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Numerical /Analytical Investigations of Stability of Stratified Rotatory Elastico – Viscous Oldroyd B Fluid In The Presence Of Variable Magnetic Field, Suspended Particles Saturating Porous Media

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ABSTRACT. *The influence of viscosity, viscoelasticity, medium permeability, medium porosity and suspended particles on the stability of a stratified elastic-viscous fluid is examined for viscoelastic polymeric solutions in the simultaneous presence of a variable horizontal magnetic field* $H\left(H_0(z),0,0\right)$ *and uniform horizontal rotation* $\Omega\left(\Omega,0,0\right)$ *in porous medium. These solutions are known as oldroyd B fluid and their rheology is approximated by the oldroyd B fluid constitutive relations, proposed by Oldroyd. The effects of coriolis force on the stability are chosen along the direction of the magnetic field and transverse to that of the gravitational field* $g\left(0,\ 0,-g\right)$*. Assuming the exponential stratifications in density, viscosity and viscoelasticity, the appropriate solution for the case of free boundaries is obtained using a linearized stability theory and normal mode analysis method. The dispersion relation is obtained and the behaviour of growth rates with respect to kinematic viscosity, kinematic viscoelasticity, medium permeability, dust particles and medium porosity is examined numerically using Newton-Raphson method through the software Fortran-90 and Mathcad. In contrast to the Newtonian fluids, the system is found to be unstable, for stable stratifications, for small wavelength perturbations. It is found that the magnetic field stabilizes the certain wave number band, for unstable stratification in the presence of rotation, suspended particles and this wave number range increases with the increase in magnetic field and decreases with the increase in kinematic viscoelasticity implying thereby the stabilizing effect of magnetic field,kinematic viscoelasticity , suspended particles and the kinematic viscosity has a stabilizing effect on the system for the low wave number range. The medium permeability has enhancing effect on the growth rates with its increase for a fixed wave number. These results are shown graphically.*

KEYWORDS: *Oldroyd B fluid; magnetic field; rotation; viscosity; viscoelasticity, medium permeability, medium porosity, suspended particles.*

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I. INTRODUCTION

The flow through porous medium has been of considerable interest in recent years particularly among geophysical fluid dynamics. When we consider flow in porous medium, some additional complexities arise which are due to the interaction between the fluids and the porous medium. Here we consider those fluid flows for which Darcy's law is applicable. This law is empirical in nature and is usually considered valid for creeping flows where the Reynolds's number as defined for a porous medium is less than one. Darcy's law states that the gross effect , as the fluid slowly percolated through the pores of rock , is that usual viscous term in the equation of elastic-viscous fluid motion will be replaced by the resistance terms $\left[-\frac{1}{k_1}\left(\mu-\mu'\frac{\partial}{\partial t}\right)q\right]$, where μ and μ' are the coefficients of viscosity and viscoelasticity , of oldroyd B fluid , k_1 is the medium permeability and q is the Darcian (filter) velocity (seepage) of the fluid.The stability of flow of a single component fluid through porous medium taking into account the Darcy's resistance has been studied by Lapwood [1] and Wooding [2]. The effect of the Earth's magnetic field on the stability of such a flow is of interest in geophysics particularly in the study of Earth's core where the Earth's mantle, which consists of conducting fluid, behaves like a porous medium which can become convectively unstable as a result of differential diffusion. The physical properties of comets and meteorites strongly suggest importance of porosity in astrophysical context (McDonnell [3]).The stability derived from the character of the equilibrium of an incompressible heavy fluid of variable density (i.e. of a heterogeneous fluid) was investigated by Rayleigh [4]. He demonstrated that the system is stable or unstable according as the density decreases everywhere or increases everywhere. An experimental demonstration of the development of the Rayleigh–Taylor instability was performed by Taylor [5]. The effect of a vertical magnetic field on the development of Rayleigh–Taylor instability was considered by Hide [6]. Reid [7] studied the effect of surface tension and viscosity on the stability of two superposed fluids. The Rayleigh–Taylor instability of a Newtonian fluid has been studied by several authors accepting varying assumptions of hydrodynamics and hydromagnetics and Chandrasekhar [8] in his celebrated monograph has given a detailed account of these investigations. Bellman and Pennington [9] further investigated in detail illustrating the combined effects of viscosity and surface tension. Gupta [10] again studied the stability of a horizontal layer of a perfectly conducting fluid with continuous density and viscosity stratifications in the presence of a horizontal magnetic field. The Rayleigh–Taylor instability problems arise in oceanography, limnology and engineering.

