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Introduce to New Kirchhoff-type Problem with Negative Modulus

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Abstract
This paper address some typical results of mine on the nonlocal new Kirchhoff-type problems with negative modulus in details.
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I. INTRODUCTION

Consider for *a*, *b*>0,the problem of the new Kirchhoff-type equation[1] as following:

$$-\left(a-b\int_{\Omega}|\nabla u|^2\,\mathrm{d}x\right)\Delta u=g(x,u),\quad x\in\Omega\subseteq\mathbf{R}^N(N\ge 1),\tag{P}$$

whereΩ is a bounded domain in $\mathbf{R}^N$ with smooth boundary ∂Ω or Ω=$\mathbf{R}^N$, $f:\Omega\times\mathbf{R}\to\mathbf{R}$ is a measurable function. We have been point out that this problem is the Kirchhoff-type problem with negative modulus when *a*,*b*>0 in [2,3] firstly based on the meaning of physics, and the same method may be use to solving the problem

$$-\left(b\int_{\Omega}|\nabla u|^2\,\mathrm{d}x-a\right)\Delta u=g(x,u),\quad x\in\Omega\subseteq\mathbf{R}^N(N\ge 1),$$

but we are not the first researcher on this kind of problems. Mentioned that Yin and Liu [4] studied this kind of problem(P) firstly, and included with Lei et al [5-7], all the reasons of research for the Eq. (P) that the nonlocal coefficient $\left(a-b\int_{\Omega}|\nabla u|^2\,\mathrm{d}x\right)$ isn't bounded below. We consider the problem (P) and explored their physical significance in [2-3], and we point out that this kind of problem (P) is the Kirchhoff-type problem with negative modulus in details [3]. We believe that the theoretical description in this thesis is helpful to the follow-up workers. Indeed, based on the pure research, this problem is related with Kirchhoff-type equation

$$-\left(a+b\int_{\Omega}|\nabla u|^2\,\mathrm{d}x\right)\Delta u=f(x,u),\ in\ \Omega,\tag{1.1}$$

where $a\ge 0, b>0,\Omega$ is a bounded domain in $\mathbf{R}^N$ with smooth boundary∂Ω or Ω=$\mathbf{R}^N$, $f:\Omega\times\mathbf{R}\to\mathbf{R}$ is a measurable function. Mentioned that Eq.(1.1) is related with the model

$$\varrho h\frac{\partial^2 u}{\partial t^2}-\left(p_0+\frac{Eh}{2L}\int_0^L\left|\frac{\partial u}{\partial x}\right|^2\mathrm{d}x\right)\frac{\partial^2 u}{\partial x^2}=f(x,u),\tag{1.2}$$

where *u*=*u*(*x*, *t*) is the lateral displacement, 0<x<*L*, the time $t\ge 0$, ϱ the mass density, *E* the Young's modulus, *h* the cross-section area, *L* the length, p_0 the initial axial tension. Eq.(1.2) named the Kirchhoff problem as an extension of classical D'Alembert's wave equations for free vibration of elastic strings by Kirchhoff[1] in 1876.

When finding the existence of stationary solution, Eq. (1.2) may be express as the Eq. (1.1) and therefore problem (1.1) was named the Kirchhoff-type problem. Eq.(1.1) has been studied by many researchers on whole space $\mathbf{R}^N$ and bounded domain with some boundary conditions, we omit it here.

