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Observation On The Ternary Quadratic Diophantine Equation With Three Unknowns

$$9x^2 - 24xy + 22y^2 = 7z^2$$

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Abstract:

The ternary quadratic equation given by $9x^2 - 24xy + 22y^2 = 7z^2$ *is considered and searched for its many different integer solution. Twelve different choices of integer solution of the above equations are presented. A few interesting relations between the solutions and special polynomial numbers are presented.*

Keywords: *Ternary quadratic, integer solutions*

Notation:

$$t_{m,n} = n\left[1 + \frac{(n-1)(m-2)}{2}\right]$$

$$PR_n = n(n+1)$$

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I. INTRODUCTION :

The Diophantine equations offer an unlimited field for research due to their variety [1-3]. In particular , one may refer [4-12] for quadratic equations with three unknowns. This communication concerns with yet another interesting equation $9x^2 - 24xy + 22y^2 = 7z^2$ representing homogeneous equation with three unknowns for determining its infinitely many non-zero integral points. Also,few interesting relations among the solutions are presented.

II. METHOD OF ANALYSIS

The Quadratic Diophantine equation with three unknown to be solved is given by

$$9x^2 - 24xy + 22y^2 = 7z^2 \tag{1}$$

$$\Rightarrow (3x - 4y)^2 + 6y^2 = 7z^2 \tag{2}$$

Assume

$$U = 3x - 4y \tag{3}$$

Substituting in (2) we get,

$$U^2 + 6y^2 = 7z^2 \tag{4}$$

(4) is solved through different approaches and the different patterns of solutions of (1) obtained are presented below.

PATTERN: 1

Assume as

$$z = a^2 + 6b^2 \tag{5}$$

Write '7' as

$$7 = \left(1 + i\sqrt{6}\right)\left(1 - i\sqrt{6}\right) \tag{6}$$

Substituting (5) & (6) in (4) and employing the method of factorization , we get

$$\left(U + i\sqrt{6}y\right)\left(U - i\sqrt{6}y\right) = \left(1 + i\sqrt{6}\right)\left(1 - i\sqrt{6}\right)\left(a + i\sqrt{6}b\right)^2\left(a - i\sqrt{6}b\right)^2$$

Equating the positive factor

$$\left(U + i\sqrt{6}y\right) = \left(1 + i\sqrt{6}\right)\left(a + i\sqrt{6}b\right)^2$$

$$\Rightarrow \left(U + i\sqrt{6}y\right) = \left(a^2 - 6b^2 - 12ab\right) + i\sqrt{6}\left(a^2 - 6b^2 + 2ab\right)$$

Equating real and imaginary parts

$$U = a^2 - 6b^2 - 12ab$$

$$y = a^2 - 6b^2 + 2ab$$

Assume a=3a, b=3b in the above equation and in view of (3),We obtain the non-zero distinct integral solution of (1) as

$$x(a,b) = 15a^2 - 90b^2 - 12ab$$

$$y(a,b) = 9a^2 - 54b^2 + 18ab$$

$$z(a,b) = 9a^2 + 54b^2$$

PROPERTIES:

- $.x(a,1) + y(a,1) - 90t_{4,a} + 30P_{ra} \equiv 0(\bmod 3)$
- $.x(a,1) + z(a,1) - 36t_{4,a} + 12P_{ra} + 36 = 0$
- $y(a,1) + z(a,1) - 96t_{4,a} + 36P_{ra} \equiv 0(\bmod 3)$

PATTERN: 2

'7' can also written as

$$7 = \frac{\left(11 + 3i\sqrt{6}\right)\left(11 - 3i\sqrt{6}\right)}{5^2} \tag{7}$$

Substituting (5) & (7) in (4) and employing the method of factorization, we get

$$(U+i\sqrt{6}y)(U-i\sqrt{6}y)=(a+i\sqrt{6}b)^2(a-i\sqrt{6}b)^2\,\frac{(11+3i\sqrt{6})(11-3i\sqrt{6})}{5^2}$$

Equating the positive factor

$$U+i\sqrt{6}y=\frac{(11+i3\sqrt{6})(a+i\sqrt{6}b)^2}{5}$$

$$\Rightarrow U+i\sqrt{6}y=\frac{\left(11a^2-66b^2-36ab\right)+i\sqrt{6}\left(3a^2-18b^2+22ab\right)}{5}$$