Generally, the magnetic field has a stabilizing effect on the instability, but there are a few exceptions also. For example, Kent [11] has studied the effect of a horizontal magnetic field which varies in the vertical direction on the stability of parallel flows and has shown that the system is unstable under certain conditions, while in the absence of magnetic field the system is known to be stable. In stellar atmospheres and interiors, the magnetic field may be (and quite often is) variable and may altogether alter the nature of the instability. Coriolis force also plays an important role on the stability of the system. In all the above studies the fluid has been assumed to be Newtonian. Generally, suspended particles number density has a destabilizing effect on the thermal convection on the fluids. From the physical point of view the effect of rotation on the micropolar fluids in the presence of suspended particles is interesting because there is a competition between the large enough stabilizing effect of rotation and destabilizing effect (to a smaller extent) of suspended particles. Moreover, rotation introduces Coriolis acceleration which plays an important role on the stability on the system and a centrifugal force which is neglected due to its small magnitude.

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With the growing importance of non–Newtonian fluids in modern technology and industries, oil recovery, petroleum refining and chemical technology. Much attention is being paid to the investigation of such fluids. One such class of viscoelastic fluids is Oldroyd-B fluids with constitutive- equations proposed by Oldroyd [14], we are interested there in. Attention has recently been drawn by calculations of the theological behavior of dilute suspensions and emulsions, whose behavior at small variable shear stresses is characterized by Oldroyd-B model. An experimental demonstration by Toms and Strawbridge [15] reveals that a dilute solution of methyl methacrylate in n-butylacetate agrees well with the theoretical model of the Oldroyd-B fluid. Boffetta et al. [16] have studied Rayleigh-Taylor instability in a viscoelastic binary fluid and found that in polymer solutions, the growth rate of the instability speeds up with elasticity which is confirmed by the numerical simulations of the viscoelastic binary fluid. Sharma and Devi [17] have studied the numerical investigations of stability of stratified viscoelastic Oldroyd-B fluid in the presence of variable magnetic field and have found that the critical wavenumber *kc* and *kmax* for the stability of the system remains unchanged in the presence of stress-relaxation time parameter, strain-retardation time parameter, kinematic viscosity whereas the critical wavenumber *kc* goes on decreasing with the increase in magnetic field.

Keeping in mind the importance of non–Newtonian fluids , medium permeability in modern technology and their various applications mentioned above, the present paper is devoted to consider the stability of rotating stratified elastico–viscous Oldroyd B fluid in the presence of variable magnetic field and rotation in porous medium.

II. RELATED WORK

The initial stationary state whose stability we wish to examine is that of an incompressible, heterogeneous infinitely extending and conducting $\left(\sigma \rightarrow \propto\right)$ elastico–viscous oldroyd B fluid of thickness d bounded by the planes $z = 0, d$ and of variable density, kinematic viscosity and viscoelasticity, arranged in horizontal strata in a porous medium of variable porosity and medium permeability so that the free surface is almost horizontal and the electrical conductivity $\eta = \frac{1}{4\pi\mu_e\sigma}$ is zero. The fluid is acted on by gravity force $g\left(0,0,-g\right)$, a uniform horizontal rotation $\Omega\left(\Omega,0,0\right)$ and a variable horizontal magnetic field

$H(H_0(z),0,0)$.The character of the equilibrium of this stationary state is determined by supposing that the system is slightly disturbed and then, following its further evolution.
The equations expressing conservation of momentum, mass, incompressibility and Maxwell's equations for the elastico–viscous Walters' (model B') fluid are

$$\frac{\rho}{\in}\left(1+\lambda\frac{\partial}{\partial t}\right)\left[\frac{\partial q}{\partial t}+(q\cdot\nabla)q\right]=\left(1+\lambda\frac{\partial}{\partial t}\right)\left(-\nabla p+\rho g+\frac{\mu_e}{4\pi}[(\nabla\times H)\times H]+K\frac{2\rho}{\in}(q\times\Omega)+\frac{KN}{\varepsilon}(q_d-q)\right)-\left(1+\lambda_0\frac{\partial}{\partial t}\right)\frac{\mu}{k_1}q \quad (1)$$

$$\nabla\cdot q=0, \quad (2)$$

$$\nabla\cdot H=0, \quad (3)$$

$$\in\frac{\partial H}{\partial t}=\nabla\times(q\times H), \quad (4)$$

$$\in\frac{\partial\rho}{\partial t}+(q\cdot\nabla)\rho=0, \quad (5)$$

where ρ, μ, p, μ_e, λ, λ_0 and u denote , respectively, the density, the viscosity, pressure, the medium permeability, the magnetic permeability, stress-relaxation time parameter, strain retardation time parameter and the fluid velocity (initially zero). q_d and $N(\bar{x},t)$ denote the velocity and number density of particles. $K'=6\pi\mu\eta'$, η' being theparticle radius, is the Stoke's drag and $\bar{x}=(x,y,z)$ .Equation (5) represents the fact that the density of a particle remains unchanged as we follow it with its motion. Assuming dust particles of uniform particle size, spherical shape and small relative velocities between the two phases (fluid and particles), then the net effect of the particles on the fluid is equivalent to an extra body force term per unit volume $KN(q_d-q)$, as has been taken in equation (1). The presence of particles adds an extra force term, proportional to the velocity difference between particles and fluid which appears in equations of motion
(1).This force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid. The distance between the particles is assumed to be so large compared with their diameter that interparticle reactions are ignored.