II. BASIC KNOWLEDGE

Assume that a, b>0, $u^+=\max\{0,u\}$, $u^-=\min\{0,u\}$, $2^*=\infty$ with $N=1,2$, $2^*=2N/(N-2)$ with $N\ge 3$. $L^q(\Omega)(1\le q<+\infty):=\left\{u:\int_{\Omega}|u|^q\mathrm{d}x<+\infty\right\}$, L^q-norm $\|u\|_q=\left(\int_{\Omega}|u|^q\mathrm{d}x\right)^{1/q}$; $L^\infty(\Omega)$ is a space of essential bounded functions. The dual space of $L^q(\Omega)(1<q<+\infty)$ is $L^{\frac{q}{q-1}}(\Omega)$.

$$W^k(\Omega):=\left\{u:\Omega\to R|1\le|\alpha|\le k,\exists D_i^\alpha u,i=1,\cdots,N\right\},$$
$$W^{k,q}(\Omega):=\left\{u\in L_{loc}^k(\Omega)\,|\,1\le|\alpha|\le k,\exists D_i^\alpha u\in L^q(\Omega),i=1,\cdots,N\right\}.$$

$W_0^{k,q}(\Omega)$ is the closure for $C_0^k(\Omega)$ on $W^{k,q}(\Omega)$. $W^{k,q}(\Omega)$, $W_0^{k,q}(\Omega)$ and their duals.

$W^{k,\frac{q}{q-1}}(\Omega)$, $W_0^{k,\frac{q}{q-1}}(\Omega)$, are complete. $W^{k,q}(\Omega)$-norm $\|\cdot\|$, and the dual action with $W^{k,\frac{q}{q-1}}(\Omega)$,

$$\|u\|=\left[\int_\Omega\sum_{|\alpha|\le k}|D^\alpha u|^q\,\mathrm{d}x\right]^{1/q},\langle u,v\rangle=\int_\Omega\sum_{|\alpha|\le k-1}(D^\alpha u)^{q-1}\cdot\sum_{|\alpha|\le k-1}D^\alpha v\mathrm{d}x.$$

Case $q\ge 2$, this is an equivalence that the norm and dual with follows:

$$\|u\|=\left[\int_\Omega(|\nabla u|^q+u^q)\mathrm{d}x\right]^{1/q},\ \langle u,v\rangle=\int_\Omega(|\nabla u|^{q-2}\nabla u\cdot\nabla v+|u|^{q-2}u\cdot v)\mathrm{d}x;$$

$\|u\|=\left[\int_\Omega|\nabla u|^q\,\mathrm{d}x\right]^{1/q}$ is the equivalence-norm of $W_0^{k,q}(\Omega)$, $\langle u,v\rangle=\int_\Omega|\nabla u|^{q-2}\nabla u\cdot\nabla v\mathrm{d}x$ is the equivalence-dual. $W^{k,2}(\Omega)=H^k(\Omega)$, $W_0^{k,2}(\Omega)=H_0^k(\Omega)$, and for N≥3, the mathematical space

$$D^{1,2}(\mathbf{R}^N)=\left\{u\in L^{2^*}(\mathbf{R}^N)\,\Big|\,D_iu\in L^2(\mathbf{R}^N),i=1,\cdots,N\right\},$$

Their norm $\|u\|=\left[\int_{\mathbf{R}^N}|\nabla u|^2\,\mathrm{d}x\right]^{1/2}$, dual action $\langle u,v\rangle=\int_{\mathbf{R}^N}\nabla u\cdot\nabla v\mathrm{d}x$, and for $D_0^{1,2}(\Omega)$: the closure of $C_0^\infty(\Omega)$ on norm $\|u\|=\left[\int_\Omega|\nabla u|^2\,\mathrm{d}x\right]^{1/q}$ and dual action $\langle u,v\rangle=\int_\Omega\nabla u\cdot\nabla v\mathrm{d}x$ Denote by H the $H_0^1(\Omega)$, $D^{1,2}(\mathbf{R}^N)$, $B_r(0):=\{u\in H\,\big|\,\|u\|\le r\}$, $B_R(\mathbf{0}):=\{x\in\mathbf{R}^N\,\big||x|\le R\}$.