Equating real and imaginary parts

$$U=\frac{\left(11a^2-66b^2-36ab\right)}{5}$$

$$y=\frac{\left(3a^2-18b^2+22ab\right)}{5}$$

Assume a=15a, b= 15b in the above equation and in view of (3),we obtain the non-zero distinct integral solution of (1) as

$$x(a,b)=345a^2-2070b^2-780ab$$

$$y(a,b)=135a^2-810b^2+990ab$$

$$z(a,b)=225a^2+1350b^2$$

PROPERTIES:

- $.x(a,1)+y(a,1)+1290t_{4,a}-1770P_{ra}\equiv 0(\bmod 2)$
- $.x(a,1)+Z(a,1)+210t_{4,a}-780P_{ra}+720=0$
- $.y(a,1)+z(a,1)-630t_{4,a}+990P_{ra}\equiv 0(\bmod 3)$

PATTERN: 3

'7' can also written as

$$7=\frac{\left(17+3i\sqrt{6}\right)\left(17-3i\sqrt{6}\right)}{7^2} \qquad (8)$$

Substituting (5) & (8) in (4) and employing the method of factorization, we get

$$(U+i\sqrt{6}y)(U-i\sqrt{6}y)=(a+i\sqrt{6}b)^2(a-i\sqrt{6}b)^2\,\frac{(17+3i\sqrt{6})(17-3i\sqrt{6})}{7^2}$$

Equating the positive factor

$$U+i\sqrt{6}y=\frac{(11+i3\sqrt{6})(a+i\sqrt{6}b)^2}{5}$$

$$\Rightarrow U+i\sqrt{6}y=\frac{\left(11a^2-66b^2-36ab\right)+i\sqrt{6}\left(3a^2-18b^2+22ab\right)}{5}$$

Equating real and imaginary parts

$$U=\frac{\left(17a^2-102b^2-36ab\right)}{7}$$

$$y=\frac{\left(3a^2-18b^2+34ab\right)}{7}$$

Assume a=21a, b= 21b in the above equation and in view of (3),we obtain the non-zero distinct integral solution of (1) as

$$x(a,b)=609a^2-3654b^2+2100ab$$

$$y(a,b)=189a^2-1134b^2+2142ab$$

$$z(a,b)=441a^2+2646b^2$$

PROPERTIES:

- $.x(a,1)+y(a,1)+1290t_{4,a}-1770P_{ra}\equiv 0(\bmod 2)$
- $.x(a,1)+Z(a,1)+210t_{4,a}-780P_{ra}+720=0$
- $.y(a,1)+z(a,1)-630t_{4,a}+990P_{ra}\equiv 0(\bmod 3)$

PATTERN: 4

(4) can also written as

$$U^2+6y^2=7\times 1\times z^2 \qquad (9)$$

1’ can also be written as

$$1=\frac{\left(1+2i\sqrt{6}\right)\left(1-2i\sqrt{6}\right)}{5^2} \qquad (10)$$

Write '7' as

$$7=\left(1+i\sqrt{6}\right)\left(1-i\sqrt{6}\right) \tag{11}$$

Substituting (9) ,(10) & (11) in (4) and employing the method of factorization, we get

$$(U+i\sqrt{6}y)\left(U-i\sqrt{6}y\right)=\frac{\left(1+i\sqrt{6}\right)\left(1-i\sqrt{6}\right)\left(1+2i\sqrt{6}\right)\left(1-2i\sqrt{6}\right)\left(a+i\sqrt{6}b\right)^2\left(a-i\sqrt{6}b\right)^2}{5^2}$$

Equating the positive factor

$$\left(U+i\sqrt{6}y\right)=\frac{\left(1+i\sqrt{6}\right)\left(1+i\sqrt{6}\right)\left(a+i\sqrt{6}b\right)^2}{5}$$

$$\Rightarrow\left(U+i\sqrt{6}y\right)=\frac{(-11a^2+66b^2-36ab)+i\sqrt{6}\left(3a^2-18b^2-22ab\right)}{5}$$