$$mN\left(\frac{\partial q_d}{\partial t}+\frac{1}{\varepsilon}(q_d\cdot\nabla)q_d\right)\ u=KN\ (q-q_d) \quad (6)$$

$$\varepsilon\frac{\partial N}{\partial t}+\nabla\cdot(N\cdot q_d)=0 \quad (7)$$

Let $\delta\rho$, δp, $q\ (u,v,w)$, N, $q_d\ (l,r,s)$and $h\ (h_x,h_y,h_z)$ denote, respectively, the perturbations in density $\rho(z)$, pressure $p(z)$, filter velocity $q\ (0,0,0)$, particle velocity $q_d\ (0,0,0)$, suspended particle number density and horizontal magnetic field $H(H_0\ (z),0,0)$. Then the equations (1)–(5) after perturbations in the cartesian form become

$$\frac{\rho}{\in}\left(1+\lambda\frac{\partial}{\partial t}\right)\left[\frac{\partial q}{\partial t}+(q\cdot\nabla)q\right]=\left(1+\lambda\frac{\partial}{\partial t}\right)\left(-\nabla p+\delta\rho\, g+\frac{\mu_e}{4\pi}[(\nabla\times H)\times H]+K\frac{2\rho}{\in}(q\times\Omega)+\frac{KN}{\varepsilon}(q_d-q)\right)-\left(1+\lambda_0\frac{\partial}{\partial t}\right)\frac{\mu}{k_1}q, \quad (8)$$

$$\nabla\cdot q=0, \quad (9)$$

$$\nabla\cdot h=0, \quad (10)$$

$$\varepsilon\frac{\partial h}{\partial t}=(H\cdot\nabla)u-(u\cdot\nabla)H, \quad (11)$$

$$\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)q_d=q\cdot \quad (12)$$

Operating equation (8) by $\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)$ to eliminate q_d between equations (9) and (12), we get

$$\frac{\rho}{\in}\left(1+\lambda\frac{\partial}{\partial t}\right)\left[\frac{\partial q}{\partial t}+(q\cdot\nabla)q\right]=\left(1+\lambda\frac{\partial}{\partial t}\right)\left(-\nabla p+\delta\rho\, g+\frac{\mu_e}{4\pi}[(\nabla\times H)\times H]+K\frac{2\rho}{\in}(q\times\Omega)+\frac{KN}{\varepsilon}(q_d-q)\right)-\left(1+\lambda_0\frac{\partial}{\partial t}\right)\frac{\mu}{k_1}q,$$

Equations (8)-(12) in the Cartesian form are

$$\frac{1}{\in}\left(1+\lambda\frac{\partial}{\partial t}\right)\left[\frac{\partial}{\partial t}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)+Nm\frac{\partial}{\partial t}\right]u=\left(1+\lambda\frac{\partial}{\partial t}\right)\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\left[-\frac{\partial}{\partial x}\delta p+\frac{\mu_e}{4\pi}\left(h_z\frac{\partial H_0}{\partial z}\right)\right]-\frac{\mu}{k_1}\left(1+\lambda_0\frac{\partial}{\partial t}\right)\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)u, \quad (13)$$

$$\frac{1}{\in}\left(1+\lambda\frac{\partial}{\partial t}\right)\left[\frac{\partial}{\partial t}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)+Nm\frac{\partial}{\partial t}\right]v=\left(1+\lambda\frac{\partial}{\partial t}\right)\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\left[-\frac{\partial}{\partial y}\delta p+\frac{\mu_e}{4\pi}\left(\frac{\partial h_y}{\partial x}-\frac{\partial h_x}{\partial y}\right)\right]-\frac{\mu}{k_1}\left(1+\lambda_0\frac{\partial}{\partial t}\right)\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)v, \quad (14)$$

$$\frac{1}{\in}\left(1+\lambda\frac{\partial}{\partial t}\right)\left[\frac{\partial}{\partial t}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)+Nm\frac{\partial}{\partial t}\right]w=\left(1+\lambda\frac{\partial}{\partial t}\right)\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\left[-\frac{\partial}{\partial z}\delta p+\frac{\mu_e}{4\pi}\left(\frac{\partial h_y}{\partial x}-\frac{\partial h_x}{\partial y}\right)\right]-\frac{\mu}{k_1}\left(1+\lambda_0\frac{\partial}{\partial t}\right)\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)w, \quad (15)$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \quad (16)$$

$$\in\frac{\partial}{\partial t}(\delta\rho)+w\frac{\partial\rho}{\partial z}=0, \quad (17)$$

$$\frac{\partial}{\partial x}h_x+\frac{\partial}{\partial y}h_y+\frac{\partial}{\partial z}h_z=0, \quad (18)$$

$$\in\frac{\partial}{\partial t}h_x=H_0\frac{\partial}{\partial x}u-w\frac{\partial}{\partial z}H_0, \quad (19)$$

$$\in\frac{\partial}{\partial t}h_y=H_0\frac{\partial}{\partial x}v, \quad (20)$$

$$\in\frac{\partial}{\partial t}h_z=H_0\frac{\partial}{\partial x}w\cdot \quad (21)$$