As we know, $H\underset{\to}{\subset}L^{2^*}(\Omega)$ not compact, but $H\underset{\to}{\subset}L^p(\Omega)\underset{\to}{\subset}L^q(\Omega)(1\le q\le p\le 2^*)$ continuously, S_1 is the best constant of $H\underset{\to}{\subset}L^q(\Omega)(1\le q\le 2^*)$, $S_1:=\inf\limits_{u\in H\setminus\{0\}}\dfrac{\|u\|^q}{\|u\|_q^q}$, which is corresponding with the first eigen-function ϑ_1; S is the best constant of $H\underset{\to}{\subset}L^{2^*}(\Omega)$, e.g.

$$S:=\inf_{u\in H\setminus\{0\}}\frac{\|u\|^2}{\|u\|_{2^*}^{2/2^*}},$$

then, is achieved by

$$u_\varepsilon(x)=\left[N(N-2)\varepsilon^2\right]^{\frac{N-2}{4}}\left(\varepsilon^2+|x|^2\right)^{\frac{2-N}{2}}\tag{2.1}$$

on $\mathbf{R}^N(N\ge 3)$ and $\varepsilon>0$ is arbitrary. $-\Delta u_\varepsilon=u_\varepsilon^{2^*-1}(N\ge3)$ and

$$S\left(\int_{\mathbf{R}^N}|u_\varepsilon|^{2^*}\mathrm{d}x\right)^{\frac{2}{2^*}}=\int_{\mathbf{R}^N}|\nabla u_\varepsilon|^2\,\mathrm{d}x=\int_{\mathbf{R}^N}|u_\varepsilon|^{2^*}\mathrm{d}x.$$

Hence $\int_{\mathbf{R}^N}|\nabla u_\varepsilon|^2\,\mathrm{d}x=\int_{\mathbf{R}^N}|u_\varepsilon|^{2^*}\,\mathrm{d}x=S^{\frac{N}{2}}$, and $S=\left(\int_{\mathbf{R}^N}|\nabla u_\varepsilon|^2\,\mathrm{d}x\right)^{\frac{2}{N}}=\left(\int_{\mathbf{R}^N}|u_\varepsilon|^{2^*}\,\mathrm{d}x\right)^{\frac{2}{N}}$.

Consider the problem with Dirichlet and Neumann's boundary:

$$\begin{cases}-\Delta u=\lambda u, & x\in\Omega\ ,\\ \ \ u=0, & x\in\partial\Omega\ ;\end{cases}\quad\begin{cases}-\Delta u=\mu u, & x\in\Omega,\\ \dfrac{\partial u}{\partial n}=0, & x\in\partial\Omega,\end{cases}\tag{2.2}$$

then, there exist $\{\lambda_i\}_{i=1}^\infty$ ($\{\mu_i\}_{i=1}^\infty$) and $\{\varphi_i\}_{i=0}^\infty$ ($\{\phi_i\}_{i=1}^\infty$) $\in C^2(\Omega)$, such that

$$\begin{cases} -\Delta\varphi_i = \lambda_i\varphi_i, \varphi_1 > 0, & x \in \Omega, \\ \varphi_i = 0 & x \in \partial\Omega, \\ 0 < \lambda_1 < \lambda_2 \le \cdots, \lim_{i\to\infty} \lambda_i = +\infty; \end{cases} \quad \begin{cases} -\Delta\phi_i = \mu_i\phi_i, \phi_1 \equiv \forall cons\tan t, x \in \Omega, \\ \dfrac{\partial\phi_i}{\partial n} = 0, & x \in \partial\Omega, \\ 0 = \mu_1 < \mu_2 \le \cdots, \lim_{i\to\infty} \mu_i = +\infty, \end{cases} \tag{2.3}$$

$\varphi_i^{\pm}, \phi_i^{\pm} \in H^1(\Omega)$, $-\Delta\varphi_i^{\pm} = \lambda_i\varphi_i^{\pm}$, $-\Delta\phi_i^{\pm} = \mu_i\phi_i^{\pm}$.