Equating real and imaginary parts of the above equation, we get

$$U=\frac{11a^2-66b^2+36ab}{5}$$

$$y=\frac{3a^2-18b^2-22ab}{5}$$

Assume a=15a, b=15b in the above equation, we obtain the non-zero integral solution of (1) is obtained as

$$x(a,b)=345a^2-2070b^2+780ab$$

$$y(a,b)=135a^2-810b^2-990ab$$

$$z(a,b)=225a^2+1350b^2$$

PROPERTIES:

- $.x(a,b)+y(a,b)-2550t_{4,a}+1770P_{ra}\equiv 0(\mathrm{mod}\,5)$
- $.x(a,b)+z(a,b)-1350t_{4,a}+780P_{ra}\equiv 0(\mathrm{mod}\,2)$
- $.y(a,b)+z(a,b)-1350t_{4,a}+990P_{ra}-540=0$

PATTERN: 5

(4) can also written as

$$U^2+6y^2=7\times 1\times z^2 \tag{12}$$

Write '7' as

$$7=\frac{\left(11+3i\sqrt{6}\right)\left(11-3i\sqrt{6}\right)}{5^2} \tag{13}$$

'1' can also written as

$$1=\frac{\left(1+2i\sqrt{6}\right)\left(1-2i\sqrt{6}\right)}{5^2} \tag{14}$$

Substituting (12),(13)& (14) in (4) and employing the method of the factorization , we get

$$\left(U+i\sqrt{6}y\right)\left(U-i\sqrt{6}y\right)=\frac{\left(11+3i\sqrt{6}\right)\left(11-3i\sqrt{6}\right)\left(1+2i\sqrt{6}\right)\left(1-2i\sqrt{6}\right)\left(a+i\sqrt{6}b\right)^2\left(a-i\sqrt{6}b\right)^2}{25^2}$$

Equating the positive factor

$$U+i\sqrt{6}y=\frac{\left(11+3i\sqrt{6}\right)\left(1+2i\sqrt{6}\right)\left(a+i\sqrt{6}b\right)^2}{25}$$

$$U+i\sqrt{6}y=\left(a^2-6b^2-12ab\right)+i\sqrt{6}\left(a^2-2ab-6b^2\right)$$

Equating real and imaginary parts of the above equations, we get

$$U=a^2-6b^2+12ab$$

$$y=a^2-2ab-6b^2$$

Assume a=3a, b=3b in the above equation,we obtain the non-zero distinct integral solution of (1) is obtained as

$$x(a,b)=15a^2+12ab-90b^2$$

$$y(a,b)=9a^2-18ab-54b^2$$

$$z(a,b)=9a^2+54b^2$$

PROPERTIES

- $.x(a,b)+y(a,b)-30t_{4,a}-6P_{ra}\equiv 0(\bmod 2)$
- $x(a,b)+y(a,b)-12_{4,a}-12P_{ra}\equiv 0(\bmod 3)$
- $y(a,b)+z(a,b)-36t_{4,a}+18P_{ra}=0$

PATTERN : 6

Case: 1

Equating (4) can be written as

$$\frac{(U+z)}{z+y}=\frac{6(z-y)}{U-z}=\frac{\alpha}{\beta} \tag{15}$$

Which is equivalent to the system of double equation as

$$\left.\begin{aligned}-\alpha y+U\beta+z(-\alpha+\beta)=0\\-\alpha U-6y\beta+z(\alpha+6\beta)=0\end{aligned}\right\} \tag{16}$$

Solving (16) by the method of cross multiplication, we get

$$\left.\begin{aligned}U&=-\alpha^2-12\alpha\beta+6\beta^2\\y&=-\alpha^2+2\alpha\beta+6\beta^2\\z&=-\alpha^2-6\beta^2\end{aligned}\right\} \tag{17}$$

Substituting (17) in (3), the non-zero distinct integral solution of (1) is given by

$x(\alpha,\beta) = -15\alpha^2 - 12\alpha\beta + 90\beta^2$

$y(\alpha,\beta) = -9\alpha^2 + 18\alpha\beta + 54\beta^2$

$z(\alpha,\beta) = -9\alpha^2 - 54\beta^2$

PROPERTIES:

- $.x(\alpha,1) + y(\alpha,1) + 30t_{4,a} - 6P_{ra} \equiv (\text{mod}\, 2)$
- $x(\alpha,1) + z(\alpha,1) + 12t_{4,a} + 12P_{ra} \equiv 0(\text{mod}\, 3)$
- $y(\alpha,\beta) + z(\alpha,\beta) + 36t_{4,a} - 18P_{ra} = 0$

