

## A Pd-Type Fuzzy Logic Control Approach For Vibration Control Of A Single-Link Flexible Manipulator

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**ABSTRACT:** This paper presents the design of PD-type fuzzy logic controller (PDFLC) for vibration control of a single-link flexible manipulator system. A flexible manipulator system is a SIMO system with motor torque as the applied input and the hub angle and tip deflection as its two outputs. The system is modelled using the finite element method. The PDFLC have two inputs, the hub angle error and its derivatives, the output of the controller is fed to the flexible manipulator model as the control signal which successfully suppressed the vibration and achieved a precise tip deflection at the tip end. The tracking performance and robustness due to payload variation were investigated via simulation in Simulink. The Simulation results show that the PDFLC provides a robust control to both internal and external disturbances.

**Keywords**— single link flexible manipulator, vibration control, PD-type fuzzy logic controller.

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### I. Introduction

An increasing demand in modern industrial robot manipulator systems shows that flexibility is an important parameter to satisfy some special industrial applications. A flexible manipulator system (FMS) is one of the modern industrial robot that offer a great advantages over the fixed heavy industrial robot, some of which are; i) high ratio of payload weight to robot weight ii) less power consumption iii) faster operation iv) cheaper compared to fixed heavy robots [2,4]. However the flexibility nature of these systems leads to greater difficulties in the control aspect as it generated a lot of vibration and high oscillation at the tip end.

Many control techniques have been reported in literature for the vibration and deflection control in these systems, literature survey shows that most of the control technique developed for the control of flexible structures has lots of accuracy and precision limitations. Two different approaches have been applied in literature for the control of the flexible robot arms these are; i) linear control approach and ii) nonlinear control approach. Linear controllers such as H-infinity [1], linear quadratic regulator (LQG)[2], conventional PID control[3] and integral resonant control (IRC)[4,5], have been applied in the control of FMS. Flexible manipulator is quite difficult to be accurately controlled by linear control approach due to their nonlinear dynamic structure of the system. Nonlinear control approach such as: Adaptive control technique [6], fuzzy logic control technique [7] and observer-based fuzzy- control [8] have also been applied in the control of FMS.

#### 1.1 Related Work

A fuzzy logic controller (FLC) is mutually exclusive to conventional dynamic model based controllers in which mathematical model is not require for the control of the plant, and is applicable to both linear and nonlinear system. Fuzzy control is a control way of applying expert knowledge to control a plant without having detail information of the plant [9]. A FLC has three main component namely i) Fuzzifier which convert the input signal into fuzzy signal ii) fuzzy interface engine which process the fuzzified signal using decision rules, and iii) Defuzzifier which convert the fuzzy controller output signal to a signal used as the control input signal to the system model.

In [9] three different fuzzy logic controllers (FLCs) are developed to control vibration and end point deflection. [10] Presented a Hybrid fuzzy logic control with genetic optimisation for vibration control of a single-link flexible manipulator. In [11] a PD-type fuzzy logic controller with non-collocated proportional integral derivative (PID) is developed for the control of vibration of flexible manipulator. [12] Presented an input shaping with PD-type fuzzy logic control for vibration and trajectory tracking of flexible arm robot. In [13] a controller is developed using fuzzy Lyapunov synthesis (FLT) to control vibration of a flexible manipulator, in [14] an experimental study using fuzzy logic and neural networks tools is presented for active vibration control of a single link flexible manipulator system. In [16] an adaptive network based interval type-2

fuzzy logic controller was developed for the control of a single flexible link carrying a pendulum. A Cascade fuzzy logic control is implemented for the vibration control of a single-link flexible-joint manipulator in [17].

### 1.2 Main work

In this work, a PD-type fuzzy logic controller is designed for a rigid body (hub joint) vibration control and tip deflection control for flexible motion of a single link flexible manipulator, with seven membership functions and 49 rule based.

The rest of this paper are organized as follows; section II presented the system model, section III presented fuzzy logic controller design, implementation and results are presented in section IV and finally section V give the conclusion.

## II. MODEL DESCRIPTION

### 2.0 Description of the system

This section explained the model description of the single-link flexible manipulator system considered in this work, is as derived by [15] in which the dynamic model is obtained by using finite element (FE) methods and the system description is as in figure 1. In this work, the motion of the manipulator is on **XOY** plane. The transverse shear and rotary inertia effects of the flexible link are neglected since, it is long and slender. Based on these assumptions a Bernoulli-Euler beam theory is used to model the elastic behaviour of the manipulator. It is also assumed that the manipulator to be stiff in vertical bending and torsion, in order to allow it to vibrate permanently in the horizontal direction and thus, the gravity effects are neglected. Moreover, the manipulator is considered to have constant cross section and uniform material properties throughout. In this study, an aluminium type flexible manipulator of dimensions  $900 \times 19.008 \times 3.2004$  mm [15]. The parameters of the system are given in table 1.

