International Journal of Research in Engineering and Science (IJRES)
ISSN (Online): 2320-9364, ISSN (Print): 2320-9356
www.ijres.org Volume 9 Issue 5 ǁ 2021 ǁ PP. 55-57

Semi-Analytic Solution of Riccati Equation Using Homotopy Perturbation Method (HPM) and Adomian Decomposition Method (ADM).

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Abstract
This research work investigates the series solution of Riccati equation by employing Homotopy Perturbation Method (HPM) and Adomian Decomposition Method (ADM).
The ordinary differential equation was solved semi-analytically by employing HPM and ADM. This method was carefully employed and examined, the results show excellent agreement with existing literature.
Key words and Phrases: *Riccati, series solution, Homotopy Perturbation Method (HPM),Adomian Decomposition Method (ADM).*

Date of Submission: 16-05-2021 Date of acceptance: 31-05-2021

I. INTRODUCTION

There are several numerical methods to solve problem ranging from ADM,HAM,DTM,LTDA,HPM,VIM and among several others.Using the above listed semi-analytical methods may give similar results. In this article, we would consider the HPM and ADM to solve the below Riccati equation

$$\frac{dy(t)}{dt} = 2y(t) - y^2(t) + 1 \quad : \quad y(0) = 0 \qquad (1)$$

II. BASIC IDEA OF HPM AND ADM

The basic idea of the two semi-analytic method used in this article can be seen in [16] for Homotopy perturbation method (HPM) and also in [17] which demonstrate the basic idea of Adomian decomposition method can be seen in this article.

III. SOLUTION SCHEME

We shall show that both methods converge rapidly.

To use ADM for the Riccati equation. Consider the Riccati (1). Denoting $\frac{d}{dt}$ by L, we can say L^{-1} is an integral from 0 to t. Operating with L^{-1} and using the initial boundary condition in (1), we obtain

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$$y(t) = 2\int_0^t y(t)dt - \int_0^t y^2(t)dt + \int_0^t dt \qquad (2)$$

Applying ADM to (2), let $y(t) = \sum_{n=0}^{\infty} y_n(t)$ and $y^2(t) = \sum_{n=0}^{\infty} A_n(t)$, where $A_n(t)'s$ are the Adomian polynomials depending on $y_0(t), y_1(t), y_2(t) \cdots y_n(t)$ On substitution, (2) can be written as

$$\sum_{n=0}^{\infty} y_i(t) = 2\int_0^t \sum_{n=0}^{\infty} y_i(t)dt - \int_0^t \sum_{n=0}^{\infty} A_i(t)dt + t \qquad (3)$$

Based on the recursion scheme of ADM, we say

$$y_0(t) = t \cdots\cdots$$
$$y_{n+1}(t) = 2\int_0^t y_n(t)dt - \int_0^t A_n(t)\,dt \;for\, n \geq 0 \cdots\cdots$$

Note that the Adomian polynomials $A_n(t)'s$ for the quadratic non-linearity can be written as [13]

$$A_0(t) = y_0^2(t) \qquad (4)$$
$$A_1(t) = 2y_0(t)y_1(t) \qquad (5)$$
$$A_2(t) = y_1^2(t) + 2y_0(t)y_2(t) \qquad (6)$$
$$A_3(t) = 2y_1(t)y_2(t) + 2y_0(t)y_3(t)\cdots\cdots \qquad (7)$$

The solution components $y_n(t)$ from $y_{n+1}(t)$ can be calculated as $y_1(t) = 2\int_0^t y_0\,(t)dt - \int_0^t A_0\,(t)\,dt = 2\int_0^t t\,dt - \int_0^t y_0^2\,(t)\,dt$

$$y_1(t) = 2\frac{t^2}{2} - \int_0^t t^2\,dt = t^2 - \frac{t^3}{3}$$
$$y_2(t) = 2\int_0^t y_1\,(t)dt - \int_0^t A_1\,(t)\,dt$$
$$= 2\int_0^t \left(t^2 - -\frac{t^3}{3}\right)dt - \int_0^t 2\,y_0(t)y_1(t)\,dt$$
$$y_2(t) = \frac{2t^3}{3} - \frac{2t^4}{3} + \frac{2t^5}{15}$$
$$y_3(t) = 2\int_0^t y_2\,(t)dt - \int_0^t A_2\,(t)\,dt$$

where on simplification we have that

$$y_3(t) = \frac{t^4}{3} - \frac{t^5}{5} - \frac{288t^6}{1620} + \frac{357t^5}{6615}$$

We can get better approximations, which has been done formally and justified in [14,15].
Using HPM for the Riccati equation (1), We construct an homotopy $y(r,p)\colon \Omega \times [0,1] \to R$ which satisfies

$$(1-p)\left(\frac{d}{dt}y(t)\right) + p\left(\frac{d}{dt}y(t) - 2y(t) + \big(y(t)\big)^2 - 1\right) = 0 \qquad (8)$$

The solution for (8) is assumed in the form of a power series,

$$y = y_0(t) + y_1(t)p + y_2(t)p^2 + y_3(t)p^3 + \cdots \qquad (9)$$
$$Where \quad y(t) = \lim_{p\to 1} y = y_0(t) + y_1(t) + y_2(t) + y_3(t) + \cdots \qquad (10)$$

Substituting (9) in (8) we get

$$\frac{d}{dt}y_0(t) + \left[\big(y_0(t)\big)^2 - 2y_0(t) + \frac{d}{dt}y_1(t) - 1\right]p + \left[2y_0(t)y_1(t) - 2y_1(t) + \frac{d}{dt}y_2(t)\right]p^2 + \left[2y_0(t)y_2(t) + \big(y_1(t)\big)^2 - 2y_2(t) + \frac{d}{dt}y_3(t)\right]p^3 + \cdots$$

Then we have it that:

$$p^0\colon \frac{d}{dt}y_0(t)$$
$$p^1\colon \left[\big(y_0(t)\big)^2 - 2y_0(t) + \frac{d}{dt}y_1(t) - 1\right]$$
$$p^2\colon \left[2y_0(t)y_1(t) - 2y_1(t) + \frac{d}{dt}y_2(t)\right]$$
$$p^3\colon \left[2y_0(t)y_2(t) + \big(y_1(t)\big)^2 - 2y_2(t) + \frac{d}{dt}y_3(t)\right]$$

And so on.
Applying initial conditions

$$y_0(0) = 0, y_1(0) = 0, y_2(0) = 0, y_3(0) = 0, y_4(0) = 0$$
$$y_0(t) = 0 \qquad y_1(t) = t \qquad y_2(t) = t^2 \qquad y_3(t) = \frac{1}{3}t^3 \qquad y_4(t) = -\frac{t^4}{3}$$
$$y(t) = y_0(t) + y_1(t) + y_2(t) + y_3(t) + y_4(t)$$
$$y(t) = t + t^2 + \frac{1}{3}t^3 - \frac{t^4}{3} + \cdots$$

IV. CONCLUSION

In this article, we employed the powerful and efficient HPM and ADM for solving the Riccati equation. This study shows that the series solution $y(t)$ obtained for HPM is same as ADM, although the solution component $y_0(t)$,$y_1(t)$,$y_2(t)$ and $y_3(t)$ looks different.

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