International Journal of Research in Engineering and Science (IJRES)
ISSN (Online): 2320-9364, ISSN (Print): 2320-9356
www.ijres.org Volume 9 Issue 6 ǁ 2021 ǁ PP. 14-33

R-Posterior Control Charts for Process Variation

Sharada V. Bhat[1*], Chetan Malagavi[2]
[1,2]Department of Statistics. Karnatak University, Dharwad. Karnataka State, India.
[*]*Corresponding Author*: bhat_sharada@yahoo.com

Abstract
As variation is one of the important characteristics of process monitoring, the construction of control charts for process variation becomes prominent in manufacturing. The variation is expressed in terms of process standard deviation (σ) and process variance (σ^2). The objective of this paper is to make use of available prior information and develop posterior control charts for σ and σ^2 based on range, a simple measure. We propose R-posterior control charts using Bayesian point of view. The proposed control charts are studied by obtaining their control limits or probability limits, power, average run length (ARL), standard deviation of run length (SDRL), coefficient of variation of run length (CVRL) and process capability ratio (PCR). They are competent, practically handy in manufacturing industry as they demand few samples to detect ongoing process is out of control, if so and simple to operate. An illustrative example is provided.
Keywords: *Process variation, control limits, probability limits, power, ARL, SDRL*

Date of Submission: 06-06-2021 Date of acceptance: 20-06-2021

I. INTRODUCTION

Quality is a distinctive character which expects a state of zero significant variations in the process exhibition. Statistically, the process variation is pronounced in terms of process variance and process standard deviation. It is measured in terms of sample variance, sample standard deviation and also sample range where range is a simple prefatory measure of variability. The information readily available in the production industry over a period of time is helpful as it contains earlier knowledge about the variability of a specific manufacturing process. This can be expressed in terms of probability distribution known as prior distribution. Using the knowledge of prior and sample information, one can construct effective control charts using Bayesian approach.

Montgomery (1996) contains an elaborative discussion on the construction of Shewhart's S control chart based on control limits (S_1 control chart) for σ and Shewhart's S^2 control chart based on probability limits for σ^2. Chen (1998) gives the run length distribution for Shewhart's range, S and S^2 control charts when σ is estimated. Ryan (2000) developed probability limits for Shewhart's S control chart (S_2 control chart). To supervise shifts in σ, Huang and Chen (2005) suggests synthetic control chart consisting of variable sampling interval scheme under normality. Zhang et. al (2005) carries out study on ARL-unbiased S^2 control chart along with its properties. Zhang and Govindaraju (2007) studied phase II S control chart and recommends to adopt two-point design or ARL-unbiased design. Shahriari et.al. (2009) proposes outlier resistant control chart based on Huber's (1981) M estimate and Schoonhoven et al. (2011) gives robust control chart based on absolute mean deviation from median for σ. Schoonhoven and Does (2012) develops robust S control chart. Rajmanya and Ghute (2014) proposes synthetic D chart for σ by combining conforming run length and salient features of Downton's estimator.

Both S and S^2 control charts for the control of process variation have caught the attention of researchers with Shewhart's S control chart being first of its kind. But many quality engineers prefer S^2 control chart for its simplicity in practice.

The traces of Bayesian methods to quality control is due to Girshick and Rubin (1952). Berger (1986) and Colosimo and Del Castillo (2006) provide detailed discussion on Bayesian methods and Bayesian process monitoring. Menzefricke (2007) constructs multivariate control chart based on predictive distribution for generalised variance. Bhat and Gokhale (2014) delineate posterior S^2 control chart (PS^2 control chart) for σ^2 considering conjugate, non-conjugate and non-informative priors. Also, they propose Posterior S control chart based on probability limits (PS_2 control chart) for σ. Bhat and Gokhale (2016) study posterior S control chart based on control limits (PS_1 control chart) under conjugate prior.

If σ is unknown and is to be estimated in terms of range, Tippett (1925) shows that

$$\hat{\sigma} = r/d \tag{1}$$

where $\hat{\sigma}$ is estimator of σ, r is sample range, $d = \int_{-\infty}^{+\infty}[1-(1-\Phi(\mathrm{x}))^n - (\Phi(\mathrm{x}))^n]dx$, n is sample size and $\Phi(\cdot)$ is the cumulative distribution function of standard normal variate. Nelson (1975) discusses the use of

$d^* = \sqrt{\frac{d'^2}{k} + d^2}$ where $d'^2 = V(R|\sigma)$ and k is number of subgroups. The comparison of use of d^* and d using mean squared error is described in Luko (1996).

As prior information throws light on the cognizance about the intrinsic process variation of a particular production process and range being a preliminary measure of variation is simple to compute, developing range-based control charts for σ and σ^2 is desirable in process monitoring.

In this paper, we propose posterior control charts based on range for process variation, namely, for σ and σ^2. We construct R_1 control chart based on posterior control limits, R_2 control chart based on posterior probability limits for σ and R^* control chart for σ^2. In section 2, we derive posterior distribution of σ^2 and σ in terms of range. We propose R_1, R_2 and R^* control charts in section 3. In section 4, we evaluate and record our observations on performance of the proposed control charts. An illustration of the control charts is provided with an example in section 5 and concluding remarks are given in section 6. Tables containing computations in support of our study are presented in appendix.

II. POSTERIOR DISTRIBUTION OF σ^2 AND σ

In this section, we derive posterior distribution of σ^2 and σ in terms of r. Suppose $X_1, X_2 \ldots X_n$ is a random sample of size n from $N(\mu, \sigma^2)$, then the density function of X_i is given by

$$f_{X_i}(x|\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad -\infty < x, \mu < \infty \ , \sigma > 0. \qquad (2)$$

Since $z = \frac{ns^2}{\sigma^2}$ where $s^2 = \frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n}$ when μ is known, has central chi-square distribution with n degrees of freedom, we have

$$f(z) = \frac{1}{2^{\frac{n}{2}}\Gamma\frac{n}{2}} e^{-\left(\frac{z}{2}\right)} z^{\left(\frac{n}{2}-1\right)}, z > 0. \qquad (3)$$

In the light of the fact that, s^2 estimates σ^2, from (1), we have $s^2 = \frac{r^2}{d^2}$, therefore $z = \frac{nr^2}{\sigma^2 d^2}$.

The distribution of $(r^2|\sigma^2)$ is given by

$$f(r^2|\sigma^2) = \frac{\left(\frac{n}{2\sigma^2 d^2}\right)^{\frac{n}{2}}}{\Gamma\frac{n}{2}} e^{-\frac{nr^2}{2\sigma^2 d^2}} (r^2)^{\frac{n}{2}-1}, r^2 > 0. \qquad (4)$$

We assume that, σ^2 has $Inverse\ Gamma(\eta, \delta)$ distribution where η is shape parameter and δ is scale parameter, which is a conjugate prior and is given by

$$\pi(\sigma^2) = \frac{\eta^\delta}{\Gamma\eta} e^{-\frac{\delta}{\sigma^2}} \left(\frac{1}{\sigma^2}\right)^{\eta+1}, \sigma^2 > 0. \qquad (5)$$

The posterior distribution of $(\sigma^2|r^2)$ is given by

$$\pi(\sigma^2|r^2) = \frac{\left(\frac{nr^2}{2d^2} + \delta\right)^{\frac{n}{2}+\eta}}{\Gamma\left(\frac{n}{2} + \eta\right)} e^{-\frac{\left(\frac{nr^2}{2d^2}+\delta\right)}{\sigma^2}} \left(\frac{1}{\sigma^2}\right)^{\frac{n}{2}+\eta+1}. \qquad (6)$$

Therefore, $\pi(\sigma^2|r^2) \sim Inverse\ gamma\ (\psi, \zeta)$ where $\psi = \frac{n}{2} + \eta, \zeta = \frac{nr^2}{2d^2} + \delta$. Under squared error loss (SEL) function, the mean and variance of σ^2 respectively are given by

$$E(\sigma^2|r^2) = \frac{\zeta}{\psi - 1} \qquad (7)$$

$$\text{and } V(\sigma^2|r^2) = \frac{\zeta^2}{(\psi-2)(\psi-1)^2}. \qquad (8)$$

Using appropriate transformation, $\pi(\sigma|r)$ is given by

$$\pi(\sigma|r) = \frac{2\left(\frac{nr^2}{2d^2}+\delta\right)^{\frac{n}{2}+\eta}}{\Gamma\left(\frac{n}{2}+\eta\right)} e^{-\frac{\left(\frac{nr^2}{2d^2}+\delta\right)}{\sigma^2}} \left(\frac{1}{\sigma^2}\right)^{\frac{n}{2}+\eta+\frac{1}{2}}. \qquad (9)$$

Under SEL function, mean and variance of σ respectively are given by

$$E(\sigma|r) = \frac{\Gamma\left(\psi - \frac{1}{2}\right)\sqrt{\zeta}}{\Gamma(\psi)} \qquad (10)$$

$$\text{and } V(\sigma|r) = \zeta\left\{\frac{1}{\psi-1} - \left(\frac{\Gamma\left(\psi-\frac{1}{2}\right)}{\Gamma(\psi)}\right)^2\right\}. \quad (11)$$

Also, we observe that, $\pi(\sigma|r) = \pi(\sigma|r^2) = 2\sigma\, \pi(\sigma^2|r^2) = 2\sigma\, IG(\psi,\zeta)$. When μ is unknown, it is estimated by $\bar{x}$. Hence z is replaced by $z = \frac{(n-1)s^2}{\sigma^2}$ that follows $\chi^2{}_{(n-1)}$. Subsequently, in all the expression through (3) to (11), n is replaced by (n-1).

III. R-POSTERIOR CONTROL CHARTS

In this section, we propose posterior control charts based on the range for process variation. We furnish the control limits of R_1, probability limits of R_2 and probability limits of R^* control charts. The limits of any control chart say A control chart for process characteristic is given by

$$Upper\ Limit\ of\ A, UL_A = E(A) + \gamma\sqrt{V(A)}, \quad (12)$$

$$Central\ Limit\ of\ A, CL_A = E(A) \quad (13)$$

$$\text{and } Lower\ Limit\ of\ A, LL_A = E(A) - \gamma\sqrt{V(A)}. \quad (14)$$

where γ is a multiplicative constant. For $\gamma = 3$ we get control limits, for $\gamma = 2$, we get warning limits and γ as specified by the consumer, we get specification limits. The probability limits of A is given by

$$Upper\ Probability\ Limit(UPL)_A = F(\cdot)_{1-\frac{\alpha}{2}}, \quad (15)$$

$$Central\ Probability\ Limit(CPL)_A = F(\cdot)_{0.5} \quad (16)$$

$$\text{and } Lower\ Probability\ Limit(LPL)_A = F(\cdot)_{\frac{\alpha}{2}} \quad (17)$$

where α is the probability of type I error and $F(\cdot)$ is the cumulative distribution function of the underlying process variate for control chart A. Width of the control chart A is given by $W_A = UCL_A - LCL_A$ or $W_A = UPL_A - LPL_A$ as the case may be.

Proceeding on similar lines of Bhat and Gokhale (2014), the probability limits of R_2 control chart is derived and expressed in terms of chi-square distribution. We provide the control limits of R_1 and probability limits of R_2 and R^* control charts along with their width in exhibit 1. Also, the control limits and probability limits of their competitors along with their width are given in exhibit 2.