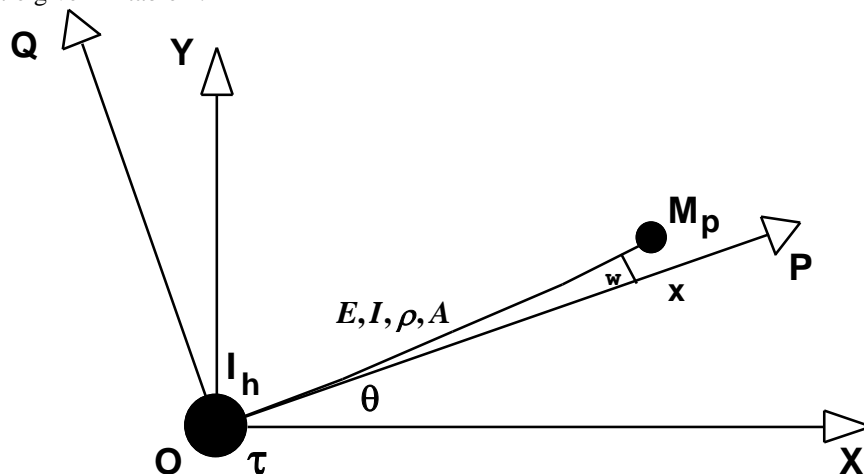


Fig.1 Description of the flexible manipulator system [15]

Table 1 system parameters

Parameters	Symbols	values	units
Young modulus	$E$	$71 \times 10^9$	$N/m^2$
Mass density per unit volume	$\rho$	2710	$Kg/m^3$
Second moment of inertia	$I$	5.1924	$m^4$
Flexible link length	$L$	0.9	$m$
Beam inertia	$Lb$	$0.04 \times 3$	$g/m^2$

### 2.1 Modelling of flexible manipulator system

The modelling of the flexible manipulator system is briefly describes in this section, the model is basically needed for simulation environment for development of control strategies for input tracking and vibration suppression of the system. In this work, the FE method with 10 elements is considered in characterising the dynamic behaviour of the manipulator incorporating structural damping, hub inertia and

payload [15]. A FE method is a process of breaking down a structure into number of small pieces or elements. This method is based on the assumption that the elements are interconnected at a joint, often called node. The equation describing the behaviour of the system which is obtained by approximation technique depends on the number of elements. These elemental equations are combined together to give the system equation [15].

The steps involves in FE method include (1) structural discretisation into number of elements; (2) Result interpolation by an approximating function selection; (3) formulation of the element equation; (4) calculating system equation from element equations; (5) boundary conditions selection and (6) solving system equation with the boundary conditions. In this way, the manipulator system is treats as an assembly of n elements and the algorithm can be developed in three main parts: i) FE analysis, ii) state-space representation and iii) obtaining and analysing the system transfer function [15].

For an angular displacement (hub angle)  $\theta(t)$  and an elastic deflection  $w(x,t)$ , the total displacement  $y(x,t)$  of a point along the manipulator at a distance  $x$  from the hub can be described as a function of both the rigid body motion  $\theta(t)$  and elastic deflection  $w(x,t)$  measured from the line  $OX$  as

$$y(x,t) = x\theta(t) + w(x,t) \quad (1)$$

Using the FE method and kinetic and potential energies of an element, the element mass matrix,  $M_n$  and stiffness matrix,  $K_n$  can be obtained as [15]

$$M_n = \frac{\rho Al}{420} \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} & m_{15} \\ m_{21} & 156 & 22l & 54 & -13l \\ m_{31} & 22l & 4l^2 & 13l & -3l^2 \\ m_{41} & 54 & 13l & 156 & -22l \\ m_{51} & -13l & -3l^2 & -22l & 4l^2 \end{bmatrix}$$

$$K_n = \frac{EI}{l^3} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 12 & 6l & -12 & 6l \\ 0 & 6l & 4l^2 & -6l & 2l^2 \\ 0 & -12 & -6l & 12 & -6l \\ 0 & 6l & 2l^2 & -6l & 4l^2 \end{bmatrix}$$

Where

$$\begin{aligned} m_{11} &= 140l^2(3n^2 - 3n + 1) \\ m_{12} &= m_{21} = 21l(10n - 7) \\ m_{13} &= m_{31} = 7l^2(5n - 3) \\ m_{14} &= m_{41} = 21l(10n - 3) \\ m_{15} &= m_{51} = -7l^2(5n - 3) \end{aligned}$$

In this paper, ten number of elements ( $n = 10$ ) were used for this control purpose.

