound shipments carrier m in period t.  
 $Y_{jqnt}$  = Amount of product l shipped from DC j to customer q when using outbound shipments carrier n in period t.  
 $Z_{ilt}$  = 1 if product l is produced at plant i in period t; 0 otherwise  
 $P_{ilt}$  = Amount of product l produced at plant i in period t.  
 $V_{ilt}$  = Inventory level of product l at plant i in period t.  
 $W_{jlt}$  = Inventory level of product l in DC j in period t.

**4.4 Objective Function**

Objective function is to minimize the total cost.

Total Cost (Z) = Fixed Cost(Z1) + Variable Cost(Z2) + Inventory Cost at plant(Z3) + Inventory Cost at DC(Z4) + Shipping Cost from plant to DC(Z5) + Shipping Cost from DC to Customers(Z6)

Objective function

Min Z = Z1 + Z2 + Z3 + Z4 + Z5 + Z6

Z1 = Fixed cost = Fixed Production Cost( $A_{ilt}$ ) x  $Z_{ilt}$  ( 1 if plant is working; otherwise 0)

Z2 = Variable cost = Variable cost( $B_{ilt}$ ) x Amount of product produced( $P_{ilt}$ )

Z3 = Inventory cost at plant = Inventory cost for plant ( $C_{ilt}$ ) x Inventory level of product at plant ( $V_{ilt}$ )

Z4 = Inventory cost at DC = Inventory cost for DC( $E_{jlt}$ ) x Inventory level of product at DC ( $W_{jlt}$ )

Z5 = Shipping cost from plant to DC = Transportation cost from plant to DC ( $F_{ijlmt}$ ) x Amount of product shipped from plant to DC ( $x_{ijlmt}$ )

Z6 = Shipping cost from DC to customers = Transportation cost from DC to customers ( $G_{jqnt}$ ) x Amount of product shipped from DC to customers ( $y_{jqnt}$ )

**4.5 CONSTRAINTS**

1.  $\sum_{j=1}^3 \sum_{n=1}^3 y_{jqnt} = D_{qt}$ , for all q,l,t
2.  $P_{ilt} \leq H_{ilt} \times Z_{ilt}$ , for all i,l,t
3.  $V_{ilt} \leq I_{ilt}$ , for all i,l,t
4.  $W_{jlt} \leq J_{jlt}$ , for all j,l,t
5.  $L_{jt} \leq \sum_{q=1}^3 \sum_{l=1}^3 \sum_{n=1}^3 y_{jqnt} \leq K_{jt}$ , for all j,t
6.  $P_{ilt} + V_{ilt-1} - V_{ilt} = \sum_{j=1}^3 \sum_{m=1}^3 x_{ijlmt}$ , for all i,l,t
7.  $\sum_{i=1}^3 \sum_{m=1}^3 x_{ijlmt} + W_{jlt-1} - \sum_{q=1}^3 \sum_{n=1}^3 y_{jqnt} \times \beta_{qnt} = W_{jlt}$ , for all j,l,t
8.  $\sum_{i=1}^3 \sum_{j=1}^3 \sum_{l=1}^3 x_{ijlmt} \leq M_{mt}$ , for all m,t
9.  $\sum_{j=1}^3 \sum_{q=1}^3 \sum_{l=1}^3 y_{jqnt} \leq N_{nt}$ , for all n,t
10.  $\frac{\sum_{i=1}^3 \sum_{j=1}^3 \sum_{l=1}^3 x_{ijlmt}}{\sum_{l=1}^3 R_{lmt}} \leq \sum_{l=1}^3 T_{lmt} \times O_{mt}$ , for all m,t

11. 
$$\frac{\sum_{j=1}^3 \sum_{q=1}^3 \sum_{l=1}^3 y_{jq\ln t}}{\sum_{l=1}^3 S_{\ln t}} \leq \sum_{l=1}^1 U_{\ln t} \times Q_{nt}, \text{ for all } n,t$$
12.  $x_{ij\ln t} \geq 0, \text{ for all } i,j,l,m,t$
13.  $y_{jq\ln t} \geq 0, \text{ for all } j,q,l,n,t$
14.  $P_{ilt} \geq 0, \text{ for all } i,l,t$
15.  $V_{ilt} \geq 0, \text{ for all } i,l,t$
16.  $W_{jt} \geq 0, \text{ for all } j,l,t$
17.  $Z_{ilt}$  is binary (either 0 or 1)

## V. LINGO CODING

We would be formulating the above scenario as a linear integer programming problem and analyzing its optimum output (minimum cost of production, inventory and distribution) through six output variables and twenty three input variables. The 23 input variables and 6 output variables were listed earlier. The problem would be solved by coding in LINGO (Optimization software) and input data ( for these 23 variables) over three months would be given. The output obtained (values of the 6 output variables for three months, obtained as solution report in LINGO) would be analyzed and interpreted to give recommendations regarding production, inventory and distribution of the earlier stated scenario of the firm General Rubbers to get the optimum cost(minimum cost) for Production, Inventory and Distribution costs combined as given by the Solver status window of the software LINGO. The 4 output variables that would be discussed are

1. If a product would be produced at a plant during a particular month
2. Amount of a particular product produced at a plant during a particular month
3. Inventory level of a particular product at a plant during a month
4. Inventory level of a particular product at a DC during a month

Industrial data for 3 months corresponding to the variables in the model has been given as input. Output deals with schedules of distribution and quantity of product need to be shipped from plant to DC and from DC to customer zones. Also optimum inventory levels are obtained for the industry.