Exhibit 1: Control limits of R_1 and probability limits R_2 and R^* control charts

Control Charts	Control Limits/ Probability Limits	Width
R_1	$UCL_{R_1} = E(\sigma\|r) + 3\sqrt{V(\sigma\|r)}$ $CL_{R_1} = E(\sigma\|r)$ $LCL_{R_1} = E(\sigma\|r) - 3\sqrt{V(\sigma\|r)}$	$W_{R_1} = 6\sqrt{\zeta\left\{\frac{1}{\psi-1} - \left(\frac{\Gamma\left(\psi-\frac{1}{2}\right)}{\Gamma(\psi)}\right)^2\right\}}$
R_2	$UPL_{R_2} = F(2\sigma\, IG(\psi,\zeta))_{1-\frac{\alpha}{2}} = \sqrt{\frac{2\zeta}{\chi^2{}_{2\zeta,\frac{\alpha}{2}}}}$ $CPL_{R_2} = F(2\sigma\, IG(\psi,\zeta))_{0.5} = \sqrt{\frac{2\zeta}{\chi^2{}_{2\zeta,0.5}}}$ $LPL_{R_2} = F(2\sigma\, IG(\psi,\zeta))_{\frac{\alpha}{2}} = \sqrt{\frac{2\zeta}{\chi^2{}_{2\zeta,1-\frac{\alpha}{2}}}}$	$W_{R_2} = \sqrt{2\zeta}\left(\frac{1}{\sqrt{\chi^2{}_{2\zeta,\frac{\alpha}{2}}}} - \frac{1}{\sqrt{\chi^2{}_{2\zeta,1-\frac{\alpha}{2}}}}\right)$
R^*	$UPL_{R^*} = F(IG(\psi,\zeta))_{1-\frac{\alpha}{2}} = \frac{2\zeta}{\chi^2{}_{2\zeta,\frac{\alpha}{2}}}$ $CPL_{R^*} = F(IG(\psi,\zeta))_{0.5} = \frac{2\zeta}{\chi^2{}_{2\zeta,0.5}}$ $LPL_{R^*} = F(IG(\psi,\zeta))_{\frac{\alpha}{2}} = \frac{2\zeta}{\chi^2{}_{2\zeta,1-\frac{\alpha}{2}}}$	$W_{R^*} = 2\zeta\left(\frac{1}{\chi^2{}_{2\zeta,\frac{\alpha}{2}}} - \frac{1}{\chi^2{}_{2\zeta,1-\frac{\alpha}{2}}}\right)$

Exhibit 2: Control/probability limits of Shewhart's and posterior variation control charts

Control Charts	Control Limits/ Probability Limits	Width
S_1	$UCL_{S_1} = c\sigma + 3\sigma\sqrt{1-c^2}$ $CL_{S_1} = c\sigma$ $LCL_{S_1} = c\sigma - 3\sigma\sqrt{1-c^2}$ where $c = \frac{4(n-1)}{4n-3}$	$W_{S_1} = 6\sigma\sqrt{1-c^2}$

S_2	$UPL_{S_2} = \sigma\sqrt{\frac{\chi^2{}_{n,\frac{\alpha}{2}}}{n}}$ $CPL_{S_2} = \sigma$ $LPL_{S_2} = \sigma\sqrt{\frac{\chi^2{}_{n,1-\frac{\alpha}{2}}}{n}}$	$W_{S_2} = \frac{\sigma}{\sqrt{n}}\left(\sqrt{\chi^2{}_{n,\frac{\alpha}{2}}} - \sqrt{\chi^2{}_{n,1-\frac{\alpha}{2}}}\right)$
S^2	$UPL_{S^2} = \frac{\sigma^2}{n}\chi^2{}_{n,\frac{\alpha}{2}}$ $CPL_{S^2} = \sigma^2$ $LPL_{S^2} = \frac{\sigma^2}{n}\chi^2{}_{n,1-\frac{\alpha}{2}}$	$W_{S^2} = \frac{\sigma^2}{n}\left(\chi^2{}_{n,\frac{\alpha}{2}} - \chi^2{}_{n,1-\frac{\alpha}{2}}\right)$
PS_1	$UCL_{PS_1} = E(\sigma\|s) + 3\sqrt{V(\sigma\|s)}$ $CL_{PS_1} = E(\sigma\|s)$ $LCL_{PS_1} = E(\sigma\|s) - 3\sqrt{V(\sigma\|s)}$ where $E(\sigma\|s) = \frac{\Gamma(\psi-\frac{1}{2})\sqrt{\tau}}{\Gamma(\psi)}$ $\mathrm{V}(\sigma\|s) = \tau\left\{\frac{1}{\psi-1} - \left(\frac{\Gamma\left(\psi-\frac{1}{2}\right)}{\Gamma(\psi)}\right)^2\right\}$ and $\tau = \frac{ns^2}{2} + \delta$.	$W_{PS_1} = 6\sqrt{\tau\left\{\frac{1}{\psi-1} - \left(\frac{\Gamma\left(\psi-\frac{1}{2}\right)}{\Gamma(\psi)}\right)^2\right\}}$
PS_2	$UPL_{PS_2} = F(2\sigma\ IG(\psi,\tau))_{1-\frac{\alpha}{2}} = \sqrt{\frac{2\tau}{\chi^2{}_{2\tau,\frac{\alpha}{2}}}}$ $CPL_{PS_2} = F(2\sigma\ IG(\psi,\tau))_{0.5} = \sqrt{\frac{2\tau}{\chi^2{}_{2\tau,0.5}}}$ $LPL_{PS_2} = F(2\sigma\ IG(\psi,\tau))_{\frac{\alpha}{2}} = \sqrt{\frac{2\tau}{\chi^2{}_{2\tau,1-\frac{\alpha}{2}}}}$	$W_{PS_2} = \sqrt{2\tau}\left(\frac{1}{\sqrt{\chi^2{}_{2\tau,\frac{\alpha}{2}}}} - \frac{1}{\sqrt{\chi^2{}_{2\tau,1-\frac{\alpha}{2}}}}\right)$
PS^2	$UPL_{PS^2} = F(IG(\psi,\tau))_{1-\frac{\alpha}{2}} = \frac{2\tau}{\chi^2{}_{2\tau,\frac{\alpha}{2}}}$ $CPL_{PS^2} = F(IG(\psi,\tau))_{0.5} = \frac{2\tau}{\chi^2{}_{2\tau,0.5}}$ $LPL_{PS^2} = F(IG(\psi,\tau))_{\frac{\alpha}{2}} = \frac{2\tau}{\chi^2{}_{2\tau,1-\frac{\alpha}{2}}}$	$W_{PS^2} = 2\tau\left(\frac{1}{\chi^2{}_{2\tau,\frac{\alpha}{2}}} - \frac{1}{\chi^2{}_{2\tau,1-\frac{\alpha}{2}}}\right)$

IV. EVALUATION OF PROPOSED CONTROL CHARTS

In this section, we evaluate R_1, R_2 and R^* control charts in terms of their power, ARL, SDRL, CVRL and PCR. For any control chart A, its power is given by

$$P_A = 1 - \beta_A \tag{18}$$

where β_A is probability of not detecting the shift $(b > 0)$ in process characteristic q viz., σ or σ^2

$$\text{and } \beta_A = \int_{LCL_A}^{UCL_A} \pi(\mathrm{q}|v)d\mathrm{q} \text{ or } \beta_A = \int_{LPL_A}^{UPL_A} \pi(\mathrm{q}|v)d\mathrm{q} \tag{19}$$

where v is the sample measure of process characteristic q.

The ARL indicates average sample number required to detect assignable cause, SDRL measures spread in run length distribution, CVRL gives variability with respect to ARL and PCR gives the performance of a process within desirable specification. All these measures of control chart A are given by

$$ARL_A = \frac{1}{1-\beta_A} = \frac{1}{P(A)}, \tag{20}$$

$$SDRL_A = \frac{\sqrt{\beta_A}}{1-\beta_A}, \tag{21}$$

$$CVRL_A = \frac{SDRL_A}{ARL_A} \tag{22}$$

$$\text{and } PCR_A = \frac{USL_A - LSL_A}{6\sigma_A} \ or\ PCR_A = \frac{USL_A - LSL_A}{F(\cdot)_{1-\alpha}}. \tag{23}$$

The performance measures of the control charts are computed for $\eta = \delta = 1$ and are given in the form of tables in appendix. Using (19), (20) and (21) respectively, the computation of power, ARL and SDRL of S_1, PS_1 and R_1 control charts are given in table 1, S_2, PS_2 and R_2 control charts in table 2 and S^2, PS^2 and R^* control charts in table 3 for various values of n. For computing power of R_1, R_2 and R^* control charts, from (19), using $r + b$ for r in $\pi(\sigma|r,b)$ given by (9) we get β_{R_1}, β_{R_2} and using $r^2 + b$ in $\pi(\sigma^2|r^2,b)$ given by (6) we get β_{R^*}.

The power and ARL of various control charts are presented in figure 1, 2 and 3 respectively using table 1, 2 and 3. The power and ARL of R_1, R_2 and R^* control charts are given in figure 4. For various values of n, CVRL of the control charts under consideration are computed using (22) and PCR using (23) and are respectively provided in table 4 and table 5. The power computations of R_1, R_2 and R^* control charts for various values of n, η and δ are given in table 6. To see the nature of the power of the proposed control charts for varying hyper parameters, using table 6, we plot figure 5. In figure 5(a), we provide power of R_1, in (b), power of R_2 and in (c) power of R^* for n=6, 12, η=1, 5, 10 and δ=1, 5, 10. In tables and figures, we take $\sigma_b = \sigma + b$ and $\sigma_b^2 = \sigma^2 + b$. We use R-program to generate 1,00,000 random samples from the distribution of the process variates and obtain the values of various measures.

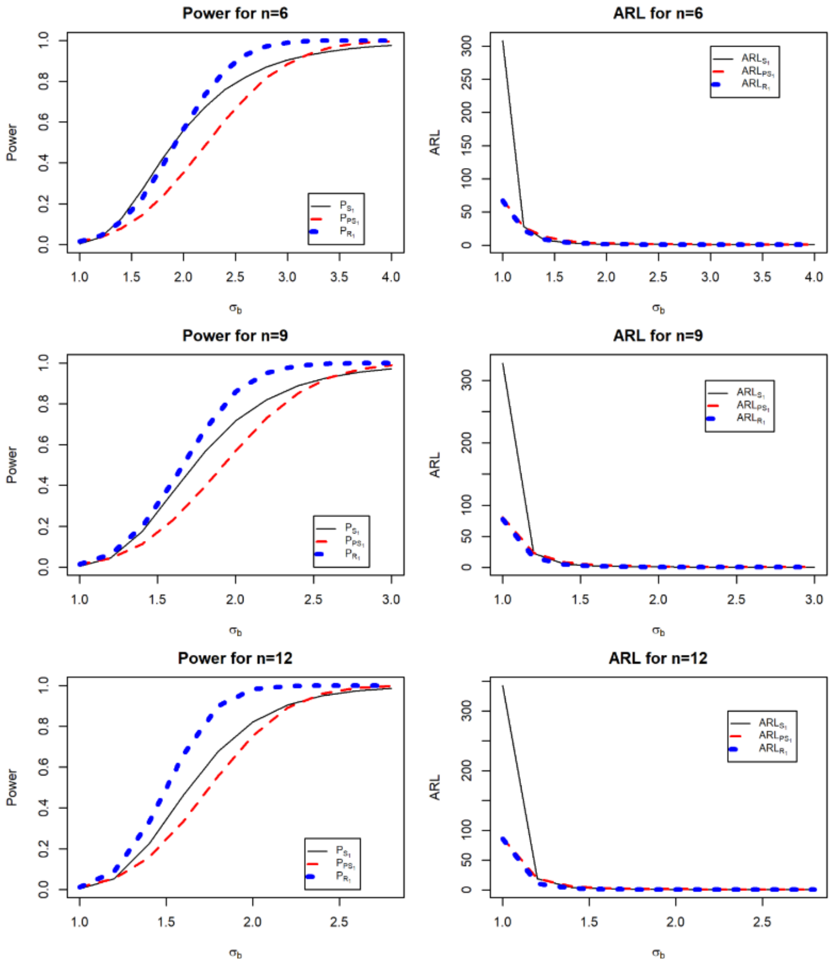


Figure 1: Power and ARL of S_1, PS_1 and R_1 control charts

From figure 1 and table 1, we observe that, for S_1, PS_1 and R_1 control charts, power increases, ARL and SDRL decrease with increasing values of σ_b. This means that the power increases, ARL and SDRL decrease for increase in shift. Also $P_{R_1} > P_{PS_1} > P_{S_1}$ and $ARL_{R_1} < ARL_{PS_1} < ARL_{S_1}$. From table 1, we see that the power of S_1, PS_1 and R_1 control charts increase, ARL and SDRL decrease for increasing values of n. Also, power converges to unity for comparatively lower shifts for higher values of n, when compared with lower values of n.