Recall that $u \in H$ is a weak solution of problem (P), if for every $v \in H$, there is

$$\left(a - b\int_\Omega |\nabla u|^2 \, \mathrm{d}x\right)\left(\int_\Omega \nabla u \cdot \nabla v \mathrm{d}x + \int_{\partial\Omega} \psi u \cdot v \mathrm{d}S\right) - \int_\Omega g(x,u) \cdot v \mathrm{d}x = 0. \tag{2.4}$$

III. Many contributions on new Kirchhoff-type problem with negative modulus

Consider the following problem with $p \in (2, 2^*)$:

$$\begin{cases} -\left(a - b\int_\Omega |\nabla u|^2 \, \mathrm{d}x\right)\Delta u = |u|^{p-2} u, & x \in \Omega; \\ u = 0, & x \in \partial\Omega, \end{cases} \tag{3.1}$$

then, by using the Mountain Pass Lemma, Yin and Liu got that:

Theorem3.1.[4]Assume that *a*,*b*>0, $p \in (2, 2^*)$, then problem(3.1)possesses at least a nontrivial weak solution.

Theorem3.2.[4]Assume that *a*,*b*>0, $p \in (2, 2^*)$, then problem(3.1)possesses at least a nontrivial non-negative solution and a nontrivial non-positive solution.

For $\lambda > 0$, $f_{\pm} = \pm\max\{\pm f, 0\} \not\equiv 0$, $f_\lambda = \lambda f_+ + f_-$ and $1 < q < 2$,

$$\begin{cases} -\left(a - b\int_\Omega |\nabla u|^2 \, \mathrm{d}x\right)\Delta u = f_\lambda(x)|u|^{q-2} u, & x \in \Omega \subset \mathbf{R}^3; \\ u = 0, & x \in \partial\Omega \end{cases} \tag{3.2}$$

by using Ekeland's variational principle, Harnack inequality, Lei et al got that.

Theorem3.3.[5]Assume that $a, b > 0$, $1 < q < 2$ and $f_\lambda \in L^\infty(\Omega)$. Then there exists $\lambda_* > 0$, such that for any $\lambda \in (0, \lambda_*)$, problem (3.2) has at least two positive solutions.

We studied in [6] for the problem with λ arbitrary, $p \in (2,4](N = 1,2,3)$ or $p \in (2,2^*)(N \ge 4)$ as follows:

$$\begin{cases} -\left(a - b\int_\Omega |\nabla u|^2 \, \mathrm{d}x\right)\Delta u = \lambda|u|^{p-2} u, & x \in \Omega; \\ u = 0, & x \in \partial\Omega \end{cases} \tag{3.3}$$

Theorem3.4.[3,6]Set $B_R(\mathbf{0}) \subset \mathbf{R}^N$ is an open ball with radius R, then, for all $R > 0$, the problem(3.3) has an unique positive solution with $\int_\Omega |\nabla u|^2 \, \mathrm{d}x < \dfrac{a}{b}$ when $\Omega = B_R(\mathbf{0})$.

Based on the variational and perturbation methods, Lei at al researched

$$\begin{cases} -\left(a - b\int_\Omega |\nabla u|^2 \, \mathrm{d}x\right)\Delta u = \dfrac{\lambda}{u^\gamma}, & x \in \Omega \subset \mathbf{R}^3; \\ u = 0, & x \in \partial\Omega \end{cases} \tag{3.4}$$

Theorem3.5.[7]Assume $a, b > 0$, $0 < \gamma < 1$, there exists $\lambda_* > 0$ such that $0 < \lambda < \lambda_*$, then, problem (3.4)has at least two positive solutions.

We explored their physical significance and considered the problem with $f(x) \in L^{\frac{4}{3}}(\mathbf{R}^4)$

$$-\left(a - b\int_{\mathbf{R}^4} |\nabla u|^2 \, dx\right)\Delta u = |u|^2 u + \mu f(x), \quad x \in \mathbf{R}^4, \tag{3.5}$$

Theorem3.6.[2,3]Set $a,b>0$, then problem(3.5)has infinitely many positive solutions when $\mu=0$.