Analyzing the disturbances into normal modes, we seek solutions whose dependence on x, y, z and time t is given by

$$f(z)\exp\left(ik_x x+ik_y y+nt\right), \quad (22)$$

where $f(z)$ is the some function of z –only; k_x, k_y are the wave-numbers in the x – and y –directions, respectively, $k=\left(k_x^2+k_y^2\right)^{1/2}$ is the resultant wave-number and n is the growth rate of the disturbance which is, in general, a complex constant.

Equations (13)–(21) using expression (22) become

$$\frac{1}{\in}(1+\lambda n)\left[\rho n\left(\frac{mn}{K'}+1\right)+Nmn\right]u=(1+\lambda n)\left(\frac{mn}{K'}+1\right)\left[-ik_x\delta p+\frac{\mu_e}{4\pi}\left(h_z DH_0\right)\right]-\frac{\mu}{k_1}\left(1+\lambda_0\frac{\partial}{\partial t}\right)\left(\frac{mn}{K'}+1\right)\left(D^2-k^2\right)u, \quad (23)$$

$$\frac{1}{\in}(1+\lambda n)\left[\rho n\left(\frac{mn}{K'}+1\right)+Nmn\right]v=(1+\lambda n)\left(\frac{mn}{K'}+1\right)\left[-ik_y\delta p+\frac{\mu_e H_0}{4\pi}\left(ik_x h_y-ik_y h_x\right)\right]-\frac{\mu}{k_1}(1+\lambda_0 n)\left(\frac{mn}{K'}+1\right)\left(D^2-k^2\right)v, \quad (24)$$

$$\frac{1}{\in}(1+\lambda n)\left[\rho n\left(\frac{mn}{K'}+1\right)+Nmn\right]w=(1+\lambda n)\left(\frac{mn}{K'}+1\right)\left[-D\delta p-g\delta\rho+\frac{\mu_e H_0}{4\pi}\left(ik_x h_z-Dh_x-\frac{h_x}{H_0}DH_0\right)\right]-\frac{\mu}{k_1}(1+\lambda_0 n)\left(\frac{mn}{K'}+1\right)\left(D^2-k^2\right)w, \quad (25)$$

$$ik_x u+ik_y v+Dw=0, \quad (26)$$

$$\in n\delta\rho+wD\rho=0, \quad (27)$$

$$ik_x h_x+ik_y h_y+Dh_z=0, \quad (28)$$

$$\in nh_x=ik_x H_0 u-w DH_0, \quad (29)$$

$$\in nh_y=ik_x H_0 v, \quad (30)$$

$$\in nh_z=ik_x H_0 w, \quad (31)$$

Now substituting the values of h_x, h_y and h_z from equations (29)–(31) in equations (23)–(25), we get

$$\frac{\rho}{\in}nu=-ik_x\delta p-\frac{1}{k_1}(\mu-\mu' n)\left(D^2-k^2\right)u+\frac{\mu_e}{4\pi}\left(\frac{ik_x H_0 w}{n}\right)DH_0+\frac{N_2}{\varepsilon}(q_d-q)+\frac{2}{\in}\rho v\Omega, \quad (32)$$

$$\frac{\rho}{\in}nv=-ik_y\delta p-\frac{1}{k_1}(\mu-\mu' n)\left(D^2-k^2\right)v+\frac{\mu_e}{4\pi}H_0\left(\frac{ik_x H_0\zeta_z}{n}+\frac{ik_y w DH_0}{n}\right)+\frac{N_2}{\varepsilon}(q_d-q)-\frac{2}{\in}\rho u\Omega, \quad (33)$$

$$\frac{\rho}{\in}nw=-D\delta p-\frac{1}{k_1}(\mu-\mu' n)\left(D^2-k^2\right)w+\frac{\mu_e H_0}{4\pi\in}\left[-\frac{k_x^2 H_0 w}{n}-D\left(\frac{ik_x H_0 u}{n}-\frac{wDH_0}{n}\right)-\left(\frac{ik_x H_0 u}{n}-\frac{wDH_0}{n}\right)\frac{DH_0}{H_0}\right]+\frac{N_2}{\varepsilon}(q_d-q)\frac{g(D\rho)w}{\in}, \quad (34)$$

where $\zeta_z=ik_x v-ik_y u$, is the z –component of vorticity.