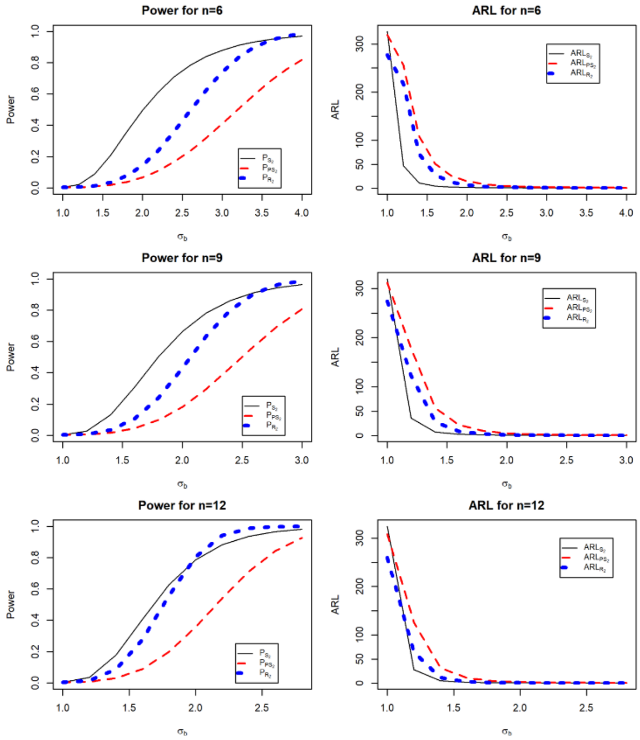


Figure 2: Power and ARL of S_2, PS_2 and R_2 control charts

Figure 2 and table 2 show that, the power of S_2, PS_2 and R_2 control charts increases, ARL and SDRL decrease with increasing σ_b. They reveal that, $P_{R_2} > P_{PS_2}$, $ARL_{R_2} < ARL_{PS_2}$ for all values of σ_b and n under consideration. Also, $P_{R_2} > P_{S_2}$ and $ARL_{R_2} < ARL_{S_2}$ when $\sigma_b \geq 3.8$ for n=6, $\sigma_b \geq 2.8$ for n=9 and $\sigma_b \geq 2.0$ for n=12. From table 2, we see that, power of S_2, PS_2 and R_2 control charts increases with increasing values of n.

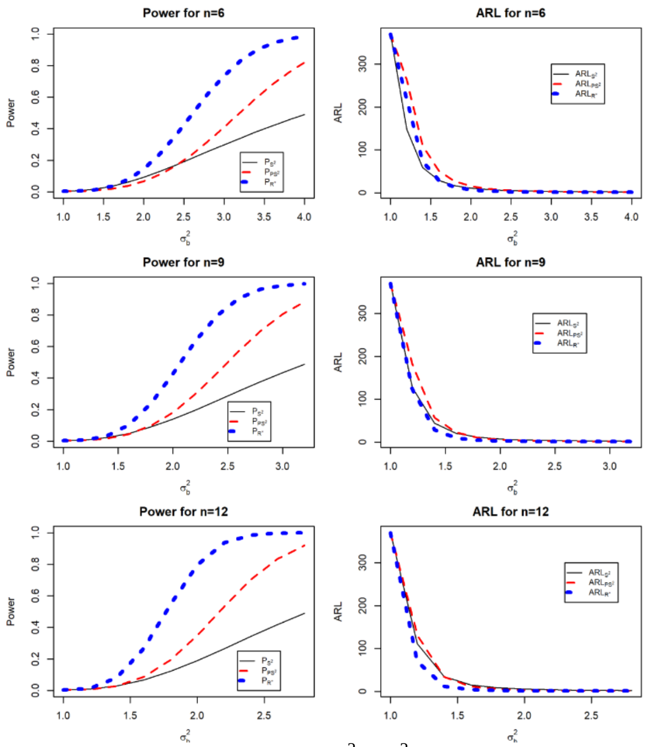


Figure 3: Power and ARL of S^2, PS^2 and R* control charts

Figure 3 makes it tangible that, $P_{S^2}, P_{PS^2}, P_{R^*}$ increase and $ARL_{S^2}, ARL_{PS^2}, ARL_{R^*}$ decrease as σ_b^2 increases. Table 3 reflects that $P_{R^*} > P_{PS^2}, ARL_{R^*} < ARL_{PS^2}$ for all values of σ_b^2, n, $P_{R^*} > P_{S^2}$, $ARL_{R^*} < ARL_{S^2}$ when $\sigma_b^2 \geq 1.8$ for n=6 and for all values of σ_b^2 when n=9 and n=12. As n increases, power of S^2, PS^2 and R^* control charts increases, ARL and SDRL decrease with increasing σ_b^2.

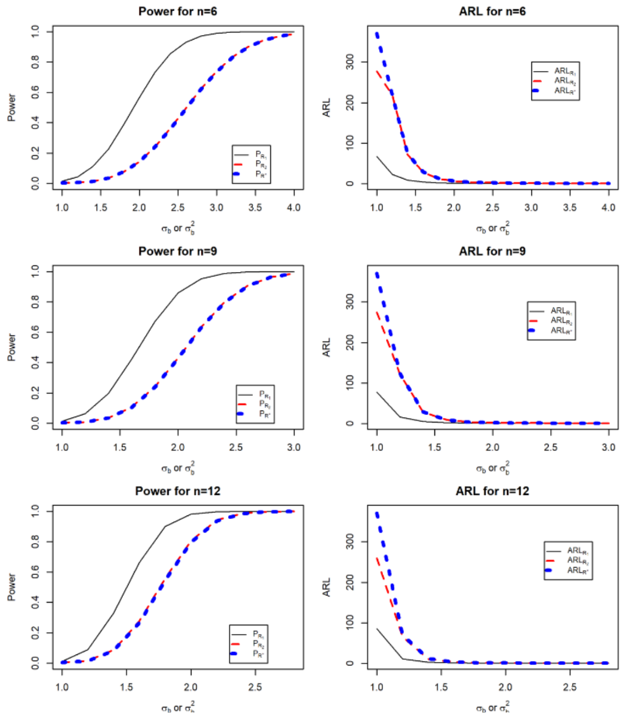


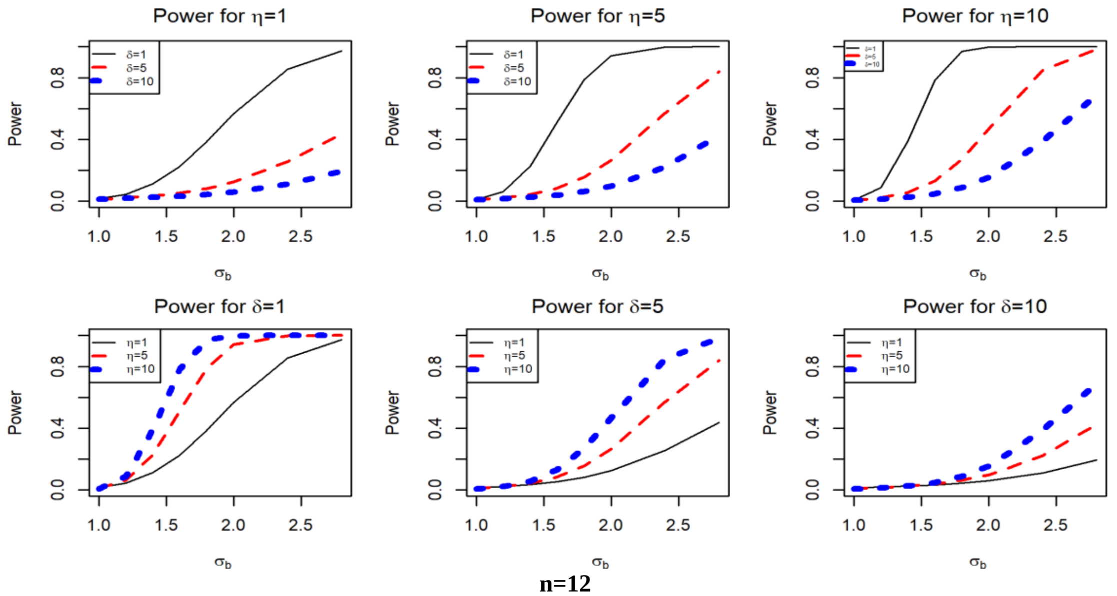
Figure 4: Power and ARL of R_1, R_2 and R* control charts

In figure 4, we compare the proposed R_1, R_2 and R^* control charts for variation. From figure 4 and tables 1, 2 and 3, we observe that, P_{R_1} is higher than P_{R_2}, P_{R^*} and P_{R_2} is slightly higher than P_{R^*}.

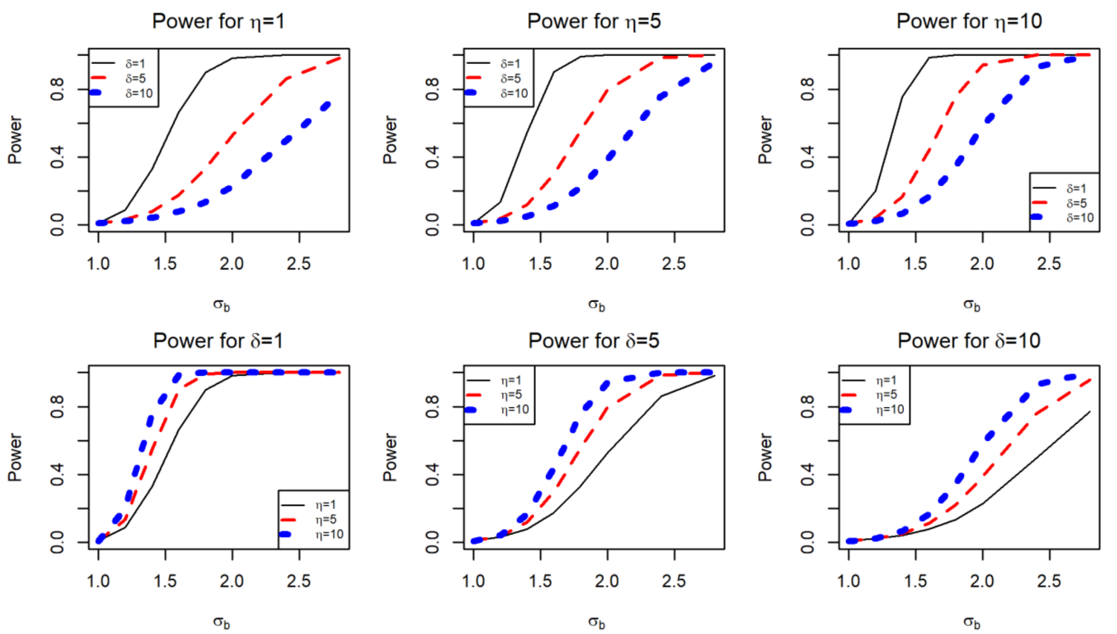
Table 4 shows that, for no shift in σ or σ^2, for increasing values of n, CVRL of S_1, PS_1, R_1 increase, S_2, PS_2, R_2 decrease and S^2, PS^2, R^* remain the same. It is observed that, $CVRL_{R_1}$ is lesser than $CVRL_{PS_1}$ and $CVRL_{S_1}$ for n=6, $\sigma_b > 1.8$ and n=9, 12, $\sigma_b \geq 1.2$. $CVRL_{R_2}$ is lesser than $CVRL_{PS_2}$ and $CVRL_{S_2}$ for n=6, $\sigma_b \geq 3.8$, n=9, $\sigma_b \geq 2.8$ and n=12, $\sigma_b \geq 2.0$. And $CVRL_{R^*}$ is lesser than $CVRL_{PS^2}$ and $CVRL_{S^2}$ for n=6, $\sigma_b \geq 1.8$ and for n=9, 12, all values of σ_b.