Theorem3.7.[2,3]Assume that $f(x)\in L^{4/3}(\mathbf{R}^4)$ is a positive function, then there is $\mu_*>0$ such that problem(3.5)has at least two positive solutions when $\mu\in(0,\mu_*]$.

The problem (P) which contains the different source. Case $f(x,u)$ is measurable. We studied their nearly resonant solution for $N=1,2,3$ in[8] as

$$\begin{cases}-\left(a-b\int_\Omega|\nabla u|^2\,\mathrm{d}x\right)\Delta u+b\lambda u^3=f(x,u), & x\in\Omega,\\ u=0, & x\in\partial\Omega,\end{cases} \tag{3.6}$$

Theorem3.8.[3,8]Denote by $F(x,\tau)=\int_0^\tau f(x,s)ds$. S_1 is the best constant for $H\underset{\rightarrow}{\subset}L^4(\Omega)$,and which corresponding with ϑ_1.

(F1) $0\le f(x,t)<b\lambda|t|^3+a\left(\lambda/|\Omega|\right)^{1/2}|t|$ for $(x,t)\in\bar{\Omega}\times(\mathbf{R}\setminus\{0\})$ a.e.;

(F2) $\lim\limits_{|t|\to\infty}\dfrac{f(x,t)}{b|t|^3}=0$, for all $(x,t)\in\bar{\Omega}\times\mathbf{R}$ uniformly;

(F3) $\lim\limits_{|t|\to\infty}\left[\dfrac{a}{2}S_1t^2-\int_\Omega F(x,t\vartheta_1)\mathrm{d}x\right]\to+\infty$, $x\in\Omega$.

Then: (1) case (F1), we have for every $\lambda\in(0,S_1)$. Problem (3.6) has at least a nontrivial weak solution;
(2) case (F2) and (F3), we have that problem(3.6)has at least three nontrivial weak solutions when $\lambda<S_1$ and $\lambda\to S_1$.

We researched the following problem:

$$-\left(a-b\int_\Omega|\nabla u|^2\,\mathrm{d}x\right)\Delta u=\lambda|u|^{q-1}u+\mu f(x),x\in\Omega\subseteq\mathbf{R}^N, \tag{3.7}$$

there a,b is the same mathematical symbol, $\lambda,\mu\in\mathbf{R}$, $0\le q\le 2^*$, $f(x)>0$ a.e., $\Omega\subset\mathbf{R}^N$ or $\Omega=\mathbf{R}^N$.

Theorem3.9.[9]Assume that $a,b>0$, $\lambda=1$, $0<f\in L^{\frac{q+1}{q}}(\Omega)$ a.e., then, case $\Omega=\mathbf{R}^N(N\ge3)$ and $q\in(1,2^*-1]$, there is a $\mu_*>0$,such that for all $\mu\in(0,\mu_*]$, (3.7) have at least two solutions.

Theorem3.10.[9]Assume that $a,b>0$, $N\ge3$, $\Omega=\mathbf{R}^N$, $q=2^*-1$, if $\mu=0$,we have for all $\lambda\in\mathbf{R}$,(3.7)has infinitely many solutions.

Theorem3.11.[9] $a,b>0$, $N\ge1$, $\Omega=\mathbf{R}^N$, $\mu=0$.Case $\lim\limits_{|x|\to\infty}u(x)=0$, then

(1) $q\in(1,2^*-1)$, for all $\lambda>0$,(3.7)has at least two solutions;

(2) $q\in(0,1]$, there is a $\lambda_*>0$ such that for all $\lambda\in(0,\lambda_*)$,(3.7)has at least two solutions;

Theorem3.12.[9]Assume that $a,b>0$, $\Omega\subseteq\mathbf{R}^N(N\ge1)$ is whole space or $\partial\Omega$ smoothly, $f\in L^{\frac{2^*}{2^*-1}}(\Omega)$, $\lambda=0$, then there is a $\mu_*>0$ such that(3.7)has at least a solution when $\mu\ge\mu_*$, and has at least three solutions when $\mu\in(0,\mu_*)$.