Multiplying equations (32) and (34) by $-ik_y$ and ik_x, respectively, and then adding we get

$$\frac{\rho\, n}{\in}\zeta_z = -\frac{\rho}{k_1}(\upsilon+\upsilon' n)\left(D^2-k^2\right)\zeta_z - \frac{\mu_e k_x^2 H_0^2}{\in 4\pi\, n}\zeta_z + \frac{N_2}{\varepsilon}(q_d-q) + \frac{2}{\in}\Omega\, Dw,$$

or
$$\zeta_z = \frac{2\, n\, \Omega\, Dw}{n^2 - \frac{\in n}{k_1}(\upsilon-\upsilon' n)(D^2-k^2) + k_x^2 V_A^2}, \tag{35}$$

where $N_2 = KN_0 \dfrac{d^2}{\mu}$, $\upsilon = \dfrac{\mu}{\rho}$, $\upsilon' = \dfrac{\mu'}{\rho}$ and $V_A^2 = \dfrac{\mu_e H_0^2}{4\pi\rho}$ (square of the Alfvén's velocity).

Substituting the value of ζ_z in equation (33), we get

$$\frac{\rho}{\in} n\, v = -ik_y \delta p - \frac{1}{k_1}(\mu-\mu' n)\left(D^2-k^2\right)v - \frac{\mu_e H_0}{4\pi \in n}\left(\frac{2\,\Omega\, n\, Dw\; ik_x}{n^2 - n(\mu-\mu' n)\left(D^2-k^2\right) + k_x^2 V_A^2}\right) + \frac{\mu_e H_0}{4\pi\, n} ik_y\, w\, D\left(H_0\right) - 2\frac{\rho}{\in}u\Omega\cdot \tag{36}$$

Multiplying equations (32) and (34) by $-ik_x$ and $-ik_y$, respectively, and then adding and using (35), we obtain

$$\frac{\rho}{\in} n\, Dw = -k^2\delta p - \frac{\rho}{k_1}(\upsilon-\upsilon' n)\left(D^2-k^2\right)Dw + \left(\frac{2n\Omega}{n^2 - \frac{\in n}{k_1}(\upsilon-\upsilon' n)\left(D^2-k^2\right) + V_A{}^2 k_x^2}\right)\left(\frac{\mu_e H_0^2}{4\pi \in n}k_x^2 k_y - \frac{2}{\in}\rho\right)Dw \tag{37}$$

Eliminating u, v and δp from equations (32)–(34) using equations (35), after little algebra, we get

$$\in \frac{n\rho}{k_1}(\upsilon-\upsilon' n)\left(D^2-k^2\right)^2 w - \frac{n\rho}{k_1}\left(n^2+k_x^2V_A^2\right)\left(D^2-k^2\right)w - \left[n^2(D\rho)\left(1+\frac{4\Omega^2}{n^2 - \frac{n\in}{k_1}(\upsilon-\upsilon' n)\left(D^2-k^2\right) + k_x{}^2V_A^2}\right) - \frac{\mu_e k_x^2 D\left(H_0^2\right)}{4\pi \in n}\right]$$

$$Dw - \frac{n\rho}{k_1}\left(n^2+k_x^2V_A^2\right)\left(D^2-k^2\right)w - \left[n^2(D\rho)\left(1+\frac{4\Omega^2}{n^2 - \frac{n\in}{k_1}(\upsilon-\upsilon' n)\left(D^2-k^2\right) + k_x{}^2V_A^2}\right) - \frac{\mu_e k_x^2 D\left(H_0^2\right)}{4\pi \in n}\right] gk^2(D\rho)w = 0\cdot \tag{38}$$

III. METHODOLOGY

In order to obtain the solution of the stability problem of a layer of Oldroyd B fluid, we suppose that the density ρ, viscosity μ, viscoelasticity μ' medium porosity $\in$ and medium permeability μ' vary exponentially along the vertical direction i.e.

$$\rho = \rho_0\, e^{\beta_1 z},\quad \mu = \mu_0\, e^{\beta_1 z},\quad \mu' = \mu_0'\, e^{\beta_1 z},\quad \in = \in_0 e^{\beta_1 z}, k_1 = k_{10} e^{\beta_1 z} \tag{39}$$

where ρ_0, μ_0, μ_0', H_1, $\in_0$, k_{10} and β_1 are constants and so the kinematic viscosity $\upsilon\left(=\dfrac{\mu}{\rho}=\dfrac{\mu_0}{\rho_0}\right)$, the kinematic viscoelasticity $\upsilon'\left(=\dfrac{\mu'}{\rho}=\dfrac{\mu_0'}{\rho_0}\right)$ and the Alfvén velocity $V_A = \left(\dfrac{\mu_e H_0^2}{4\pi\rho}\right)^{1/2} = \left(\dfrac{\mu_e H_1^2}{4\pi\rho}\right)^{1/2}$ are constant everywhere.