From table 5, we observe that, the PCR of all the control charts under consideration decrease as the width of the specification limits decrease and increases as n increases. We also see that $PCR_{R_1} > PCR_{S_1} > PCR_{PS_1}$, $PCR_{R_2} > PCR_{S_2} > PCR_{PS_2}$ for n=6, 9, 12, $PCR_{R^*} > PCR_{S^2} > PCR_{PS^2}$ when n=9, 12, 15 and $PCR_{R_1} > PCR_{R_2} > PCR_{R^*}$ for n=6, 9, 12. R-posterior control charts use less specification band than their competitors and R_1 control chart uses lesser specification band than R_2 and R^* control charts.

n=6

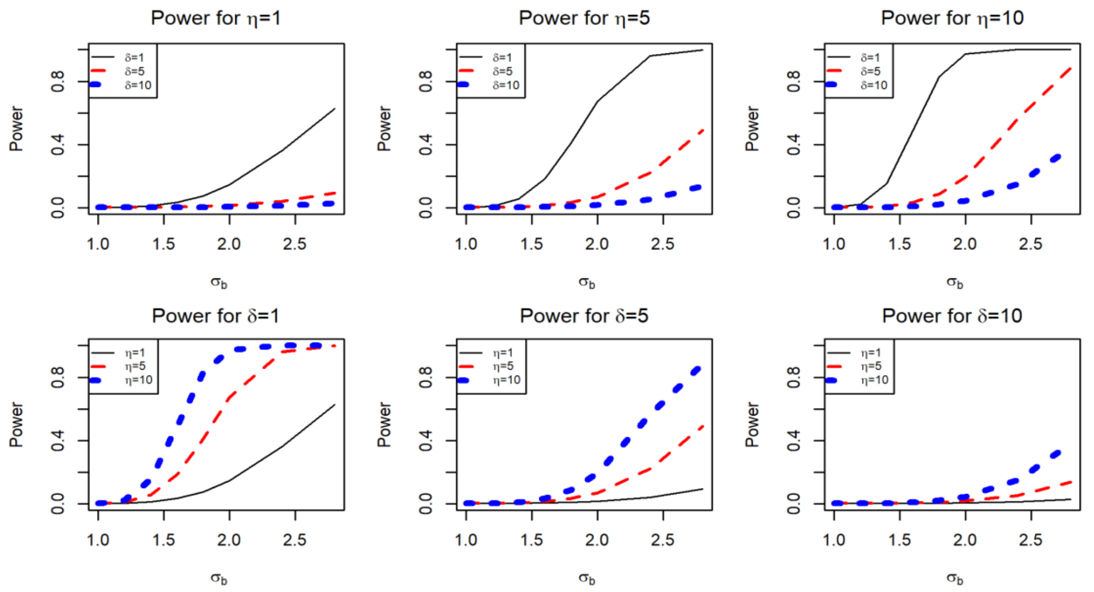


n=12

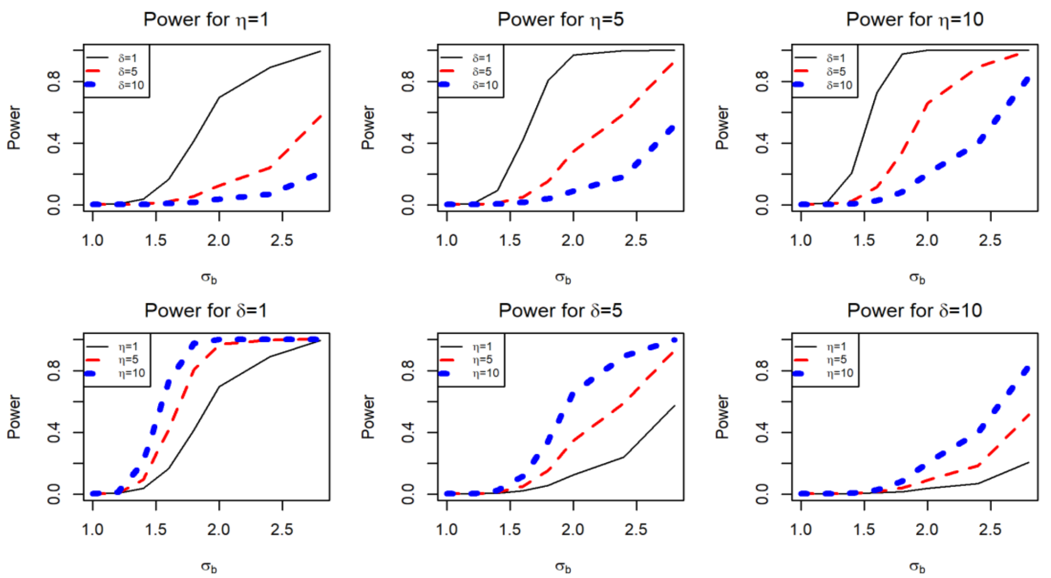


(a): Power of R_1 control chart

n=6

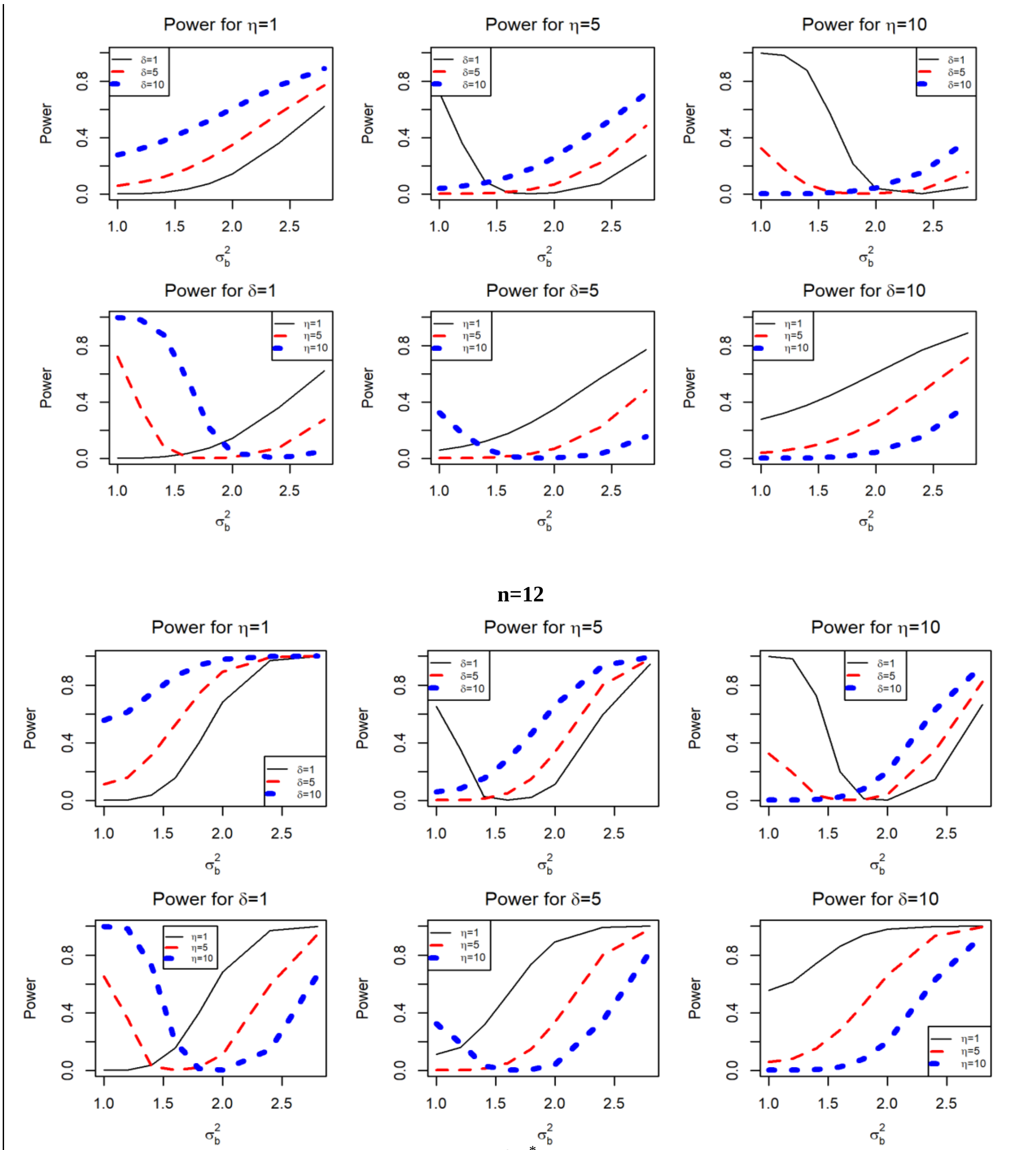


n=12



(b): Power of R_2 control chart

n=6



(c): Power of R* control chart

Figure 5: Power of R-posterior control charts for various values of n, η and δ

From figure 5, table 1, 2, 3 and 6, it is found that, P_{R_1} and P_{R_2} are higher for lower values of δ when η is fixed and higher values of η when δ is fixed. The power increases for lower shifts as n increases. Also, P_{R^*} decreases up to certain shift and then increases when $\eta = 5, 10,\ \delta = 1$ and $\delta = 1, 5,\ \eta = 10$ for n=6, 12.

V. ILLUSTRATION

In this section, we provide an example to illustrate the proposed control charts along with its competitors. We consider partial data pertaining to piston ring of automobile engine given in Montgomery (1996) with n=10 sample subgroups each of size 5. The data and necessary computation are given in exhibit 3.

Exhibit 3: Inside diameter (in mm) of automobile piston ring

n	Sample observations					r_i	r_i^2	s_i	s_i^2
1	74.030	74.002	74.019	73.992	74.008	0.038	0.001444	0.013212	0.000174
2	73.995	73.992	74.001	74.011	74.004	0.019	0.000361	0.006711	0.000045
3	73.988	74.024	74.021	74.005	74.002	0.036	0.001296	0.013191	0.000174
4	74.002	73.996	73.993	74.015	74.009	0.022	0.000484	0.008124	0.000066
5	73.992	74.007	74.015	73.989	74.014	0.026	0.000676	0.010929	0.000119
6	74.009	73.994	73.997	73.985	73.993	0.024	0.000576	0.007787	0.000060
7	73.995	74.006	73.994	74.000	74.005	0.012	0.000144	0.00494	0.000024
8	73.985	74.003	73.993	74.015	73.988	0.030	0.000900	0.010962	0.000120
9	74.008	73.995	74.009	74.005	74.004	0.014	0.000196	0.004956	0.000024
10	73.998	74.000	73.990	74.007	73.995	0.017	0.000289	0.005621	0.000031

Here, μ is unknown and we assume that the hyper parameters η=0.001 and δ=0.001. The control charts S_1, PS_1 and R_1 are illustrated in figure 6, S_2, PS_2 and R_2 control charts in figure 7, S^2, PS^2 and R^* control charts in figure 8 and R_1, R_2 and R^* control charts in figure 9. The limits of the control charts are obtained using exhibit 1 and exhibit 2.