The model can also be represented in state space form as described

$$\begin{aligned} \dot{v} &= Av + Bu \\ y &= Cv + Du \end{aligned} \quad (2)$$

$$\begin{aligned} \text{Where } A &= \begin{bmatrix} O_m & I_m \\ -M_n^{-1}K_n & -M_n^{-1}D \end{bmatrix} \quad B = \begin{bmatrix} O_{m \times 1} \\ M_n^{-1} \end{bmatrix} \\ C &= [O_m \quad I_m] \quad D = [O_{2m \times 1}] \end{aligned}$$

Where the subscript  $n$  indicate number of elements,  $O_m$  is  $m \times m$  null matrix,  $I_m$  is  $m \times m$  identity matrix,  $O_{m \times 1}$  is  $m \times 1$  null vector [15],

$$\begin{aligned} u &= [\tau \ 0 \ \dots \ \dots \ 0]^T, \quad v = [\theta \ w_1 \ \theta_1 \ \dots \ w_n \ \theta_n \ \dot{\theta} \ \dot{w}_1 \ \dot{\theta}_1 \ \dots \ \dot{w}_n \ \dot{\theta}_n]^T \\ M_n &= \begin{bmatrix} M_{\theta\theta} & M_{\theta w} \\ M_{\theta w} & M_{ww} \end{bmatrix} \end{aligned}$$

In which  $M_{ww}$  presents matrix relates to the elastic degrees of freedom (residual motion),  $M_{\theta w}$  represents the coupling between the hub angle  $\theta$  and elastic degrees of freedom and  $M_{\theta\theta}$  is the terms relates to the system inertia about the motor axis. Similarly, the global stiffness matrix is as follows

$$K_n = \begin{bmatrix} 0 & 0 \\ 0 & K_{ww} \end{bmatrix}$$

Where  $K_{ww}$  is relates to the elastic degrees of freedom (residual motion).As can be seen that the elastic degrees of freedom is not related to the hub angle via the stiffness matrix. The global damping matrix D is as follows [15]

$$D = \begin{bmatrix} 0 & 0 \\ 0 & D_{ww} \end{bmatrix}$$

Where  $D_{ww}$  represents the sub-matrix associated with the material damping of the system. It obtained as

$$D_{ww} = \alpha M_{ww} + \beta K_{ww} \quad (3)$$

Where

$$\alpha = \frac{2f_1 f_2 (\varepsilon_2 f_2 - \varepsilon_1 f_1)}{f_2^2 - f_1^2} ; \beta = \frac{2(\varepsilon_2 f_2 - \varepsilon_1 f_1)}{f_2^2 - f_1^2}$$

Where  $\varepsilon_1, \varepsilon_2, f_1$  and  $f_2$  representing the damping ratios and natural frequencies of modes 1 and 2 respectively [15]. The dynamic equations of the motion of the system is represented as

$$M\ddot{Q}(t) + D\dot{Q}(t) + KQ(t) = F(t) \quad (4)$$

Where  $F(t) = [\tau \ 0 \ \dots \ 0]^T$  represented the vector associated with the applied forces and torque,  $Q(t) = [\theta \ w_0 \ \theta_0 \ \dots \ w_n \ \theta_n]^T$  and D is the global damping matrix, usually obtained by experimentation [15].

### III. CONTROLLER DESIGN

#### 3.0 PD-Type Fuzzy Logic Controller (PDFLC) Design

In this section a detail design of the PD-type fuzzy logic controller (PDFLC) is presented. The fuzzy controller has two inputs and one output, the inputs are the hub angle error (e) and its derivatives ( $\dot{e}$ ) as shown in block diagram in figure 2, the output is the fuzzy control signal generated based on decisions as design using rule base. The controller is designed to control vibration of the flexible manipulator system (FMS) due to rigid motion and minimize tip deflection due to elastic (flexible) motion. To achieve these, the hub angle should track the applied torque and the tip deflection should be regulated close to zero value with zero steady state error.

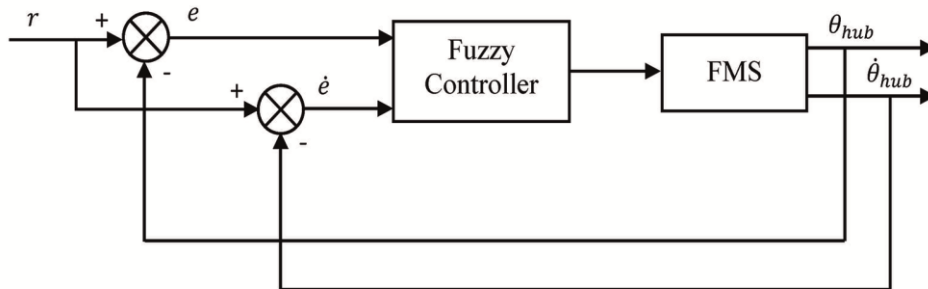


Fig. 2 PD-type fuzzy logic controller block diagram

A Fuzzy logic controller design process involves selection of type and number of membership function, selection of rule base, inference mechanism and defuzzification process. In this paper a triangular membership function is used. The rule base are developed using the symbols NB (negative Big), NM (Negative Medium), NS (Negative Small), ZE (Zero), PS (Positive Small), PM (Positive Medium) and PB (Positive Big).

