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A New Concept of Sets and Topologies in Ideal Topological Space

Himanshu Singh Rajput*[1], Dr Bhawna Agarwal*[2], Dr Chitra Singh*[3]
**1 M.sc mathematics Rabindra Nath Tagore University, M.P, Bhopal*
**2 Associated professor Rabindra Nath Tagore University, M.P, Bhopal*
**3 Associated professor Rabindra Nath Tagore University, M.P, Bhopal.*

ABSTRACT
Topology is the key technology of mathematical modelling of engineering and science, It is important for accurate reconstructon of the entity model that conforms to the design. In order to improve the measurement accuracy of the workpiece, The method of recognition based on topological transformation is proposed in this paper. Firstly The information of scanning point is expressed as the form of topological space. Based on this, to discover a new concept of Sets and Topologies in Ideal Topological space, finite union and intersection of ∗*-perfect sets are again* ∗*-perfect set and obtain a new topology for the finite topological spaces which is finer than* ∗*-topology.*

OBJECTIVE
The objective of this presentation is to discover a new concept of Sets and Topologies in Ideal Topological space.

KEYWORDS
Topological space,Ideal Topological space, ∗*-perfect sets.*

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I. INTRODUCTION

Ideal topological space

The concept of **ideal topological spaces** was introduced by Kuratowski and Vaidyanathswamy . An **ideal** I as we know is a non-empty collection of subsets of X closed with respect to finite union and heredity. (X, τ, I) is an **ideal topological space** and we call it an **ideal space** in this paper.

The contributions of Hamlett and Jankovic in ideal topological spaces initiated the generalization of some important properties in general topology via topological ideals. The properties like decomposition of continuity, separation axioms, connectedness, compactness, and resolvability have been generalized using the concept of ideals in topological spaces. By a space (,), we mean a topological space with a topology defined on on which no separation axioms are assumed unless otherwise explicitly stated. For a given point in a space (,), the system of open neighbourhoods of is denoted by () = { ∈ : ∈ }. For a given subset of a space (,), cl() and int() are used to denote the closure of and interior of , respectively, with respect to the topology. A nonempty collection of subsets of a set is said to be an ideal on , if it satisfies the following two conditions: (i) If ∈ I and ⊆, then ∈ I;

(ii) If ∈ I and ∈ I, then ∪ ∈ I. An ideal topological space (or ideal space) (, , I) means a topological space (,) with an ideal I defined on . Let (,) be a topological space with an ideal I defined on . Then for any subset of , ∗(I,) = { ∈ / ∩ ∉ I for every ∈ ()} is called the local function of with respect to I and . If there is no ambiguity, we will write ∗(I) or simply ∗ for ∗(I,). Also, cl∗() = ∪A'.

defines a Kuratowski closure operator for the topology ∗(I) (or simply ∗) which is finer than . An ideal I on a space (,) is said to be codense ideal if and only if ∩I = {0}. ∗ is always a proper subset of . Also, =∗ if and only if the ideal is condense.

II. METHODOLOGY

Lemma 1. Let (□, □) be a space with I1 and I2 being ideals on □, and let □ and □ be two subsets on □. Then

(i) □⊆□⇒□∗ ⊆ □∗;
(ii) I1 ⊆ I2 ⇒ □∗(I2) ⊆□∗(I1);
(iii) □∗ = □□(□∗) ⊆ □□(□) (□∗ is a closed subset of □□(□));
(iv) (□∗) ∗ ⊆ □∗;
(v) (□ ∪ □)∗ = □∗ ∪ □∗;
(vi) □∗ − □∗ = (□ − □)∗ − □∗ ⊆ (□ − □)∗;
(vii) for every □ ∈ I, (□ ∪ □) ∗ = □∗ = (□ − □) ∗

Definition 2. Let (□, □) be a space with an ideal I on □. One says that the topology □ is compatible with the ideal I, denoted by □ ∼ I, if the following holds, for every □⊆□: if for every □∈□, there exists a □ ∈ □(□) such that □∩□ ∈ I, then □ ∈ I.