Exhibit 4: Computed limits of various control charts

Charts→ Control Limits ↓	**S_1**	**PS_1**	**R_1**
UCL	0.018055	0.034214	0.033344
CL	0.008643	0.018853	0.018373
LCL	0.000000	0.003492	0.003403
Charts→ Probability Limits ↓	**S_2**	**PS_2**	**R_2**
UPL	0.014664	0.046388	0.045207
CPL	0.008354	0.017895	0.017440
LPL	0.003440	0.009931	0.009678
Charts→ Probability Limits ↓	**S^2**	**PS^2**	**R^***
UPL	0.026019	0.002152	0.002044
CPL	0.008012	0.000320	0.000304
LPL	0.001192	0.000099	0.000094

From exhibit 4, we plot the control charts in figure 6, 7, 8 and 9. The process variables are plotted against sample number.

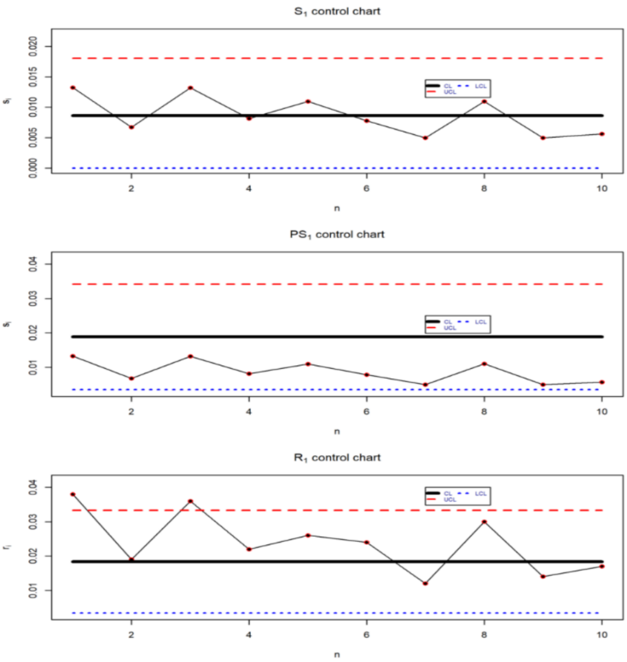


Figure 6: S_1, PS_1 and R_1 Control charts

From figure 6, it is observed that, sample points 1 and 3 in R_1 control chart fall outside the upper control limit, whereas, all sample points in S_1 and PS_1 control charts lie within control limits.

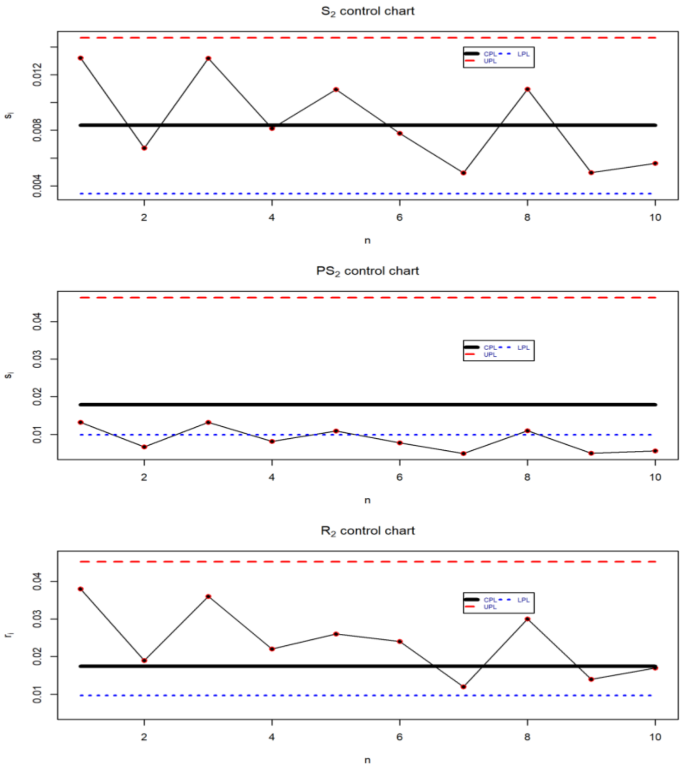


Figure 7: S_2, PS_2 and R_2 Control charts

From figure 7, it is seen that, all sample points in S_2 and R_2 control charts fall within probability limits and sample points 2, 4, 6, 7, 9, 10 fall below LPL in PS_2 control chart.

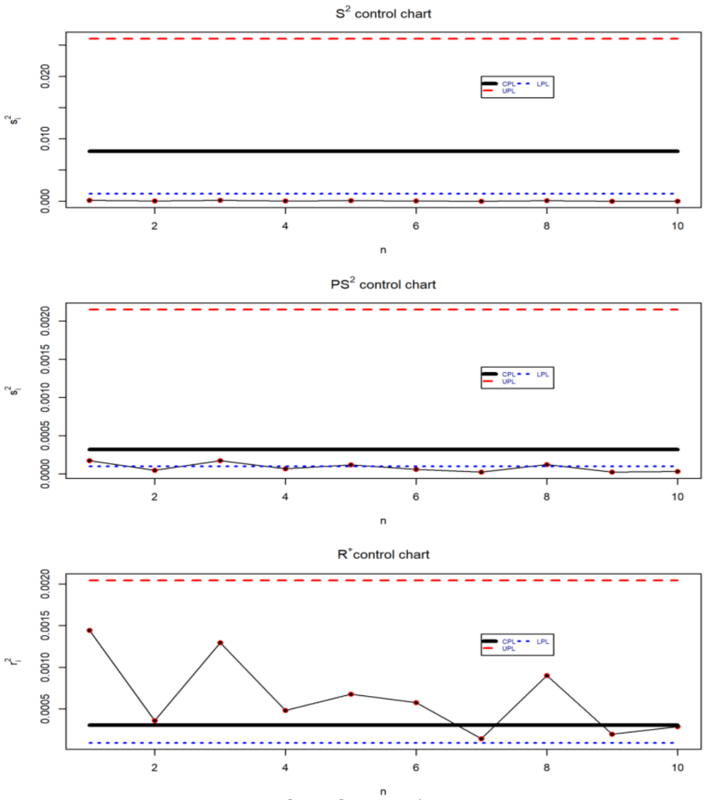


Figure 8: S^2, PS^2 and R^* Control charts

From figure 8, it is seen that, sample points fall within probability limits in R^* control chart, all sample points fall below LPL in S^2 control chart and sample points 2, 4, 6, 7, 9, 10 fall below LPL in PS^2 control chart.

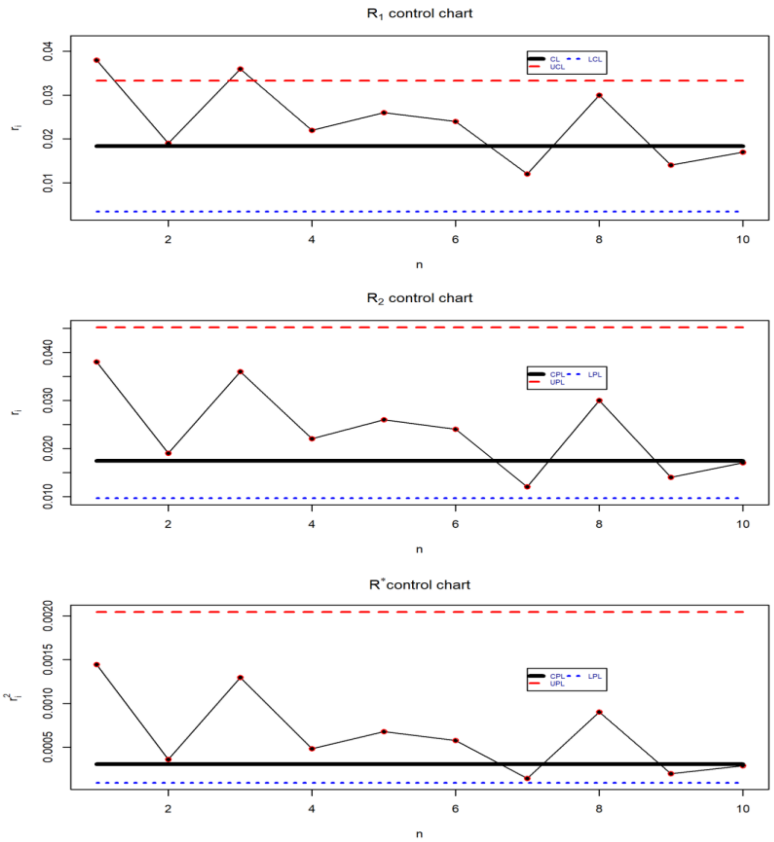


Figure 9: R_1, R_2 and R^* Control charts

From figure 9, it is noted that, all sample points corresponding to R_2 and R^* control charts fall within probability limits and sample points 1, 3 in R_1 control chart fall outside UCL.

VI. CONCLUSIONS

In this section, we furnish concluding remarks about the R-posterior control charts for process variation.

- R_1 and R_2 posterior control charts respectively based on control limits and probability limits for process standard deviation and R^* posterior control chart for process variance are proposed.
- The proposed control charts are established with the assumptions that, the underlying distribution is normal distribution and variance follows inverted gamma distribution which is a conjugate prior.
- R_1 control chart outperforms S_1, PS_1 control charts. R_2 control chart excels PS_2 control chart for copious shifts in σ and performs better than S_2 control chart for relatively larger shifts in σ.
- R^* control chart is better than S^2 and PS^2 control charts in terms of all the performance measures for smaller shift in σ^2 and smaller values of n.
- Among R_1, R_2 and R^*, it is found that, R_1 control chart is superior to other two in terms of performance measures.
- When variance follows inverted gamma prior with shape parameter η and scale parameter δ, power of R_1 and R_2 control charts is higher for lower values of δ when η is fixed and higher values of η when δ is fixed.
- Using lesser specification limits, R-posterior control charts, yield higher PCR than Shewhart's control charts and posterior control charts based on standard deviation and variance with R_1 control chart having the highest PCR among them.
- We suggest to use R_1 control chart for process standard deviation and R^* control chart for process variance whenever the process stimulates fewer samples.