Combination of these rules are used to generate control signal for example if hub angle error is negative big and derivative of the error is positive medium then the controller output should give negative medium and if the hub error is negative small and its derivative is positive small then the control output should give zero signal etc. Table 2 shows the appropriate fuzzy control signal for different hub angle error and its derivative. Different number of membership function are tested and it was found that, with seven membership functions are more appropriate for this system, which has 49 possible control signal (rule based).

Table 2 rule base table

$\dot{e}/e$	NB	NM	NS	ZE	PS	PM	PB
NB	NB	NB	NB	NB	NM	NS	ZE
NM	NB	NB	NB	NM	NS	ZE	PS
NS	NB	NB	NM	NS	ZE	PS	PM
ZE	NB	NM	NS	ZE	PS	PM	PB
PS	NM	NS	ZE	PS	PM	PB	PB
PM	NS	ZE	PS	PM	PB	PB	PB
PB	ZE	PS	PM	PB	PB	PB	PB

The membership functions of the hub angle error ( $e$ ) are implemented with seven membership function [NB,NM,NS,ZE,PS,PM,PB] and scale within the range of [-2 2]. Membership functions of the derivative of the error ( $\dot{e}$ ) and the membership functions of the output are both implemented with seven membership function [NB,NM,NS,ZE,PS,PM,PB] and scale within the range of [-10 10] each. Figure 3 shows the control surface obtained from the result of table 2 (rule base table) using Min-Max operator and centroid purification method.

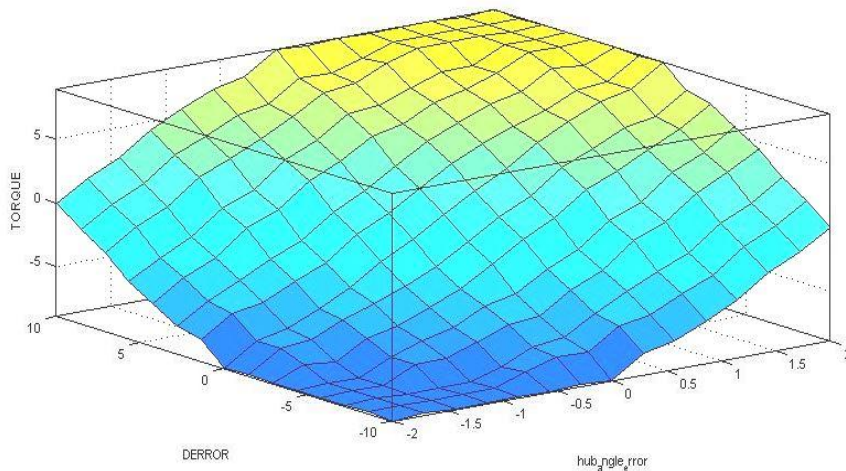


Fig. 3 control surface of the rule base

#### IV. IMPLEMENTATION AND RESULTS

In this section, the proposed control scheme is implemented and tested within the simulation environment of the flexible manipulator and the corresponding results are presented. The manipulator is required to follow a trajectory within the range of  $\pm 80$  degrees as shown in Figure 4.

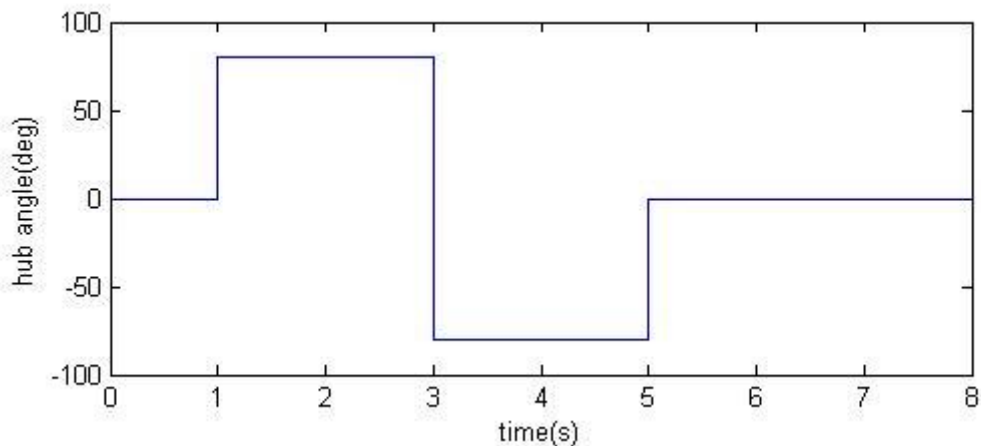


Fig.4 reference hub angle

A PD-type fuzzy logic controller have been designed and simulated using MATLAB fuzzy tool box as shown in figure 5, the purpose of the controller is to suppress the vibration and minimize tip deflection of a single link flexible manipulator system. Two types of control objectives are involves in this control, these are; 1) servo (tracking) control and 2) regulation control. Figure 6a and 6b shows the result of tracking control in which the hub angle tracked or follows the reference hub angle with zero steady state error and rise time of 0.36 second and zero overshoot. Similarly figure 7 shows the result of regulation control in which the tip deflection has been regulated close to zero deflection with maximum deflection of  $13.3 \times 10^{-3}m$ .