## VI. RESULTS AND DISCUSSION

Data given was for 3 months for 3 products involving 3 plants, 3 distribution centres and 3 customer zones. The three products are Precured tread rubber, Tread rubber and Bonding gum. The three plants are situated at Vazhoor, Ettumanoor and Pondicherry. The three DC's are situated at Manimala, Karaikal, and Kottayam. The three customer zones are Kottayam, Tamilnadu and Ernakulam. The data given was that of a period from April 2011 to June 2011.

### 6.1 BEST OBJECTIVE

The result of the Integer linear program is minimum  $Z = 581.318$  lakhs  
 Ie, minimum value of total cost (production + Inventory + transportation) = 581.318 lakhs  
 Optimum Fixed production cost  $Z1 = 35$  lakhs  
 Optimum variable cost  $Z2 = 335$  lakhs  
 Optimum inventory cost at plant  $Z3 = 94$  lakhs  
 Optimum inventory cost at DC  $Z4 = 37.218$  lakhs  
 Optimum distribution cost from plant to DC  $Z5 = 22.8$  lakhs  
 Optimum distribution cost from DC to customer zone  $Z6 = 57.2$  lakhs

### 6.2 SOLVER SOLUTION REPORT EXPLANATION

Best objective value is 581.318 lakhs. It is the objective value of the best solution found so far. It is the theoretical bound on the objective. Bound is a limit on how far the solver will be able to improve the objective. This is an integer linear programming problem. Number of steps is equal to 2. It denotes the number of branches in the branch and bound tree. In the branch and bound algorithm for solving integer problem the

simplex method of solving the LPP is followed. Only difference is that the solution obtained is round off to the nearest integer.

**6.3 RECOMMENDATIONS TO BE FOLLOWED TO OBTAIN THE OPTIMUM COST**

Only plant 2 needs to be operated during this period

Only DC 3 needs to be operated during this period

Inventory level to be kept at plant

	month 1	month 2	month 3	
product 1	0	0	0	plant 1
product 2	0	15	0	
product 3	0	4	0	
product 1	0	12	0	plant 2
product 2	0	0	0	
product 3	0	14	0	
product 1	0	2	0	plant 3
product 2	0	0	0	
product 3	0	0	0	

Amount of product needed to be produced at plant

	month 1	month 2	month 3	
product 1	0	0	0	plant 1
product 2	0	69	0	
product 3	0	65	0	
product 1	0	51	0	plant 2
product 2	0	8	0	
product 3	0	14	0	
product 1	0	21	0	plant 3
product 2	0	0	0	
product 3	0	0	0	

Inventory level to be kept at DC

	month 1	month 2	month 3	
product 1	18	16	0	DC 1
product 2	18	16	0	
product 3	16	15	0	
product 1	18	16	0	DC 2
product 2	18	0	0	
product 3	16	15	0	
product 1	18	16	0	DC 3
product 2	18	16	0	
product 3	16	15	0	

**VII. CONCLUSION**

An integrated production inventory distribution network has been developed to redesign the same of the company General rubbers. The model which is an integer programming problem is coded in standard software package LINGO and input data from industry is given. The input and output parameters are specified in the model clearly. The output of the work is production inventory and distribution schedules and quantity of product to be shipped and stored.

Amount of product to be shipped from plants to DCs and from DCs to customers during the period of three months to obtain minimum cost of distribution is tabulated. Also which all plants and DCs need to be

operated during the period is also obtained to obtain least production cost. Similarly the amount of product need to be produced at a particular plant is obtained. Optimum Inventory levels of products at plants and DCs during the period is also obtained and tabulated.

The overall cost which is optimum cost of production, inventory and distribution according to this model is 581.318 lakhs. Optimum fixed production cost is 35 lakhs. Optimum variable cost is 335 lakhs. Optimum inventory cost at plant is 94 lakhs. Optimum inventory cost at DC is 37.218 lakhs. Optimum distribution cost from plant to DC is 22.8 lakhs. Optimum distribution cost from DC to customer zone is 57.2 lakhs.

How the supply chain model behaves under variation different parameters can be done as a future development. Similarly large scale problem solving abilities of the solver LINGO for this model can be tested. Also the model can be tested with data of different industries.

#### **REFERENCES**

- [1] Erdem Eskigun etal "Outbound Supply Chain Network Design with mode selection, lead times and capacitated vehicle distribution centres" *European Journal of Operational Research* 165(2005), 182-206.
- [2] Young Hae Lee, Sook Han Kim "Production distribution planning in supply chain considering capacity constraints" *Computers and Industrial Engineering* 43(2002), 169-190.
- [3] M. A. Hoque "A Vendor buyer integrated production-inventory model with normal distribution of lead time" *International Journal of Production Economics*, 144(2013), 409-417
- [4] A. Bellabdaoui, J. Teghem "A Mixed Integer linear programming model for the continuous casting planning" *International Journal of Production Economics*, 104(2006), 260-270.
- [5] Annelie I Petterson, Anders Segerstedt "Measuring Supply Chain Cost," *International Journal of Production Economics*, 143(2013), 357-363
- [6] Henrick Andersson, Arild Hoff etal "Industrial Aspects and Literature Survey: Combined Inventory Management and Routing" *Computers and Operations Research* 37(2010), 1515-1536
- [7] Kejia Chen, Ping Ji "A mixed integer programming model for advanced planning and scheduling" *European Journal of Operations Research*, 181(2007), 515-522