VII. APPENDIX

Table 1: Power, ARL and SDRL of S_1, PS_1 and R_1 control charts for various values of n

n	σ_b	Power			ARL			SDRL		
		S_1	PS_1	R_1	S_1	PS_1	R_1	S_1	PS_1	R_1
6	1.0	0.003240	0.014610	0.014870	308.641975	68.446270	67.249496	308.141570	67.944430	66.747623
	1.2	0.035040	0.035910	0.043890	28.538813	27.847396	22.784233	28.034354	27.342825	22.278623
	1.4	0.126430	0.078010	0.111380	7.909515	12.818869	8.978273	7.392626	12.308718	8.463516
	1.6	0.267690	0.144080	0.226120	3.735664	6.940589	4.422431	3.196799	6.421151	3.890432
	1.8	0.421550	0.237060	0.388010	2.372198	4.218341	2.577253	1.804196	3.684571	2.016180
	2.0	0.559980	0.353800	0.566370	1.785778	2.826456	1.765630	1.184578	2.272091	1.162678
	2.2	0.670880	0.482600	0.731840	1.490580	2.072109	1.366419	0.855130	1.490479	0.707589
	2.4	0.7s60510	0.609460	0.855970	1.314907	1.640797	1.168265	0.643485	1.025386	0.443372
	2.6	0.822160	0.720300	0.930110	1.216308	1.388310	1.075142	0.512930	0.734231	0.284232
	2.8	0.870920	0.817790	0.972230	1.148211	1.222808	1.028563	0.412526	0.521969	0.171403
	3.0	0.904140	0.884600	0.989920	1.106023	1.130454	1.010183	0.342439	0.384022	0.101422
	3.2	0.927730	0.933210	0.996670	1.077900	1.071570	1.003341	0.289773	0.276934	0.057899
	3.4	0.945950	0.963660	0.999170	1.057138	1.037710	1.000831	0.245770	0.197819	0.028834
	3.6	0.959120	0.981880	0.999720	1.042622	1.018454	1.000280	0.210806	0.137095	0.016738
	3.8	0.968880	0.991500	0.999960	1.032120	1.008573	1.000040	0.182075	0.092986	0.006325
	4.0	0.975840	0.995970	0.999990	1.024758	1.004046	1.000010	0.159283	0.063739	0.003162
9	1.0	0.003050	0.012340	0.012890	327.868853	81.037277	77.579519	327.368471	80.535725	77.077897
	1.2	0.044150	0.042790	0.062330	22.650057	23.369946	16.043639	22.144413	22.864480	15.535595
	1.4	0.174220	0.112230	0.196970	5.739869	8.910274	5.076915	5.215959	8.395398	4.549522
	1.6	0.369200	0.231180	0.422320	2.708559	4.325634	2.367873	2.151217	3.792819	1.799708
	1.8	0.563710	0.392580	0.670840	1.773962	2.547252	1.490668	1.171742	1.985255	0.855233
	2.0	0.716000	0.568710	0.859110	1.396648	1.758365	1.163995	0.744297	1.154766	0.436909
	2.2	0.820520	0.731310	0.952990	1.218739	1.367409	1.049329	0.516320	0.708801	0.227513

	2.4	0.889610	0.852080	0.988240	1.124088	1.173599	1.011900	0.373478	0.451370	0.109734
	2.6	0.930750	0.928830	0.997820	1.074402	1.076623	1.002185	0.282733	0.287218	0.046792
	2.8	0.956670	0.970260	0.999700	1.045293	1.030652	1.000300	0.217587	0.177739	0.017326
	3.0	0.972360	0.989250	0.999990	1.028426	1.010867	1.000010	0.170979	0.104809	0.003162
12	1.0	0.002920	0.011490	0.011600	342.465753	87.032202	86.206897	341.965388	86.530757	85.705438
	1.2	0.052440	0.052040	0.088920	19.069413	19.215988	11.246064	18.562680	18.709308	10.734425
	1.4	0.222220	0.156980	0.329300	4.500045	6.370238	3.036745	3.968672	5.848906	2.486981
	1.6	0.463120	0.335610	0.664260	2.159268	2.979649	1.505435	1.582141	2.428716	0.872295
	1.8	0.677220	0.556280	0.899720	1.476625	1.797656	1.111457	0.838926	1.197460	0.351965
	2.0	0.821310	0.755360	0.981810	1.217567	1.323872	1.018527	0.514687	0.654802	0.137369
	2.2	0.904580	0.890370	0.998050	1.105485	1.123129	1.001954	0.341486	0.371873	0.044245
	2.4	0.950860	0.959330	0.999900	1.051680	1.042394	1.000100	0.233132	0.210218	0.010001
	2.6	0.974210	0.988130	0.999990	1.026473	1.012013	1.000010	0.164844	0.110258	0.003162
	2.8	0.985840	0.997250	1.000000	1.014363	1.002758	1.000000	0.120705	0.052585	0.000000

Table 2: Power, ARL and SDRL of S_2, PS_2 and R_2 control charts for various values of n

n	σ_b	Power			ARL			SDRL		
		S_2	PS_2	R_2	S_2	PS_2	R_2	S_2	PS_2	R_2
6	1.0	0.003065	0.003137	0.003604	326.264274	318.775901	277.469478	325.76389	318.27551	276.96903
	1.2	0.021126	0.003906	0.004592	47.335037	256.016385	217.770035	46.832368	255.5159	217.26946
	1.4	0.088623	0.009234	0.013927	11.283753	108.295430	71.802973	10.772155	107.79427	71.30122
	1.6	0.209446	0.019596	0.035728	4.774500	51.030823	27.989252	4.2451564	50.528349	27.484705
	1.8	0.354683	0.037662	0.076804	2.819419	26.551962	13.020155	2.2648851	26.047164	12.510167
	2.0	0.495159	0.067087	0.144697	2.019553	14.906018	6.910993	1.4349363	14.397338	6.3914658
	2.2	0.614391	0.109721	0.241614	1.627628	9.114026	4.138833	1.0107151	8.5995021	3.6043176
	2.4	0.711715	0.166651	0.363418	1.405057	6.000564	2.751652	0.7544056	5.477792	2.1954358
	2.6	0.784415	0.236944	0.494779	1.274835	4.220407	2.021104	0.5919205	3.6866549	1.4365788
	2.8	0.838714	0.320297	0.624957	1.192302	3.122102	1.600110	0.4788334	2.5739892	0.9799195
	3.0	0.879753	0.410402	0.739880	1.136683	2.436635	1.351571	0.3941635	1.8709774	0.6893275
	3.2	0.910072	0.503280	0.831416	1.098814	1.986966	1.202767	0.3295124	1.4003808	0.493844
	3.4	0.931393	0.594802	0.898336	1.073661	1.681232	1.113169	0.2812233	1.0701908	0.3549317
	3.6	0.947858	0.680804	0.942991	1.055010	1.468852	1.060456	0.2409076	0.8298634	0.2532003
	3.8	0.959935	0.756491	0.970095	1.041737	1.321893	1.030827	0.2085167	0.6523096	0.1782615
	4.0	0.968867	0.819901	0.985325	1.032133	1.219659	1.014894	0.1821153	0.5176	0.1229446
9	1.0	0.003126	0.003206	0.003641	319.897633	311.915159	274.649822	319.39724	311.41476	274.14937
	1.2	0.027711	0.005599	0.008177	36.086753	178.603322	122.294240	35.58324	178.10262	121.79321
	1.4	0.131235	0.017541	0.035049	7.619918	57.009293	28.531484	7.1023404	56.50708	28.027025
	1.6	0.308719	0.045083	0.106739	3.239192	22.181310	9.368647	2.6931711	21.675544	8.8545411
	1.8	0.502853	0.097790	0.241570	1.988653	10.225994	4.139587	1.4021722	9.7131338	3.6050787
	2.0	0.663789	0.181973	0.431373	1.506503	5.495321	2.318179	0.8735261	4.9702343	1.7480779
	2.2	0.782232	0.296463	0.632953	1.278393	3.373102	1.579896	0.5965701	2.8292607	0.9571707
	2.4	0.860877	0.431504	0.799010	1.161606	2.317476	1.251549	0.4332697	1.7473459	0.5610932
	2.6	0.911741	0.571363	0.907400	1.096803	1.750201	1.102050	0.3258427	1.145863	0.3353565
	2.8	0.944481	0.700469	0.964181	1.058783	1.427615	1.037150	0.2494753	0.7813254	0.19629
	3.0	0.964717	0.807233	0.988213	1.036573	1.238800	1.011928	0.1947076	0.543898	0.1098629
12	1.0	0.003082	0.003246	0.003850	324.464633	308.071473	259.740260	323.96425	307.57107	259.23978
	1.2	0.035379	0.007919	0.015409	28.265355	126.278571	64.897138	27.760853	125.77758	64.395197
	1.4	0.176908	0.030979	0.087138	5.652656	32.279932	11.476049	5.1283389	31.775998	10.964655
	1.6	0.407150	0.089337	0.277461	2.456097	11.193570	3.604110	1.8911151	10.681875	3.0635763
	1.8	0.627957	0.198469	0.559233	1.592466	5.038570	1.788163	0.9713297	4.5109445	1.1871668

	2.0	0.785590	0.358243	0.806471	1.272929	2.791401	1.239970	0.5894223	2.2361843	0.5454868
	2.2	0.882740	0.541814	0.941498	1.132836	1.845652	1.062137	0.38792	1.2493113	0.2569011
	2.4	0.937058	0.711674	0.987473	1.067170	1.405138	1.012686	0.2677342	0.7545027	0.1133439
	2.6	0.966296	0.842709	0.998216	1.034880	1.186649	1.001787	0.1899899	0.4706243	0.0423129
	2.8	0.982022	0.925518	0.999816	1.018307	1.080476	1.000184	0.1365367	0.2948769	0.0135672

Table 3: Power, ARL and SDRL of S^2, PS^2 and R^* control charts for various values of n

n	σ_b^2	Power			ARL			SDRL		
		S^2	PS^2	R*	S^2	PS^2	R*	S^2	PS^2	R*
6	1.0	0.002700	0.002700	0.002700	370.370370	370.370370	370.370370	369.870032	369.870032	369.870032
	1.2	0.006751	0.003790	0.004603	148.125339	263.849559	217.234185	147.625357	263.351768	216.749043
	1.4	0.017040	0.009092	0.013692	58.685936	109.981959	73.035968	58.183298	109.485660	72.533626
	1.6	0.034957	0.019588	0.034863	28.606955	51.051074	28.684068	28.102126	50.549192	28.179269
	1.8	0.060516	0.037989	0.075948	16.524627	26.323215	13.166841	16.016753	25.818568	12.657032
	2.0	0.092676	0.067185	0.143650	10.790259	14.884358	6.961377	10.278126	14.375582	6.441990
	2.2	0.129875	0.109569	0.239920	7.699714	9.126636	4.168052	7.182328	8.612167	3.633818
	2.4	0.170453	0.166376	0.359713	5.866723	6.010493	2.779994	5.343377	5.487751	2.224494
	2.6	0.212903	0.237173	0.491661	4.696967	4.216326	2.033921	4.167085	3.682543	1.450143
	2.8	0.255980	0.319711	0.621523	3.906549	3.127829	1.608952	3.369661	2.579818	0.989834
	3.0	0.298720	0.410161	0.736594	3.347614	2.438066	1.357599	2.803376	1.872459	0.696762
	3.2	0.340418	0.503716	0.828974	2.937568	1.985247	1.206311	2.385733	1.398554	0.498873
	3.4	0.380586	0.595356	0.896507	2.627530	1.679668	1.115440	2.067939	1.068464	0.358841
	3.6	0.418910	0.680611	0.941656	2.387149	1.469269	1.061959	1.819705	0.830350	0.256511
	3.8	0.455209	0.756125	0.969357	2.196794	1.322533	1.031611	1.621452	0.653116	0.180585
	4.0	0.489400	0.819939	0.985002	2.043318	1.219602	1.015226	1.460079	0.517521	0.124331
9	1.0	0.002700	0.002700	0.002700	370.370370	370.370370	370.370370	369.870032	369.870032	369.870032
	1.2	0.007864	0.005502	0.008118	127.164049	181.754822	123.189930	126.660763	181.251401	122.682031
	1.4	0.022702	0.017448	0.034473	44.049710	57.314212	29.008049	43.546112	56.810959	28.503824
	1.6	0.049969	0.045094	0.105009	20.012503	22.175803	9.522967	19.506000	21.670132	9.009129
	1.8	0.089644	0.097545	0.239353	11.155196	10.251710	4.177937	10.643498	9.738852	3.643785
	2.0	0.139470	0.181156	0.428018	7.169986	5.520105	2.336352	6.651234	4.995142	1.766970
	2.2	0.196176	0.295191	0.629214	5.097465	3.387633	1.589284	4.570194	2.844020	0.967750
	2.4	0.256466	0.430173	0.796007	3.899146	2.324644	1.256271	3.362177	1.754803	0.567402
	2.6	0.317557	0.570427	0.905496	3.149037	1.753071	1.104367	2.601426	1.148996	0.339499
	2.8	0.377354	0.699531	0.963243	2.650032	1.429529	1.038159	2.091085	0.783597	0.199037
	3.0	0.434427	0.805599	0.988000	2.301883	1.241312	1.012146	1.731121	0.547306	0.110875
12	1.0	0.002700	0.002700	0.002700	370.370370	370.370370	370.370370	369.870032	369.870032	369.870032
	1.2	0.008997	0.007605	0.014433	111.150039	131.489960	69.284196	110.647031	130.991485	68.783848
	1.4	0.028765	0.029732	0.082171	34.764110	33.634189	12.169677	34.260822	33.130022	11.659027
	1.6	0.066385	0.085963	0.265952	15.063680	11.632961	3.760075	14.555058	11.121678	3.221506
	1.8	0.121441	0.192788	0.544857	8.234430	5.187054	1.835343	7.718273	4.660299	1.238202
	2.0	0.189683	0.349629	0.795742	5.271944	2.860177	1.256689	4.745687	2.306605	0.567959
	2.2	0.265404	0.532142	0.935767	3.767834	1.879198	1.068642	3.229363	1.285374	0.270839
	2.4	0.343278	0.703814	0.985988	2.913089	1.420829	1.014211	2.360722	0.773258	0.120054
	2.6	0.419216	0.836370	0.997879	2.385405	1.195642	1.002126	1.817898	0.483652	0.046152
	2.8	0.490518	0.921437	0.999776	2.038663	1.085261	1.000224	1.455156	0.304189	0.014970