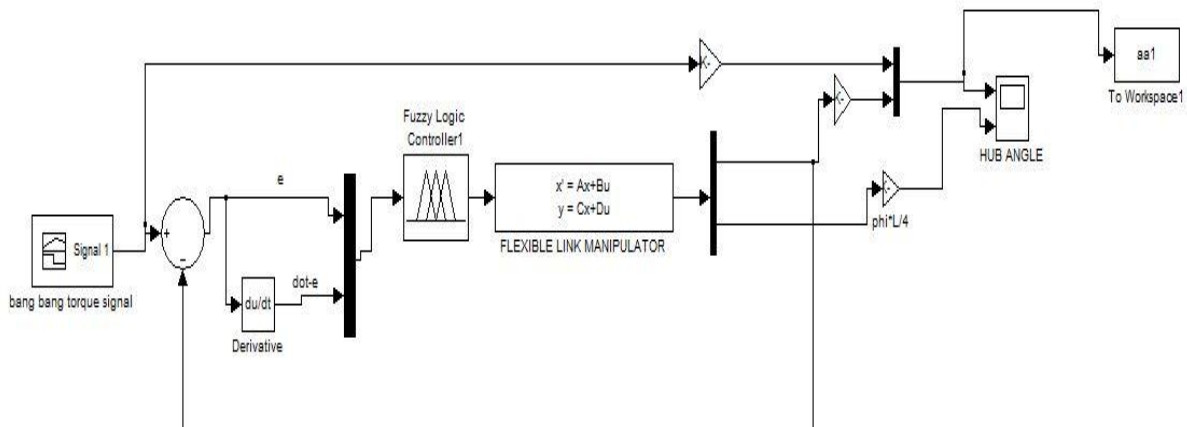


Fig. 5 MATLAB Simulink block of the fuzzy logic controller with the flexible link model

In addition, the controller has been tested to offered robustness to changes in an external influence such as change in payload. Figure 8 and Figure 9 shows the hub angle and tip deflection with the changes in payload range 20 g, 30 g and 50 g. Table 3 gives the summary of the controller results with different payload values. The hub angle error and its derivative are shown in figure 10 and 11 respectively, it's based on the error and its derivative the fuzzy logic controller was designed.