Using the stratifications of the form (39), equation (38) transforms to

$$\left(D^2-k^2\right)^3 w - \frac{2}{\frac{\in_0 n}{k_{10}}(\upsilon_0-\upsilon_0' n)}\left(n^2+k_x^2V_A^2\right)\left(D^2-k^2\right)^2 w + \frac{1}{n^2\frac{\in_0}{k_{10}}(\upsilon_0-\upsilon_0' n)^2}\left[n^4 + k_x^2V_A^2\left(2n^2+k_x^2V_A^2\right) - V_A^2\, k_x^2\,\beta_1\, n(\upsilon_0-\upsilon_0' n)\right.$$

$$\left. - g\, k^2\beta_1\, n(\upsilon_0-\upsilon_0' n)\right]\left(D^2-k^2\right)w + \frac{N_2}{\varepsilon}(q_d-q) - \frac{1}{\frac{\in_0 n}{k_{10}}(\upsilon_0-\upsilon_0' n)^2}\left[4\Omega^2 n^2 + V_A^2k_x^2\beta_1\left(n^2+k_x^2V_A^2\right) - g\, k^2\beta_1\left(n^2+k_x^2V_A^2\right)w\right] = 0. \tag{40}$$

Considering the case of two free boundaries, we must have

$$w = D^2 w = 0 \text{ at } z = 0 \text{ and } z = d. \tag{41}$$

The appropriate solution of equation (41) satisfying the above boundary condition is

$$w = A_0 \sin\frac{m\pi\, z}{d}, \tag{42}$$

where m is an integer and A_0 is a constant.
Substituting the value of w from equation (42) in equation (40) we obtain dispersion relation

$$n^4\left[\left(1-\frac{2\in_0}{k_{10}}\upsilon_0' L_3\right)^2\right]+n^3\left[2\upsilon_0 L_3\left(1-\frac{2\in_0}{k_{10}}\upsilon_0' L_3\right)\right]+n^2\left[L_3^2\upsilon_0^2+\left(2k_x^2V_A^2-\frac{g\,k^2\beta_1}{L_3}\right)(1-\upsilon_0' L_3)-\frac{1}{L_3}V_A^2k_x^2\beta_1(1-\upsilon_0' L_3)\right]$$
$$-\frac{4\in_0\Omega^2k_x^2}{L_3k_{10}^2}\frac{1}{L_3}V_A^2k_x^2\beta_1(1-\upsilon_0' L_3)\Bigg]+n\left[v_0L_3\left(2k_x^2V_A^2-\frac{\in_0^2 g\beta_1 k^2}{k_{10}^2L_3}\right)-\frac{1}{L_3}V_A^2k_x^2\beta_1(1-\upsilon_0' L_3)\right]+k_x^2V_A^2\left[k_x^2V_A^2-\frac{\beta_1}{L_3}\left(g\,k^2+V_A^2k_x^2\right)\right]=0 \quad (43)$$

where $L_3=\left[k^2+\dfrac{m_1^2\pi^2}{d^2}\right]$.

Equation (43) is biquadratic in n and is the dispersion relation governing the effects of uniform rotation, variable horizontal magnetic field, viscosity, viscoelasticity and medium porosity on the stability of stratified Oldroyd B fluid.

IV. RESULTS AND DISCUSSIONS

Case of stable stratifications (i.e. $\beta_1<0$) and $\left(k_{10}>4\in_0 v_o'\right)$, Equation (43) does not admit any positive real root or complex root with positive real part using Routh–Hurwitz criterion; therefore, the system is always stable for disturbances of all wave-number.

Case of unstable stratifications (i.e. $\beta_1>0$) and $\left(k_{10}<4\in_0 v_o'\right)$, If $\beta_1>0$, $\dfrac{k_x^2V_A^2}{k^2}\left(1-\dfrac{\beta_1}{L_3}\right)<\dfrac{\beta_1}{L_3}g$, the constant term in the equation (43) is negative and therefore has at least one root with positive real part using Routh–Hurwitz criterion; so the system is unstable for all wave-numbers satisfying the inequality

$$k^2<\frac{\beta_1 d^2 g\sec^2\theta-V_A^2\left(m_1^2\pi^2-\beta_1 d^2\right)}{V_A^2d^2}, \quad (44)$$

where θ is the angle between k_x and k i.e. $\left(k_x=k\cos\theta\right)$.

If $\beta_1>0$, (unstable stratifications) $1>\dfrac{\beta_1}{L_3}$ and $V_A^2>\dfrac{\beta_1\,g\,k^2}{L_3\,k_x^2\left(1-\dfrac{\beta_1}{L_3}\right)}$,equation (43) does not admit of any positive real root or complex root with positive real part, therefore, the system is stable. The system is clearly unstable in the absence of magnetic field, rotation and for non–viscoelastic fluid.

$$n^4\left[\left(1-\frac{2\in_0}{k_{10}}\upsilon_0' L_3\right)^2\right]+n^3\left[2\upsilon_0 L_3\left(1-\frac{2\in_0}{k_{10}}\upsilon_0' L_3\right)\right]+n^2\left[L_3^2\upsilon_0^2-\frac{g\,k^2\beta_1}{L_3}(1-\upsilon_0' L_3)+\frac{1}{L_3}V_A^2k_x^2\beta_1(1-\upsilon_0' L_3)\right]-n\left[v_0L_3\frac{\in_0^2 g\beta_1 k^2}{L_3k_{10}^2}\right]=0. \quad (45)$$