Table 4: CVRL of various control charts for different values of n

n	σ_b	S_1	PS_1	R_1	S_2	PS_2	R_2	σ_b^2	S^2	PS^2	R*
6	1.0	0.998379	0.992668	0.992537	0.998466	0.998430	0.998196	1.0	0.998649	0.998649	0.998649
	1.2	0.982324	0.981881	0.977809	0.989381	0.998045	0.997701	1.2	0.996625	0.998113	0.997767

	1.4	0.934650	0.960203	0.942666	0.954661	0.995372	0.993012	1.4	0.991435	0.995487	0.993122
	1.6	0.855751	0.925159	0.879704	0.889131	0.990154	0.981974	1.6	0.982353	0.990169	0.982401
	1.8	0.760559	0.873464	0.782298	0.803316	0.980988	0.960831	1.8	0.969266	0.980829	0.961281
	2.0	0.663340	0.803866	0.658506	0.710522	0.965874	0.924826	2.0	0.952537	0.965818	0.925390
	2.2	0.573689	0.719305	0.517842	0.620974	0.943546	0.870854	2.2	0.932805	0.943630	0.871826
	2.4	0.489377	0.624932	0.379513	0.536922	0.912880	0.797861	2.4	0.910794	0.913028	0.800179
	2.6	0.421711	0.528867	0.264367	0.464311	0.873531	0.710789	2.6	0.887186	0.873401	0.712979
	2.8	0.359277	0.426861	0.166643	0.401604	0.824441	0.612408	2.8	0.862567	0.824795	0.615204
	3.0	0.309613	0.339706	0.100400	0.346766	0.767853	0.510020	3.0	0.837425	0.768010	0.513231
	3.2	0.268831	0.258438	0.057706	0.299880	0.704784	0.410590	3.2	0.812146	0.704474	0.413553
	3.4	0.232486	0.190630	0.028810	0.261929	0.636552	0.318848	3.4	0.787028	0.636116	0.321704
	3.6	0.202188	0.134611	0.016733	0.228346	0.564974	0.238766	3.6	0.762292	0.565145	0.241545
	3.8	0.176409	0.092196	0.006325	0.200162	0.493466	0.172931	3.8	0.738099	0.493837	0.175051
	4.0	0.155435	0.063482	0.003162	0.176445	0.424381	0.121140	4.0	0.714563	0.424336	0.122466
	1.0	0.998474	0.993811	0.993534	0.998436	0.998396	0.998178	1.0	0.998649	0.998649	0.998649
	1.2	0.977676	0.978371	0.968334	0.986047	0.997197	0.995903	1.2	0.996042	0.997230	0.995877
	1.4	0.908724	0.942215	0.896119	0.932076	0.991191	0.982319	1.4	0.988568	0.991219	0.982618
	1.6	0.794229	0.876824	0.760053	0.831433	0.977199	0.945125	1.6	0.974691	0.977197	0.946042
	1.8	0.660523	0.779371	0.573725	0.705087	0.949847	0.870879	1.8	0.954129	0.949973	0.872149
	2.0	0.532917	0.656727	0.375353	0.579837	0.904448	0.754074	2.0	0.927650	0.904900	0.756294
9	2.2	0.423651	0.518353	0.216818	0.466656	0.838771	0.605844	2.2	0.896562	0.839530	0.608922
	2.4	0.332250	0.384603	0.108444	0.372992	0.753987	0.448319	2.4	0.862285	0.754870	0.451656
	2.6	0.263154	0.266777	0.046690	0.297084	0.654704	0.304302	2.6	0.826102	0.655419	0.307415
	2.8	0.208159	0.172453	0.017321	0.235625	0.547294	0.189259	2.8	0.789079	0.548150	0.191721
	3.0	0.166253	0.103682	0.003162	0.187838	0.439052	0.108568	3.0	0.752046	0.440909	0.109544
	1.0	0.998539	0.994238	0.994183	0.998458	0.998376	0.998073	1.0	0.998649	0.998649	0.998649
	1.2	0.973427	0.973632	0.954505	0.982151	0.996033	0.992266	1.2	0.995475	0.996209	0.992778
	1.4	0.881918	0.918161	0.818963	0.907244	0.984389	0.955438	1.4	0.985523	0.985010	0.958039
	1.6	0.732721	0.815101	0.579431	0.769968	0.954287	0.850023	1.6	0.966235	0.956049	0.856766
12	1.8	0.568137	0.666123	0.316670	0.609953	0.895283	0.663903	1.8	0.937317	0.898448	0.674643
	2.0	0.422718	0.494611	0.134870	0.463044	0.801097	0.439919	2.0	0.900178	0.806455	0.451949
	2.2	0.308902	0.331104	0.044159	0.342432	0.676894	0.241872	2.2	0.857087	0.684001	0.253442
	2.4	0.221676	0.201668	0.010000	0.250882	0.536960	0.111924	2.4	0.810384	0.544230	0.118372
	2.6	0.160593	0.108949	0.003162	0.183586	0.396599	0.042237	2.6	0.762092	0.404512	0.046054
	2.8	0.118996	0.052440	0.000000	0.134082	0.272914	0.013565	2.8	0.713780	0.280291	0.014967

Table 5: PCR of various control charts for different values of n

n	**Control Charts**	**Chart Limits**		**USL**	**1.6**	**1.5**	**1.4**	**1.3**	**1.2**
				LSL	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**
		UCL	**LCL**	**Width**	**PCR**				
6	$\mathbf{S_1}$	1.8058	0.1129	1.6929	0.8860	0.7679	0.6498	0.5316	0.4135
	$\mathbf{PS_1}$	2.0852	0.1304	1.9548	0.7673	0.6650	0.5627	0.4604	0.3581
	$\mathbf{R_1}$	1.4376	0.0899	1.3477	4.9547	4.2941	3.6335	2.9728	2.3122
9	$\mathbf{S_1}$	1.6694	0.2759	1.3935	1.0765	0.9330	0.7894	0.6459	0.5024
	$\mathbf{PS_1}$	1.8456	0.3051	1.5405	0.9737	0.8439	0.7141	0.5842	0.4544
	$\mathbf{R_1}$	1.2431	0.2055	1.0376	8.3592	7.2447	6.1301	5.0156	3.9009

12	**S_1**	1.5851	0.3737	1.2114	1.2382	1.0731	0.9080	0.7429	0.5778
	PS_1	1.7121	0.4036	1.3085	1.1464	0.9935	0.8407	0.6878	0.5349
	R_1	1.0548	0.2487	0.8061	13.8498	12.0031	10.1565	8.3098	6.4632
n	**Control Charts**	**UPL**	**LPL**	**Width**	**PCR**				
6	**S_2**	1.9035	0.2656	1.6379	0.9158	0.7937	0.6716	0.5495	0.4274
	PS_2	2.9320	0.5616	2.3704	0.6328	0.5484	0.4641	0.3797	0.2953
	R_2	2.0215	0.3872	1.6343	0.9179	0.7955	0.6731	0.5507	0.4283
9	**S_2**	1.7350	0.3714	1.3636	1.1000	0.9533	0.8067	0.6600	0.5133
	PS_2	2.3724	0.6011	1.7713	0.8468	0.7339	0.6210	0.5081	0.3952
	R_2	1.5980	0.4049	1.1931	1.2572	1.0896	0.9220	0.7543	0.5867
12	**S_2**	1.6348	0.4425	1.1923	1.2581	1.0904	0.9226	0.7549	0.5871
	PS_2	2.0897	0.6302	1.4595	1.0278	0.8907	0.7537	0.6167	0.4796
	R_2	1.2874	0.3883	0.8991	1.6683	1.4458	1.2234	1.0010	0.7785
n	**Control Charts**	**UPL**	**LPL**	**Width**	**PCR**				
6	**S^2**	3.6232	0.0706	3.5526	0.4222	0.3659	0.3096	0.2533	0.1970
	PS^2	8.5967	0.3154	8.2813	0.1811	0.1570	0.1328	0.1087	0.0845
	R^*	4.0863	0.1499	3.9364	0.3811	0.3303	0.2794	0.2286	0.1778
9	**S^2**	3.0103	0.1379	2.8724	0.5222	0.4526	0.3830	0.3133	0.2437
	PS^2	5.6285	0.3613	5.2672	0.2848	0.2468	0.2088	0.1709	0.1329
	R^*	2.5536	0.1639	2.3897	0.6277	0.5440	0.4603	0.3766	0.2929
12	**S^2**	2.6725	0.1958	2.4767	0.6057	0.5249	0.4442	0.3634	0.2826
	PS^2	4.3668	0.3972	3.9696	0.3779	0.3275	0.2771	0.2267	0.1763
	R^*	1.6574	0.1507	1.5067	0.9956	0.8628	0.7301	0.5973	0.4646
15	**S^2**	2.4538	0.2442	2.2096	0.6788	0.5883	0.4978	0.4073	0.3168
	PS^2	3.6767	0.4264	3.2503	0.4615	0.4000	0.3384	0.2769	0.2154
	R^*	1.2214	0.1416	1.0798	1.3893	1.2040	1.0188	0.8336	0.6483