Theorem3.13.[9]Assume that $ab>0,\lambda>0,\mu=0$, $N\ge1$, $\Omega=\mathbf{R}^N$, then for every $q\in(-1,2^*-1)$, the problem (3.7)has infinitely many solutions, extremely the classical solutions when $\Omega=\left\{x\in\mathbf{R}^N:\prod\limits_{i=1}^N x_i\ne0\right\}$.

Next, for theorem3.12 with (3.7), recall that(3.7) now

$$\begin{cases}-\left(a-b\int_\Omega|\nabla u|^2\,dx\right)\Delta u=\mu f(x), & x\in\Omega\subset\mathbf{R}^N(N\ge1),\\ u=0, & x\in\partial\Omega,\end{cases} \tag{3.8}$$

Theorem3.14.[10]Assume that $a,b>0$ and $f(x)\in L^{2^*/(2^*-1)}(\Omega)$ is positive almost everywhere, then, there is a constant $\mu_*>0$, such that the problem(3.8)has at least three nontrivial solutions for $\mu\in(0,\mu_*)$ and a nontrivial solution for $\mu\in[\mu_*,+\infty)$.

Theorem3.15.[10]Assume that $a,b>0$ and $f(x)\in L^{2^*/(2^*-1)}(\Omega)$ is positive a.e., then, there is a constant $\mu_{**}>0$ such that problem(3.8) has only three solutions for $0<|\mu|<\mu_{**}$, only two solutions for $\mu=\pm\mu_{**}$ and unique solution for $|\mu|>\mu_{**}$. Moreover, problem (3.8)has infinitely many solutions for $a,b>0,\mu=0$.

Corollary3.16. Assume that $a,b>0$ and $f(x)\in L^2(\Omega)$ is positive a.e., then, for μ_{**} defined by Theorem before, the problem (3.8) has only three solutions for every $0<|\mu|<\mu_{**}$, only two solutions for $\mu=\pm\mu_{**}$ and unique solution for every $|\mu|>\mu_{**}$.

We considered the problem with Hardy-Sobolev critical exponent:

$$\begin{cases}-\left(a-b\int_\Omega|\nabla u|^2\,\mathrm{d}x\right)\Delta u=\dfrac{\lambda u^3}{|x|}+f(x,u), & x\in\Omega\subset\mathbf{R}^3,\\ u=0, & x\notin\Omega\setminus\{\mathbf{0}\}.\end{cases}\tag{3.9}$$

$a,b,\lambda>0$, $f(x,u)$: $\Omega\times\mathbf{R}$ to $\mathbf{R}$ measurable.

Theorem3.17.[11]Assume that $a,b,\lambda>0$ and $f(x,u)$ satisfy the following conditions:

(f1) case $f(x,u)\ge 0$ and there is $s<5$ such that $\lim\limits_{|u|\to 0}\dfrac{f(x,u)}{|u|}=\lim\limits_{|u|\to+\infty}\dfrac{f(x,u)}{|u|^s}=0$ for $x\in\Omega$ uniformly;

(f2) assume that, there exist a constant $\mu>4$ such that $0<\mu F(x,u)\le f(x,u)u$ when $|u|>0$ and $x\in\Omega$, $F(x,u)=\int_0^{|u|}f(x,t)dt$.

Then, equation (3.9) has at least a positive solution if the conditions (f1) and (f2) are satisfied.