For $\beta_1>0$, the constant term in the equation (43) is negative and therefore has at least one root with positive real part therefore the system is clearly unstable. The magnetic field, therefore, stabilizes potentially unstable stratifications for small wave-length perturbations

$$k^2>\frac{\beta_1\,d^2 g\sec^2\theta-V_A^2\left(m_1^2\pi^2-\beta_1 d^2\right)}{V_A^2d^2}. \quad (46)$$

Also, it is clear that the wave-number range, for which the potentially unstable system gets stabilized, increases with the increase in magnetic field and decreases with the increase in kinematic viscoelasticity. All long wave-length perturbations satisfying equation (44) remain unstable and are not stabilized by magnetic field. The behaviour of growth rates with respect to kinematic viscosity υ_0, kinematic viscoelasticity υ_0' and square of the Alfvén velocity V_A^2 satisfying equation (43) has been examined numerically using Newton–Raphson method through the software Mathcad. Figure (1) shows the variation of growth rate n_r (positive real value of n) with respect to the wave-number k for fixed permissible value of $\beta_1=2,\ \in_0=0.5, k_{10}=6, m_1=1,\ d=6\,\text{cm},\ \Omega=1$revolution/minute, $\upsilon_0'=1$ for three values of $\upsilon_0'=2$, 3 and 4, respectively. These values are the permissible values for the respective parameters and are in good agreement with the corresponding values used by Chandrasekhar [8] while describing various hydrodynamic and hydromagnetic stability problems. The graph shows that for fixed wave-numbers, the growth rate increases for certain wave number with the increase in kinematic viscoelasticity υ_0', which indicates the destabilizing effect of viscoelasticity whereas the growth rate decreases for certain wave numbers implying thereby the stabilizing effect of kinematic viscoelasticity on the system in the presence of medium permeability and medium porosity for low wave numbers range.

Figure (2) shows the variation of growth rate n_r (positive real value of n) with respect to the wave-number k for fixed permissible values of $\beta_1 = 2,\ m_1 = 1,\ d = 6\,\text{cm},\ \Omega = 1$ revolution/minute, $\upsilon_0' = 1$, $g = 980\text{cm/s}^2,\ k_x = k\cos 45^0,\ V_A^2 = 55, \in_0 = 0.5, k_{10} = 6,$ for three values of $\upsilon_0 = 2$, 4 and 6, respectively. The graph shows that for fixed wave-numbers, the growth rate increases for certain wave number with the increase in kinematic viscosity υ_0 which indicates the destabilizing influence of kinematic viscosity, whereas the growth rate decreases for certain wave numbers, implying thereby the stabilizing effect of kinematic viscosity on the system in the presence of medium permeability and medium porosity.

Figure (3) shows the variation of growth rate n_r (positive real value of n) with respect to wave-number k for fixed permissible values of $\beta_1 = 2,\ m_1 = 1,\ d = 6\,\text{cm},\ \Omega = 1$ revolution/minute, $\upsilon_0 = 4,\ \upsilon_0' = 2$, $g = 980\,\text{cm/s}^2,\ k_x = k\cos 45^0, \in_0 = 0.5, k_{10} = 6$ for two values of $V_A^2 = 15$ and 55, respectively. The graph shows that for fixed wave-numbers, the growth rate increases with the increase in the square of the Alfvén velocity V_A^2 for certain wave number which indicates the destabilizing influence of the square of the Alfvén velocity, whereas growth rate decreases for certain wave numbers, implying thereby the stabilizing effect of the square of the Alfvén velocity on the system in the presence of medium permeability and medium porosity.

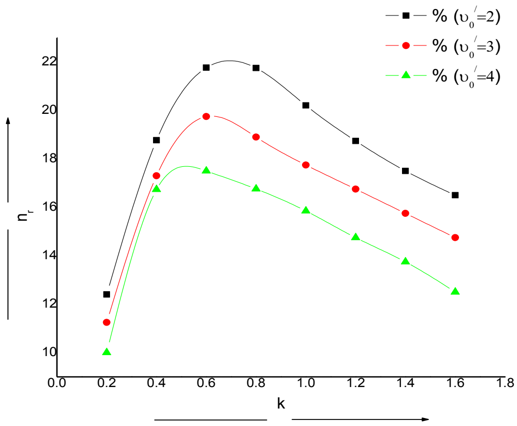

Figure 1:The variation of n_r with wave-number k for three values of $\upsilon_0' = 2{,}3{,}4$