Table 6: Power of R_1, R_2 and R^* control charts for various values n, η and δ

Control Charts	n	σ_b	$\eta = 1$		$\eta = 5$			$\eta = 10$		
			$\delta = 5$	$\delta = 10$	$\delta = 1$	$\delta = 5$	$\delta = 10$	$\delta = 1$	$\delta = 5$	$\delta = 10$
R_1	**6**	1.0	0.01458	0.01461	0.01109	0.01047	0.01047	0.00848	0.00834	0.00795
		1.2	0.02178	0.01854	0.06360	0.02117	0.01594	0.08895	0.02202	0.01447
		1.4	0.03505	0.02478	0.22899	0.04293	0.02449	0.38924	0.05659	0.02571
		1.6	0.05511	0.03331	0.51520	0.08525	0.03945	0.78303	0.13224	0.04845
		1.8	0.08241	0.04341	0.78876	0.15605	0.06206	0.96834	0.27101	0.08750
		2.0	0.12659	0.06071	0.94120	0.26502	0.09779	0.99819	0.46789	0.15478
		2.4	0.25537	0.10899	0.99880	0.56998	0.22316	1.00000	0.84691	0.38591
		2.8	0.43743	0.19241	1.00000	0.83969	0.42546	1.00000	0.98371	0.68933
	9	1.0	0.01246	0.01252	0.00996	0.00968	0.00997	0.00942	0.00780	0.00836
		1.2	0.02606	0.02006	0.09044	0.02785	0.01923	0.15061	0.03145	0.01990
		1.4	0.05323	0.03151	0.36043	0.07363	0.03681	0.60064	0.10558	0.04641
		1.6	0.10291	0.05153	0.72425	0.17117	0.07141	0.93630	0.27579	0.10308
		1.8	0.18119	0.08056	0.93917	0.33482	0.12909	0.99736	0.53571	0.20701
		2.0	0.29345	0.12568	0.99317	0.54213	0.22279	0.99999	0.78506	0.36929
		2.4	0.57858	0.26640	0.99998	0.88333	0.50062	1.00000	0.98660	0.75001
		2.8	0.82594	0.47057	1.00000	0.98857	0.78859	1.00000	0.99987	0.95630
	12	1.0	0.01141	0.01143	0.00954	0.00952	0.00912	0.00811	0.00762	0.00814
		1.2	0.03070	0.02137	0.13594	0.03665	0.02184	0.20088	0.04121	0.02374
		1.4	0.07928	0.04112	0.54524	0.11911	0.05047	0.75528	0.16707	0.06666
		1.6	0.17614	0.07711	0.90164	0.29892	0.11215	0.98590	0.44064	0.16597
		1.8	0.33298	0.13621	0.99316	0.55627	0.22205	0.99987	0.75831	0.34600
		2.0	0.52619	0.22845	0.99988	0.79522	0.38704	1.00000	0.94227	0.58349
		2.4	0.85992	0.49702	1.00000	0.98586	0.75909	1.00000	0.99956	0.92935
		2.8	0.98095	0.77142	1.00000	0.99980	0.95594	1.00000	1.00000	0.99698
R_2	**6**	1.0	0.00331	0.00313	0.00416	0.00326	0.00302	0.00501	0.00353	0.00310

		1.2	0.00293	0.00272	0.01111	0.00355	0.00301	0.02220	0.00451	0.00337
		1.4	0.00380	0.00280	0.05531	0.00703	0.00387	0.15402	0.01159	0.00516
		1.6	0.00649	0.00381	0.18337	0.01638	0.00655	0.48937	0.03488	0.01061
		1.8	0.01027	0.00476	0.41313	0.03510	0.01079	0.83021	0.08810	0.02125
		2.0	0.01720	0.00708	0.67285	0.06994	0.01863	0.97318	0.19531	0.04320
		2.4	0.04222	0.01323	0.95958	0.22333	0.05421	0.99997	0.56333	0.14998
		2.8	0.09493	0.02891	0.99877	0.48903	0.13718	1.00000	0.88410	0.38040
	9	1.0	0.00328	0.00285	0.00412	0.00330	0.00308	0.00511	0.00370	0.00338
		1.2	0.00319	0.00277	0.02051	0.00535	0.00361	0.04219	0.00731	0.00461
		1.4	0.00656	0.00377	0.12429	0.01491	0.00640	0.30493	0.02887	0.01043
		1.6	0.01541	0.00704	0.38823	0.04420	0.01418	0.74684	0.10030	0.02799
		1.8	0.03256	0.01176	0.71348	0.10803	0.02985	0.96715	0.26079	0.06749
		2.0	0.06262	0.01971	0.91858	0.22653	0.05885	0.99854	0.50548	0.14851
		2.4	0.18468	0.05326	0.99857	0.59130	0.19156	1.00000	0.91152	0.45813
		2.8	0.39700	0.12691	1.00000	0.88656	0.44361	1.00000	0.99591	0.80930
	12	1.0	0.00319	0.00303	0.00400	0.00324	0.00291	0.00505	0.00397	0.00334
		1.2	0.00298	0.00273	0.00797	0.00346	0.00279	0.01189	0.00447	0.00316
		1.4	0.00725	0.00383	0.09562	0.01340	0.00547	0.20581	0.02603	0.00836
		1.6	0.02215	0.00855	0.41686	0.05134	0.01528	0.72832	0.11521	0.02796
		1.8	0.05736	0.01731	0.80486	0.15332	0.03969	0.97699	0.34438	0.08368
		2.0	0.12673	0.03671	0.96938	0.34575	0.09154	0.99960	0.65708	0.20173
		2.4	0.24127	0.07035	0.99818	0.58883	0.18586	1.00000	0.89063	0.39866
		2.8	0.57517	0.20715	1.00000	0.93108	0.51574	1.00000	0.99774	0.82604
Control Chart	**n**	σ_b^2	$\eta = 1$		$\eta = 5$			$\eta = 10$		
			$\delta = 5$	$\delta = 10$	$\delta = 1$	$\delta = 5$	$\delta = 10$	$\delta = 1$	$\delta = 5$	$\delta = 10$
R*	**6**	1.0	0.05883	0.27881	0.71920	0.00270	0.04111	0.99898	0.32503	0.00270
		1.2	0.08364	0.32101	0.35963	0.00312	0.05681	0.98372	0.17776	0.00288
		1.4	0.12250	0.37676	0.08908	0.00660	0.08243	0.87660	0.06841	0.00494
		1.6	0.17895	0.44491	0.00987	0.01516	0.12208	0.57062	0.01714	0.01012
		1.8	0.25487	0.52272	0.00274	0.03352	0.18005	0.21454	0.00343	0.02128
		2.0	0.34899	0.60584	0.00933	0.06894	0.25921	0.04066	0.00369	0.04356
		2.4	0.56846	0.76623	0.07397	0.22123	0.47436	0.00412	0.03182	0.15201
		2.8	0.77104	0.88781	0.27637	0.48233	0.71000	0.04952	0.15760	0.38034
	9	1.0	0.07313	0.37841	0.62574	0.00270	0.04666	0.99690	0.30539	0.00270
		1.2	0.12937	0.46573	0.17746	0.00444	0.07756	0.93382	0.11381	0.00361
		1.4	0.22372	0.57272	0.01459	0.01404	0.13271	0.60744	0.02233	0.00916
		1.6	0.35910	0.68730	0.00310	0.04119	0.22183	0.17409	0.00303	0.02458
		1.8	0.52208	0.79393	0.01630	0.10386	0.34872	0.01708	0.00585	0.06158
		2.0	0.68490	0.87943	0.06288	0.21955	0.50398	0.00282	0.02409	0.13721
		2.4	0.91079	0.97234	0.35179	0.58160	0.80201	0.06272	0.19141	0.43955
		2.8	0.98649	0.99645	0.76003	0.88096	0.95708	0.37769	0.58629	0.79345
	12	1.0	0.11239	0.55523	0.65339	0.00270	0.05895	0.99761	0.32540	0.00270
		1.2	0.16048	0.61496	0.35920	0.00316	0.08042	0.98138	0.19327	0.00287
		1.4	0.31140	0.74387	0.02949	0.01287	0.15350	0.72751	0.03557	0.00805
		1.6	0.52372	0.85948	0.00308	0.04976	0.28089	0.19932	0.00319	0.02693
		1.8	0.73884	0.93876	0.02401	0.14968	0.46199	0.01229	0.00767	0.08048
		2.0	0.89168	0.97944	0.11252	0.33861	0.66259	0.00457	0.04008	0.19915
		2.4	0.99275	0.99902	0.59166	0.79787	0.93368	0.14741	0.34555	0.62679
		2.8	0.99987	0.99999	0.94457	0.98091	0.99565	0.66301	0.82484	0.93621

REFERENCES

[1]. Berger, J.O. (1986). *Statistical Decision Theory*, Universal Book Stall, New Delhi.

[2]. Bhat, S. V. and Gokhale, K. D. (2014). Posterior Control Charts for Process Variance Based on Various Priors. *Journal of the Indian Society for probability and statistics, 15*, 52-66.

[3]. Bhat, S.V. and Gokhale, K. D. (2016). Posterior Control Chart for Standard Deviation based on Conjugate Prior. *Journal of Indian Statistical Association, 54*(1&2), 157-166.

[4]. Chen, G. (1998). The Run Length Distributions of the R, S and S2 Control Charts when is Estimated. *The Canadian Journal of Statistics, 26* (2), 311 322.

[5]. Colosimo, B. M. and Del Castillo, E. (2006). *Bayesian Process Monitoring Control and Optimization*. Chapman and Hall /CRC New York.

[6]. Girshick, M. A. and Rubin, H. (1952). A Bayesian approach to a quality control model. *Annals of Mathematical Statistics, 23*(1):114–125.

[7]. Huber, P. J. (1981). *Robust statistics*. John Wiley, New York.

[8]. Huang, H. J. and Chen, F. L. (2005). A Synthetic Control Chart for Monitoring Process Dispersion with Sample Standard Deviation. *Computers and Industrial Engineering, 49* (2), 221-240.

[9]. Luko, S. N. (1996). Concerning the estimators r/d2 and r/d2* in estimating variability in a normal universe. *Quality Engineering, 8*(3):481–487.

[10]. Menzefricke, U. (2007). Control Charts for the Generalised Variance Based on its Predictive Distribution. *Communications in Statistics - Theory and Methods, 36* (5), 1031-1038.

[11]. Montgomery, D. C. (1996). *Introduction to Statistical Quality Control (6th Edition ed.)*. John Wiley and Sons, New York.

[12]. Nelson, L. S. (1975). Use of the range to estimate variability. *Journal of Quality Technology, 7*(1):46–48.

[13]. Rajmanya, S. V. and Ghute, V. B. (2014). A Synthetic Control Chart for Monitoring Process Variability. *Quality and Reliability Engineering International*, 30, 1301 1309, Published online 23 October 2013 in Wiley Online Library. DOI: 10.1002/qre.1551.

[14]. Ryan, T. P. (2000). *Statistical Methods for Quality Improvement (2nd Edition ed.)*. John Wiley and Sons, New York.

[15]. Schoonhoven, M. and Does, R. J. M. M. (2012). A Robust Standard Deviation Control Chart. *Technometrics, 54* (1), 73-82.

[16]. Schoonhoven, M., Riaz, M. and Does, R. J. M. M. (2011). Design and Analysis of Control Charts for Standard Deviation with Estimated Parameters. *Journal of Quality Technology, 43* (4), 307-333.

[17]. Shahriari, H., Maddahi, A. and Shokouhi, A. H. (2009). A Robust Dispersion Control Chart Based on M-estimate. *Journal of Industrial and Systems Engineering, 2* (4), 297-307.

[18]. Tippett, L. H. C. (1925). On the extreme individuals and the range of samples taken from a normal population. *Biometrika, 17*(1):364–387.

[19]. Zhang, L., Bebbington, M. S., Lai, C. D. and Govindaraju, K. (2005). On Statistical Design of the S2 Control Chart. *Communications in Statistics-Theory and Methods, 34* (1), 229-244.

[20]. Zhang, L. and Govindaraju, K. (2007). On the Probability Limits for Phase II S Control Chart. METRON - *International Journal of Statistics, 65* (3), 305-318.