Case : 2

(3) can be written as

$$\frac{U+z}{6(z+y)} = \frac{z-y}{U-z} = \frac{\alpha}{\beta} \tag{18}$$

which is equivalent to the system of double equation as

$$\left.\begin{aligned} U\beta - 6y\alpha - z(-6\alpha+\beta) &= 0 \\ -U\alpha - y\beta + z(\alpha+\beta) &= 0 \end{aligned}\right\} \tag{19}$$

Solving (19) by the method of cross multiplication ,we get

$$\left.\begin{aligned} U &= -6\alpha^2 - 12\alpha\beta + \beta^2 \\ y &= -6\alpha^2 + 2\alpha\beta + \beta^2 \\ z &= -6\alpha^2 - \beta^2 \end{aligned}\right\} \tag{20}$$

Substituting (20) in (3), the non zero distinct integral solution of (1) is given by

$$x(\alpha,\beta) = -90\alpha^2 - 12\alpha\beta + 15\beta^2$$

$$y(\alpha,\beta) = -54\alpha^2 + 18\alpha\beta + 9\beta^2$$

$$z(\alpha,\beta) = -54\alpha^2 - 9\beta^2$$

PROPERTIES:

- $.x(\alpha,\beta)+y(\alpha,\beta)+150t_{4,a}-6P_{ra}\equiv 0(\mod 2)$
- $.x(\alpha,\beta)+z(\alpha,\beta)+132t_{4,a}+12P_{ra}\equiv 0(\mod 3)$
- $y(\alpha,\beta)+z(\alpha,\beta)+126t_{4,a}-18P_{ra}=0$

Case:3

(4) can be written as

$$\frac{U+z}{3(z+y)}=\frac{2(z-y)}{U-z}=\frac{\alpha}{\beta} \tag{21}$$

which is equivalent to the system of double equation as

$$\left.\begin{aligned} &\beta U - 3\alpha y+z(-3\alpha+\beta)=0 \\ &-\alpha U-2\beta y+z(\alpha+2\beta)=0 \end{aligned}\right\} \tag{22}$$

Solving (22) by the method of cross multiplication, we get

$$\left.\begin{aligned} U&=-3\alpha^2-12\alpha\beta+2\beta^2 \\ y&=-3\alpha^2+2\beta^2+2\alpha\beta \\ z&=-3\alpha^2-2\beta^2 \end{aligned}\right\} \tag{23}$$

Substituting (23) in (3), the non-zero distinct integral solution of (1) is given by

$$x(\alpha,\beta)=-45\alpha^2-12\alpha\beta+30\beta^2$$

$$y(\alpha,\beta)=-27\alpha^2+18\alpha\beta+18\beta^2$$

$$z(\alpha,\beta)=-27\alpha^2-18\beta^2$$

PROPERTIES:

1. $x(\alpha,1)+y(\alpha,1)+42t_{4,a}+30P_{ra}\equiv 0(\mod 2)$

2. $x(\alpha,1)+z(\alpha,1)+60t_{4,a}+12P_{ra}\equiv 0(\mod 3)$

3. $y(\alpha,1)+z(\alpha,1)+36t_{4,a}+18P_{ra}=0$

Case: 4

(4) can be written as,

$$\frac{U+z}{2(z+y)}=\frac{3(z-y)}{U-z}=\frac{\alpha}{\beta} \tag{24}$$

Which is equivalent to the system of double equation as

$$\left.\begin{aligned}\beta U - 2\alpha y + z(-2\alpha + \beta) = 0\\ -\alpha U - 3\beta y + z(3\beta + \alpha) = 0\end{aligned}\right\} \quad (25)$$

Solving (25) by the method of cross multiplication, we get

$$\left.\begin{aligned}U &= -2\alpha^2 - 12\alpha\beta + 3\beta^2\\ y &= -2\alpha^2 + 2\alpha\beta + 3\beta^2\\ z &= -2\alpha^2 - 3\beta^2\end{aligned}\right\} \quad (26)$$