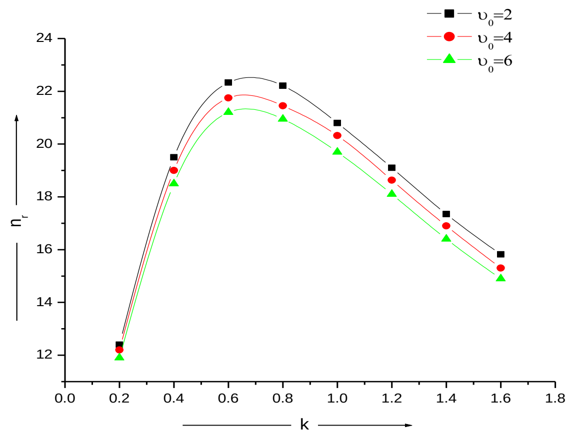

Figure 2: The variation of n_r with wave-number k for three values of $\upsilon_0 = 2, 4, 6$

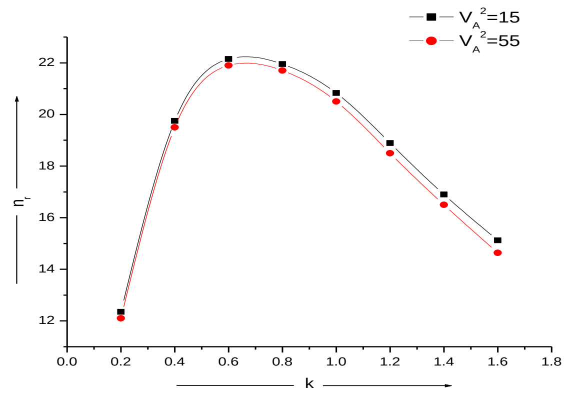

Figure 3:The variation of n_r with wave-number k for two values of $V_A^2 = 15, 55$.

V. CONCLUSION

Both the parameters such as kinematic viscosity, kinematic viscoelasticity, has the stabilizing effect on the system whereas the parameters such as suspended particles and rotation have the destabilizing effect on the system. Square of the Alfvén velocity V_A^2 also has stabilizing effect on the system in the presence of medium permeability and medium porosity. We have considered Oldroyd B fluid viscoelastic fluid in the present problem. The problem could be extended by taking different fluids such as Rivlin –Ericksen fluid, Nano fluid particles and Ferromagnetic fluids.

REFERENCES

[1]. Lapwood, E.R.(1948), "Convection of fluid in a porous medium. Proc. Camb."Phil. Soc.Vol. 45, pp. 508.
[2]. Wooding, R. A. (1960), "Rayleigh instability of a thermal boundary layer in flow through a porous medium". J. Fluid Mech. Vol. 9,pp.183.
[3]. McDonnel, J. A. M. (1978), "Cosmic Dust." John Wiley and Sons, Vol. 330.
[4]. Rayleigh, L.(1883) "Proc. London Math. Soc." Vol. 14, pp. 170.
[5]. Taylor, G. I. (1950), "The instability of liquid surfaces when accelerated in direction perpendicular to their planes". Proc. Roy. Soc. (Lon.) Vol. A 201, pp. 192.
[6]. Hide, R.(1955), "Waves in a heavy, viscous, incompressible, electrically conducting fluid of a variable density in the presence of a magnetic field." Proc. Roy. Soc. (London,) Vol. A233, pp.376.
[7]. Reid, W. H. (1960), "The effect of surface tension and viscosity on the stability of two superposed fluids." Proc. Camb. Phil. Soc.Vol. 57 ,pp. 415.
[8]. Chandrasekhar ,S. (1981), "Hydrodynamic and Hydromagnetic stability", Dover publications, New York, USA.
[9]. Bellman R. and Pennington, R. H. (1954), "Effect of surface tension and viscosity on Taylor Instability" Quart. Appl. Math., Vol. 12,pp. 151–162.
[10]. Gupta, A. S. (1963), "Rayleigh–Taylor instability of a viscous electrically conducting fluid in the presence of a horizontal magnetic field". J. Phys Soc. Japan, Vol. 18 ,pp. 1073.
[11]. Kent, A.(1966), "Instability of laminar flow of a magnetofluid. Phys. Fluids, Vol. 9pp 1286.
[12]. Fredricksen. (1964), "Principles and applications of Rheology"Prentice – Hall Inc". New Jersy.
[13]. Joseph, D. D.(1976) , "Stability of fluid motions". Springer-Verla Berlin, Heidelberg and New York.
[14]. Oldroyd, J .G. (1950), "On the formulation of rheological equations of state".Proc.Roy .Soc.(Lon.), Vol. 200, App. 523.
[15]. Toms, B.A. and Strawbridge, D. J. (1953), "Elastic and viscous properties of dilute solutions of polymethyl methacrylate in organic liquids". Trans.
[16]. Boffetta et al. (2010), "Rayleigh-Taylor instability in a viscous-elastic binary fluid". Journal of fluid mechanics, Vol.643,pp. 127-136.
[17]. Sharma,V and Devi ,S.(2012) "Numerical investigations of stability of stratified viscoelastic Oldroyd fluid in the presence of variable magnetic field."Proceedings of HEM, India,pp. 248-251.