Definition 3. A subset □ of an ideal space (□, □, I) is said to be

(i) □∗-closed [3] if □∗ ⊆ □,
(ii) ∗-dense-in-itself [10] if □⊆□∗,
(iii) □-open [11] if □ ⊆ int(□∗),
(iv) almost □-open [12] if □ ⊆ cl(int(□∗)),
(v) □-dense [7] if □∗ = □,
(vi) almost strong □-□-open [13] if □ ⊆ cl∗(int(□∗).
(vii) ∗-perfect [10] if □=□∗,
(viii) regular □-closed [14] if □=(int(□)).
(ix) an □□-set [15] if □⊆(int(□))∗

In Ideal spaces, usually □⊂□ implies □∗ ⊂ □∗. We observe that there are some sets □ and □ such that □⊂□ but □∗ = □∗. Example 1. Let (□, □, I) be an ideal space with □ = {□, □, □, □}, □ = {□, □, {□, □}, {□}, {□, □, □}}, I = {□, {□}, {□}, {□, □}}. Here the sets □ = {□} and □ = {□, □} are such that □⊆□, but □∗ = □∗ = {□, □, □}. Proposition 01. Let (□, □, I) be an ideal space. Let □ and □ be two subsets of □ such that □⊆□ and □∗ = □∗; then

(i) □ is □∗-perfect if □ is □∗-perfect;
(ii) □ is □∗-perfect if □ is □∗-perfect.

Proof. (i) Let □ be an □∗-perfect set. Then □∗ −□∈ I. Now, □∗ −□ = □∗ −□ ⊆ □∗ − □. By heredity property of ideals, □∗ −□∈ I. Hence □ is □∗-perfect.
(iii) Let □ be an □∗-perfect set. Then □−□∗ ∈ I. Now, □−□∗ = □−□∗ ⊆□−□∗. By heredity property of ideals, □−□∗ ∈ I. Hence □ is □∗-perfect.

Corollary 11. Let (□, □, I) be an ideal space. Let □ and two □ be subsets of □ such that □ ⊆ □ ⊆ □□∗□; then

(i) □ is □∗-perfect if □ is □∗-perfect,
(ii) □ is □∗-perfect if □ is □∗-perfect.

Proof. Since □⊆□⊆ cl∗□, □∗ ⊆ □∗ ⊆ (cl∗□)∗ = □∗. Hence □∗ = □∗. Therefore, the result follows from Proposition 20.

Proposition 02. Let □ be a subset of an ideal topological space (□, □, I) such that □ is □∗-perfect set and □∩□∗ is □∗-perfect; then both □ and □∩□∗ are □∗-perfect.

Proof. Since □ is □∗-perfect, □−□∗ ∈ I. By Lemma 1(vii), for every □ ∈ I, (□∪□)∗ = □∗ = (□−□)∗. Therefore, (□∪(□− □∗))∗ = □∗ = (□ − (□ − □∗))∗. This implies □∗ = (□ ∩ □∗) ∗. Therefore, we have □∩□∗ ⊆ □ with (□ ∩ □∗) ∗ = □∗. By Proposition 20, □ is □∗-perfect if □∩□∗ is □∗-perfect and □∩□∗ is □∗-perfect if □ is □∗-perfect set. Hence □ is □∗- perfect and □∩□∗ is □∗-perfect.

Proposition 03. If a subset □ of an ideal topological space (□, □, I) is □∗-perfect set and □∗ is □∗-perfect, then □∩□∗ is □∗-perfect. Proof. Since □ is □∗-perfect, □∗−□ ∈ I. By Lemma 1(vii), for every □ ∈ I, (□ ∪ □)∗ = □∗ = (□ − □)∗. Therefore, (□∗ ∪ (□∗ − □))∗ = □∗∗ = (□∗−(□∗−□))∗.This implies □∗∗ = (□∩□∗) ∗. Therefore, we have □∩□∗ ⊆ □∗ with (□ ∩ □∗) ∗ = □∗∗. By Proposition 20, □∩□∗ is □∗-perfect if □∗ is □∗-perfect set. Hence □∩□∗ is □∗-perfect.