We introduced in [3] but not research it that time. We consider it now in [12],

$$-\left(a-b\int_{\mathbf{R}^N}|\nabla u|^2\,dx\right)\Delta u=u^{2^*-1}\ \ (\text{or}=|u|^{2^*-2}u),x\in\mathbf{R}^N(N\ge 3);\tag{3.10}$$

$$-\left(a-b\int_{\mathbf{R}^N_\pm}|\nabla u|^2\,dx\right)\Delta u=u,x\in\mathbf{R}^N_\pm(N\ge 1);\tag{3.11}$$

$$-\left(a-b\int_{\mathbf{R}^N_\pm(0)}|\nabla u|^2\,dx\right)\Delta u=u^{q-1}(\text{or}=|u|^{q-2}u),q\in(2,2^*),x\in\mathbf{R}^N_\pm(0)(N\ge 1)\tag{3.12}$$

$$\mathbf{R}^N_+=\left\{x\in\mathbf{R}^N\mid x_i>0\right\},\mathbf{R}^N_-=\left\{x\in\mathbf{R}^N\mid x_i<0\right\},$$

$$\mathbf{R}^N_-=\left\{x\in\mathbf{R}^N\mid x_i<0\right\},\ \mathbf{R}^N_-(0)=\{x\in\mathbf{R}^N\mid x_i\le 0\}.$$

Theorem3.18.[12] Assume that $a>0$, $b>0$, then equation (3.10) has infinitely many solutions in $C^2(\mathbf{R}^N)$.

Theorem3.19. Assume that $a\ge 0$, $b>0$, then equation (3.10) has infinitely many solutions in $C^2(\mathbf{R}^N_+)$, $C^2(\mathbf{R}^N_-)$ and $C^2(\mathbf{R}^N_\pm)$.

Theorem3.20. Assume that $a\ge 0$, $b>0$,then equation (3.10)has infinitely many solutions in $C^2(\mathbf{R}^N_+(0))$, $C^2(\mathbf{R}^N_-(0))$ and $C^2(\mathbf{R}^N_\pm(0))$.

The references [2,3,12]} et al, whose got that $a>0>b$ and $\beta=1$

$$-\left[a+\left(b\int_{\mathbf{R}^N}|\nabla u|^2\,\mathrm{d}x\right)^\beta\right]\Delta u=|u|^{2^*-2}u,x\in\mathbf{R}^N(N\ge 3),u\in D^{1,2}(\mathbf{R}^N)\tag{3.13}$$

has infinitely many solutions. [13] addresses $b>0>a$, $\beta\in\mathbf{R}$:

Theorem3.21.[13]Assume that $a<0<b$, then: for all $\beta\in(-\infty,0)\bigcup(\frac{2}{N-2},+\infty)$; and for $\beta=0$ and $a+b>0$; $\beta=\frac{2}{N-2}$ and $b>S^{N/(2-N)}$; $\beta\in(0,\frac{2}{N-2})$ and

$$\frac{(N-2)\beta-2}{(N-2)\beta}\left[\frac{1}{2}(N-2)b\beta S^{\frac{N\beta}{2}}\right]^{\frac{2}{2-(N-2)\beta}}\le a,$$

(3.13) has infinitely many classical solutions.

We considered for a,b are not equal zero at the same time, $q\neq-1$:

$$-\left(a-b\int_\Omega|\nabla u|^2\,\mathrm{d}x\right)\Delta u=u^q,x\in\Omega. \tag{3.14}$$

Where $\Omega=[c_1,d_1]\times[c_2,d_2]\times\cdots\times[c_N,d_N]$.

Theorem3.22.[14]Assume that $q\neq-1$, then, equation (3.14) has infinitely many classical solutions. Moreover,

I. assume that $q=1$, $a\le0<b$ or $ab>0$: Exponential function type solution;

II. assume that $q=1$ and no: $a\le0\le b$ Trigonometric function solution .Especially, $ab>0$; or $b=0$; or $b=0$ and $a\sum_{i=1}^{N}\frac{n_i^2\pi^2}{(c_i-d_i)^2}=1$, the zero boundary condition can be satisfied. $n_i(i=1,\cdots,N)$.

Ⅲ. assume that $q=0$, for any non-simultaneous zero $a,b\in\mathbf{R}$, def $u^0\equiv1$: Power function type solution.