Substituting (26) in (3), the non- zero distinct integral solution of (1) is given by

$$x(\alpha,\beta) = -30\alpha^2 - 12\alpha\beta + 45\beta^2$$

$$y(\alpha,\beta) = -18\alpha^2 + 18\alpha\beta + 27\beta^2$$

$$z(\alpha,\beta) = -18\alpha^2 - 27\beta^2$$

PROPERTIES:

$$1.x(\alpha,1) + y(\alpha,1) + 54t_{4,a} - 6P_{ra} \equiv 0 \pmod 2$$

$$2..x(\alpha,1) + z(\alpha,1) + 36t_{4,a} + 12P_{ra} \equiv 0 \pmod 3$$

$$3.y(\alpha,1) + z(\alpha,1) + 54t_{4,a} - 18P_{ra} = 0$$

Case:5

(4) can be written as,

$$\frac{U-z}{2(z+y)} = \frac{3(z-y)}{U+z} = \frac{\alpha}{\beta} \quad (27)$$

Which is equivalent to the system of double equation as

$$\left.\begin{aligned}\beta U - 2\alpha y + z(-2\alpha - \beta) = 0\\ -\alpha U - 3\beta y + z(3\beta - \alpha) = 0\end{aligned}\right\} \quad (28)$$

Solving (28) by the method of cross multiplication, we get

$$\left.\begin{aligned} U &= 2\alpha^2 - 12\alpha\beta - 3\beta^2 \\ y &= 2\alpha^2 + 2\alpha\beta - 3\beta^2 \\ z &= 2\alpha^2 + 3\beta^2 \end{aligned}\right\} \tag{29}$$

Substituting (29) in (3), the non- zero distinct integral solution of (1) is given by

$$x(\alpha,\beta) = 30\alpha^2 - 12\alpha\beta - 45\beta^2$$

$$y(\alpha,\beta) = 18\alpha^2 + 18\alpha\beta - 27\beta^2$$

$$z(\alpha,\beta) = 18\alpha^2 + 27\beta^2$$

Case:6

(4) can be written as

$$\frac{U - z}{3(z - y)} = \frac{2(z + y)}{U + z} = \frac{\alpha}{\beta}, \beta \neq 0 \tag{30}$$

which is equivalent to the system of double equation as

$$\left.\begin{aligned} \beta U + 3\alpha y + z(-3\alpha - \beta) = 0 \\ -\alpha U + 2\beta y + z(-\alpha + 2\beta) = 0 \end{aligned}\right\} \tag{31}$$

Solving (31) by the method of cross multiplication, we get

$$\left.\begin{aligned} U &= -3\alpha^2 + 12\alpha\beta + 2\beta^2 \\ y &= -3\alpha^2 + 2\beta^2 - 2\alpha\beta \\ z &= -3\alpha^2 - 2\beta^2 \end{aligned}\right\} \tag{32}$$

Substituting (32) in (3), the non-zero distinct integral solution of (1) is given by

$$x(\alpha,\beta) = -45\alpha^2 + 12\alpha\beta + 30\beta^2$$

$$y(\alpha,\beta) = -27\alpha^2 - 18\alpha\beta + 18\beta^2$$

$$z(\alpha,\beta) = -27\alpha^2 - 18\beta^2$$

PROPERTIES:

$$1.x(\alpha,1)+y(\alpha,1)+42t_{4,a}+30P_{ra}\equiv 0(\mathrm{mod}\,2)$$

$$2.x(\alpha,1)+z(\alpha,1)+60t_{4,a}+12P_{ra}\equiv 0(\mathrm{mod}\,3)$$

$$3.y(\alpha,1)+z(\alpha,1)+36t_{4,a}+18P_{ra}=0$$

Case: 7

(4) can be written as

$$\frac{(U-z)}{z-y}=\frac{6(z+y)}{U+z}=\frac{\alpha}{\beta},\beta\neq 0 \tag{33}$$

Which is equivalent to the system of double equation as

$$\left.\begin{aligned}\beta U+6\alpha y+z(6\alpha+\beta)=0\\ -\alpha U+y\beta+z(\beta-\alpha)=0\end{aligned}\right\} \tag{34}$$

Solving (34) by the method of cross multiplication, we get

$$\left.\begin{aligned}U&=-6\alpha^2-\beta^2\\ y&=-6\alpha^2-\beta^2\\ z&=-6\alpha^2-\beta^2\end{aligned}\right\} \tag{35}$$