Proposition 04. If □ and □ are □∗-perfect sets, then □∪□ is an □∗-perfect set.
Proof. Let □ and □ be □∗-perfect sets. Then □∗ −□∈ I and □∗ −□∈ I. By finite additive property of ideals, (□∗ − □) ∪ (□∗ − □) ∈ I. Since (□∗ ∪ □∗) − (□ ∪ □) ⊆ (□∗ − □) ∪ (□∗ − □), by heredity property (□∗ ∪ □∗) − (□ ∪ □) ∈ I. Hence (□ ∪ □)∗ − (□ ∪ □) ∈ I. This proves the result.
Corollary 22. Finite union of □∗-perfect sets is an □∗-perfect set. Proof. The proof follows from Proposition 24.
Proposition 05. If □ and □ are □∗-perfect sets, then □∪□ is an □∗-perfect set.
Proof. Since □ and □ are □∗-perfect sets, □−□∗ ∈ I and □ − □∗ ∈ I. Hence by finite additive property of ideals,(□−□∗) ∪ (□−□∗) ∈ I. Since (□∪□)− (□∪□)∗ = (□∪□)− (□∗ ∪□∗) ⊆ (□−□∗)∪(□−□∗), by heredity property (□∪□) − (□∪□)∗ ∈ I. This proves that □∪□ is an □∗-perfect set.
Corollary 33. Finite union of □∗-perfect sets is an □∗-perfect sets. Proof. The proof follows from Proposition 26.
Proposition 06. If □ and □ are □∗-perfect sets, then □∩□ is an □∗-perfect set.
Proof. Suppose that □ and □ are □∗-perfect sets. Then □∗ − □ ∈ I and □∗ −□∈ I. By finite additive property of ideals, (□∗ − □) ∪ (□∗ − □) ∈ I. Since (□∗ ∩ □∗) − (□ ∩ □) ⊆ (□∗ − □) ∪ (□∗ − □), by heredity property (□∗ ∩ □∗) − (□ ∩ □) ∈ I. Also (□ ∩ □)∗ − (□ ∩ □) ⊆ (□∗ ∩ □∗) − (□ ∩ □) ∈ I. This proves the result.
Corollary 33. Finite intersection of □∗-perfect sets is an □∗- perfect set. Proof. The proof follows from Proposition 28. Proposition 30. Finite union of □∗-perfect sets is a □∗-perfect set. Proof. From Corollaries 27 and 29, finite union of □∗-perfect sets is a □∗-perfect set. Proposition 31. If (□, □, I) is an ideal topological space with □ being finite, then the collection R is a topology which is finer than the topology of □∗-closed sets.

III. RESULT

Some New Sets and Topologies in Ideal Topological Spaces in this research article introduce union and intersection, prove finite union and intersection of □∗-perfect sets are again □∗-perfect set. And prove that finite union and intersection of □∗-perfect sets are again □∗-perfect set. observe that there are some sets □ and □ such that □⊂□ but □∗ = □∗.

if □∪□ is an □∗-perfect set. By finite additive property of ideals, (□∗ − □) ∪ (□∗ − □) ∈ I. Since (□∗ ∩ □∗) − (□ ∩ □) ⊆ (□∗ − □) ∪ (□∗ − □), by heredity property (□∗ ∩ □∗) − (□ ∩ □) ∈ I. and □∪□ is an □∗-perfect, □∗-perfect sets, □−□∗ ∈ I and □ − □∗ ∈ I. Hence by finite additive property of ideals,(□−□∗) ∪ (□−□∗) ∈ I. Since (□∪□)− (□∪□)∗ = (□∪□)− (□∗ ∪□∗) ⊆ (□−□∗)∪(□−□∗)

IV. MAIN RESULT

In this section, we prove finite union and intersection of □∗-perfect sets are again □∗-perfect set. Using these results, we obtain a new topology for the finite topological spaces which is finer than □∗-topology.

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