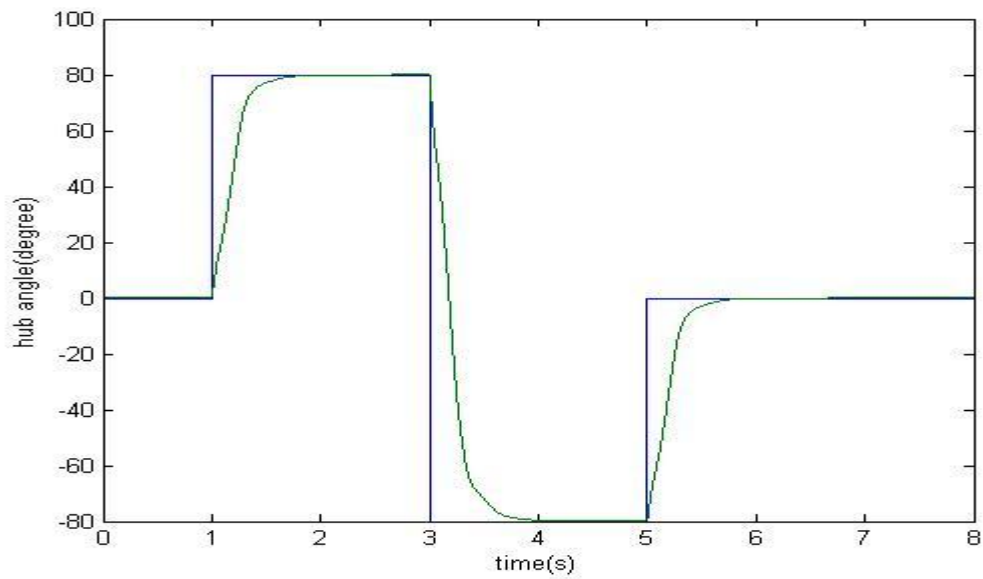


Fig. 6a hub angle tracking the reference hub angle with zero payload

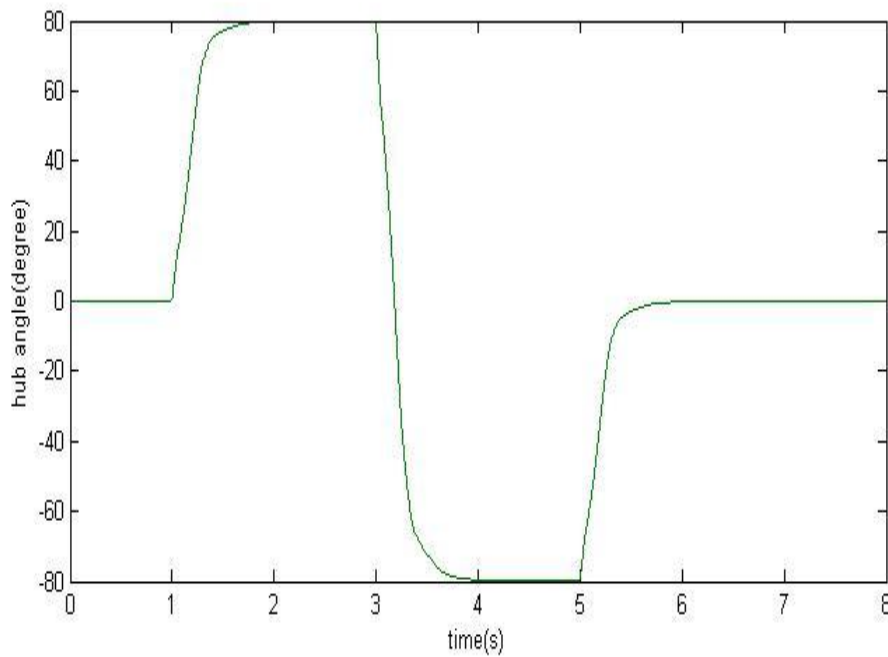


Fig. 6b hub angle with zero payload



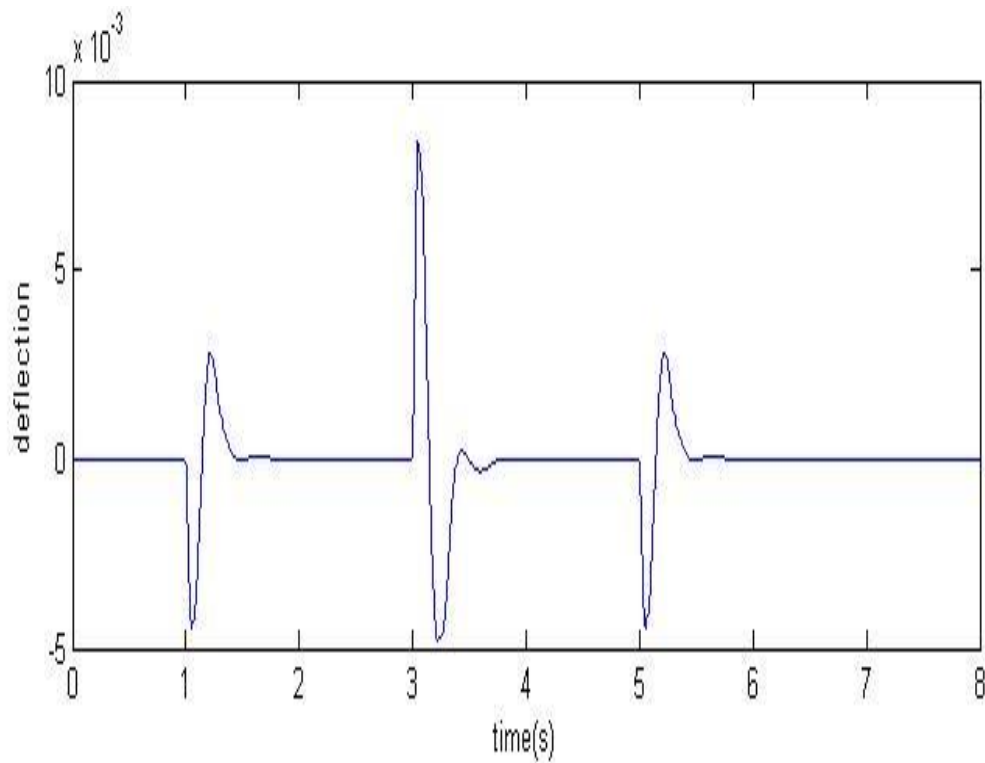


Fig. 7 regulated tip deflection at the tip end with zero payload

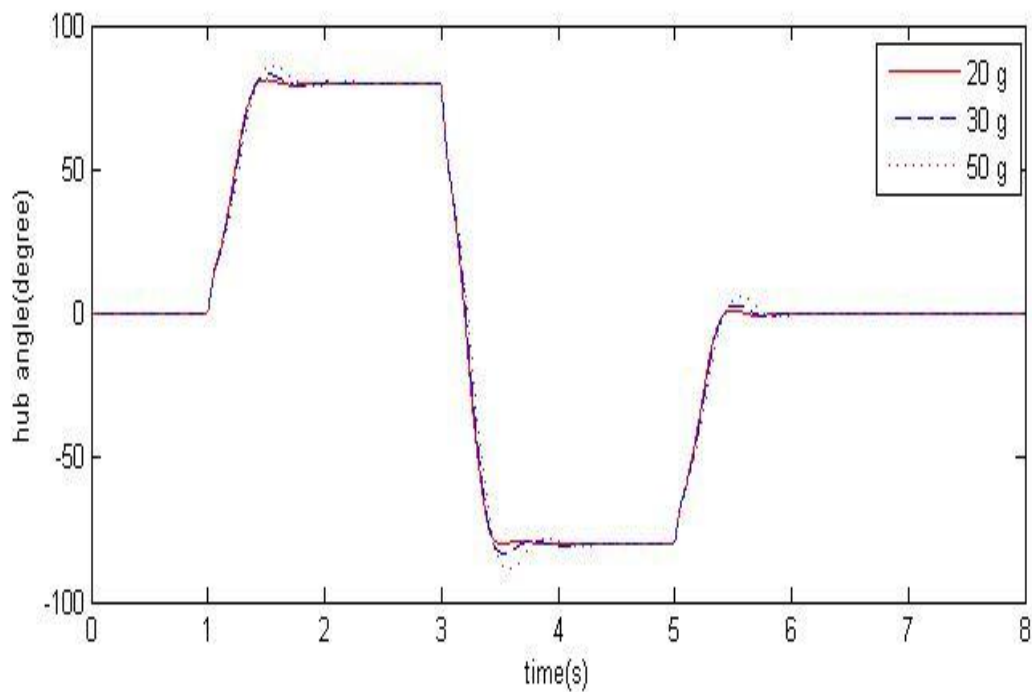


Fig. 8 hub angle with 20, 30 and 50 g payload



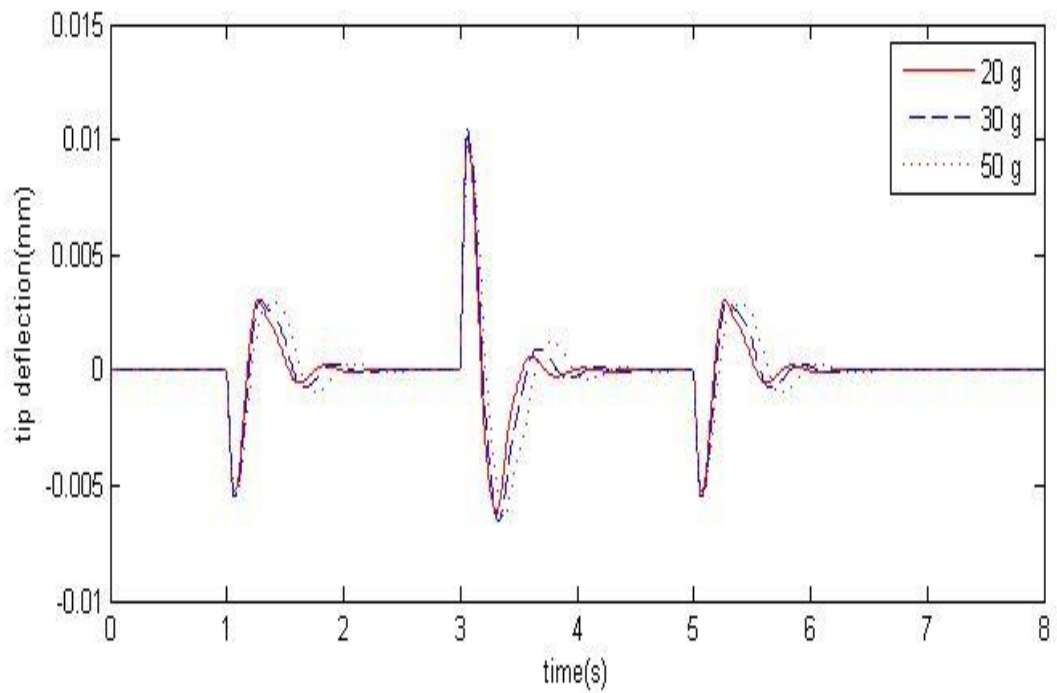


Fig. 9 tip deflection with 20, 30 and 50 g payload

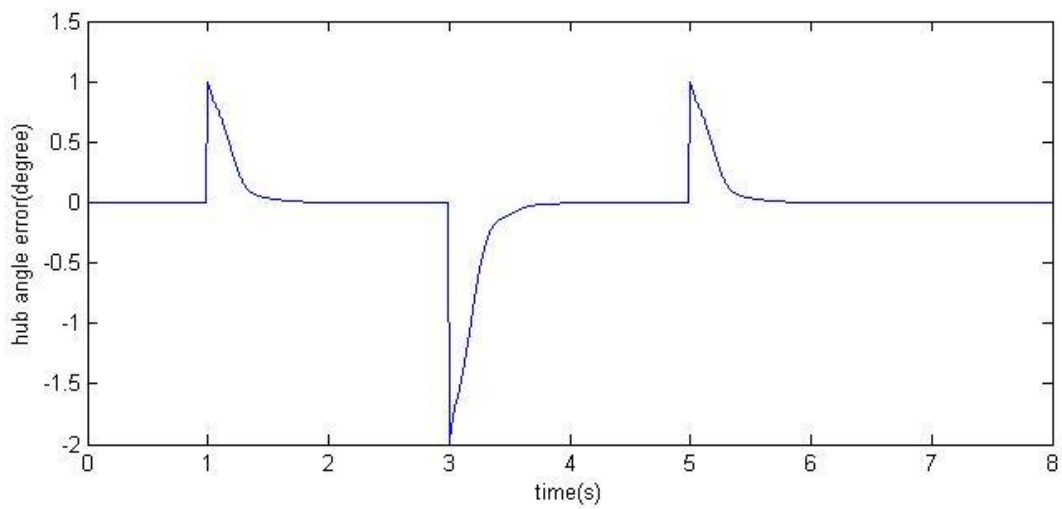


Fig. 10 Hub angle error ( $e$ )

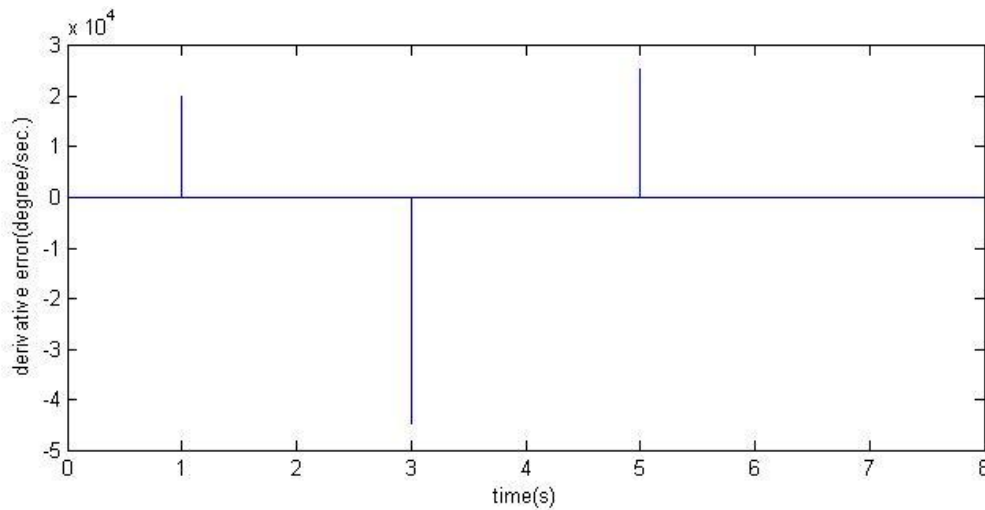


Fig. 11 derivative of error ( $\dot{e}$ )

Table 2 summary of the controller results with different payload value

Payload in gram (g)	Hub angle Overshot (degree)	Hub angle Settling time (s)	Hub angle Rise time (s)	Tip deflection Maximum deflection (mm)
0	0	0.28	0.19	13.4