Ⅳ. assume that $q<-1$ and no $a\le0\le b$; $-1<q\neq1$ and no $a\ge0\ge b$: Power function type solution.

We studied the following problem in *Applicable Anlysis*:

$$\begin{cases}-\left(a-b\int_\Omega\nabla u|^2\,\mathrm{d}x\right)\Delta u=|u|^{p-2}u, & x\in\Omega;\\ u=0, & x\in\partial\Omega,\end{cases} \tag{3.15}$$

Theorem3.23.[15]Assume that $a,b>0$, $N\ge1$, then, for every $p\in[2,2^*)$, the equation (3.15) has infinitely many solutions $\{u_k\}_{k=1}^\infty$ and $I(u_k)\to\frac{a^2}{4b}$, $\int_\Omega|\nabla u_k|^2\,\mathrm{d}x\to\frac{a}{b}$, $\int_\Omega|u_k|^p\mathrm{d}x\to0$ as $k\to\infty$, where I is the variational functional for problem (3.15).

We consider, for $a,b,\lambda\in\mathbf{R}$, $\Omega\subset\mathbf{R}^N(N\ge1)$, $\mathcal{A}u=u$ or $\frac{\partial u}{\partial\mathbf{n}}$, the problem

$$\begin{cases}-\left(a-b\int_\Omega\nabla u|^2\,\mathrm{d}x\right)\Delta u=\lambda u, & x\in\Omega;\\ \mathcal{A}u=0, & x\in\partial\Omega.\end{cases} \tag{3.16}$$

Theorem3.24.[16]Case $a>0$, then those solutions are classical:

(1) Assume that $b=0$, there is a sequence $\{a\mu_i\}_{i=1}^\infty,\lim_{i\to\infty}\mu_i=\infty$, such that the problem (3.16) has infinitely many classical solutions when $\lambda=a\mu_i$;

(2) Assume that $b>0$, for all $\lambda\in\mathbf{R}$, (3.16) has infinitely many classical solutions $\{u_n\}_{n=1}^\infty$;

(3) Assume that $b<0$, $a\mu_i<\lambda\le a\mu_{i+1}$, then, problem (3.16) has i-pair different solutions, and the solutions are constants when $\lambda<a\mu_2$;

Theorem3.25.[16]Assume that $a=0$, then problem (3.16) has infinitely many classical solutions $\{u_n\}_{n=1}^\infty$ case $b\lambda<0$, and the solution is 0 if and only if $b\lambda\ge0$;all those solutions are satisfy the $I(u_n)\to0$ case $n\to\infty$, and for constant λ, $\int_\Omega u_n^2\mathrm{d}x<<\int_\Omega|\nabla u_n|^2\,\mathrm{d}x\to0$.

Theorem3.26.[16]Case λ is constant, then for theorem 3.4(2), $\int_\Omega |\nabla u_n|^2\, \mathrm{d}x \to \frac{a}{b}$, $\lambda\int_\Omega u_n^2 \mathrm{d}x \to 0$, $I(u_n) \to \frac{a^2}{4b}$ as $n \to \infty$;case λ is a parameter, then, for the solution near resonance of the theorem 3.24(2)(3),are satisfied with $\int_\Omega |\nabla u_i|^2\, dx \to 0$, $\lambda\int_\Omega u_i^2 dx \to 0$, $I(u_i) \to 0$,and $\int_\Omega u_i^2 dx << \int_\Omega |\nabla u_i|^2\, dx$, when $\lambda \to a\mu_i^{[-b\ \text{symbol}]}$.

For more details about new Kirchhoff-type problem with negative modulus, we refer the readers to the reference [17].

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Appendix: A brief introduction to this paper in Chinese

带有负模量的基尔霍夫型问题介绍

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摘要：

本文在带有负模量的非局部新基尔霍夫型问题方面阐述了我们所做的一些典型结果.

关键词：非局部问题,负模量,新基尔霍夫型问题.

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