Substituting (35) in (3), the non-zero distinct integral solution of (1) is given by

$$x(\alpha,\beta)=-90\alpha^2-15\beta^2$$

$$y(\alpha,\beta)=-54\alpha^2-9\beta^2$$

$$z(\alpha,\beta)=-54\alpha^2-9\beta^2$$

PROPERTIES:

- $.x(\alpha,1)+y(\alpha,1)+30t_{4,a}-6P_{ra}\equiv(\mathrm{mod}\,2)$
- $x(\alpha,1)+z(\alpha,1)+12t_{4,a}+12P_{ra}\equiv 0(\mathrm{mod}\,3)$
- $y(\alpha,\beta)+z(\alpha,\beta)+36t_{4,a}-18P_{ra}=0$

III. GENERATION OF SOLUTIONS

Different formulas for generating sequence of integer solutions based on the given solution are presented below:

Let $(u_0,y_{0,}z_0)$ be any given solution to (1)

Formula:1

Let $(u_1, y_{1,} z_1)$given by

$$u_1 = u_0,\ y_1 = y_0 + h,\ z_1 = z_0 + h, \tag{36}$$

be the 2^{nd} solution to (1).Using (36) in (1) and simplifying, one obtains

$$h = 12y_0 - 14z_0$$

In view of (36), the values of y_1 and z_1 is written in the matrix form as

$$(y_1, z_1)^t = M^n (y_0, z_0)^t$$

Where

$$M = \begin{pmatrix} 13 & -14 \\ 12 & -13 \end{pmatrix}$$ and t is the transpose

The repetition of the above process leads to the n^{th} solutions y_n, z_n given by

$$(y_n, z_n)^t = M^n (y_0, z_0)^t$$

If α, β are the distinct eigenvalues of M , then

$$\alpha = 1,\ \beta = -1$$

Thus, the general formulas for integer solutions to (1) are given by

$$u_n = u_0\ ,$$
$$y_n = \left(7 + 6(-1)^n\right)y_0 + 7\left(-1 + (-1)^n\right)z_0\ ,$$
$$z_n = 6\left(1 - (-1)^n\right)y_0 + \left(-6 + 7(-1)^n\right)z_0$$

Formula: 2

Let $(u_1, y_{1,} z_1)$ given by

$$u_1 = u_0 + 3h,\ y_1 = y_0,\ \ z_1 = z_0 + h, \tag{37}$$

be the 2nd solution to (1).Using (37) in (1) and simplifying, one obtains

$$h = -6u_0 + 7z_0$$

In view of (37), the values of x_1 and z_1 is written in the matrix form as

$$(x_1, z_1)^t = M(x_0, z_0)^t$$

Where

$$M = \begin{pmatrix} -17 & 21 \\ -6 & 8 \end{pmatrix}$$ and t is the transpose

The repetition of the above process leads to the n^{th} solutions x_n, z_n given by

$$(u_n, z_n)^t = M^n (u_0, z_0)^t$$

If α, β are the distinct eigenvalues of M , then

$$\alpha = -1,\ \ \beta = 10$$

We know that

$$M^n = \frac{\alpha^n}{(\alpha-\beta)}(M-\beta I)+\frac{\beta^n}{(\beta-\alpha)}(M-\alpha I), I = 2\times 2 \text{ identity matrix}$$

Thus, the general formulas for integer solutions to (1) are given by

$$u_n = \left(\frac{7(-1)^n - 16(10)^n}{11}\right)u_0 + \left(\frac{21\left(-(-1)^n + (10)^n\right)}{11}\right)z_0 \ ,$$

$$y_n = y_0 \ ,$$

$$z_n = 6\left(\frac{(-1)^n - (10)^n}{11}\right)u_0 + 9\left(\frac{-2(-1)^n + (10)^n}{2}\right)z_0$$

IV. CONCLUSION:

In this paper, an attempt has been made to obtain non-zero distinct integer solutions to the ternary quadratic diophantine equation $9x^2 - 24xy + 22y^2 = 7z^2$ representing homogenous cone. As the Diophantine equations are rich in variety, one may search for integer solutions to higher degree Diophantine equations with multiple variables along with suitable properties

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