20	1.24	0.54	0.21	14.6
30	2.32	0.65	0.23	15.8
50	3.73	1.05	0.27	16.2

## V. CONCLUSION

In this work, a PD-type fuzzy logic controller (PDFLC) have been designed with hub angel error and its derivative as inputs of the controller, it was implemented using MATLAB fuzzy tool box to achieve two control purposes namely input tracking and tip deflection regulation. The results shows that the controller successfully achieved both the tracking and regulation control, it also the advantage of the propose control in handling various payload. This control is applicable to many industrial flexible manipulators system, In order to test the robustness of the controller different values of payload are tested and the results shows small changes in the overshoot, rise time and settling time as compared to the result with no payload, which shows that the controller is robust to some extend with external changes with zero steady state error. Some limitations of this control scheme it is difficult to design the rule base and the simulation time takes longer time (delay) but this is as a result of using many membership functions and many rule bases. The control gives better result with high number of membership function as when design with less number of membership function. The propose control scheme can be improved by introducing another controller to improve the tracking performance such as integral control in the outer loop.

## VI. ACKNOWLEDGEMENTS

My Acknowledgements goes to my supervisor in person of Assoc. Prof. Dr. zaharuddin Mohamed and also to the Faculty of Electrical Engineering, University Tecknologi Malaysia for supporting me with resources and favourable environment to conducted and complete this paper.

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