

Cross Ratios, Color Charge Force and Gravity

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Abstract

In earlier publications of the author are mentioned as spacetime extension the octonians. This is explored in the first section for presenting a color charge subspace through the use of the complex cross ratios as color charge invariants under Moebius transformations of a complex Riemannian plane. Eigenvectors of the symmetries are presented by using six octonian coordinates for six energies, having different metrical qualities. As model, the SI rotor is described for nucleons and deuteron with an own inner complex projective spacetime CP^2 . There is a third spacetime available by the $SU(3)$ coordinate extension from spacetime 1234, listed by indices of octonian coordinates as subspace of 123456 where the 4-dimensional subspace 1256 is for gravity with mass. Several projections and projective normings are described where 1256 is in superposition with 1234 or 3456 as subspace for CP^2 . The 07 octonian coordinates are reserved for a G-compass, a color charge force 0 and the electromagnetic force on 7. Extending 123456 to octonians means that also astronomy can use this investigation for dark energy and dark matter.

Date of Submission: 09-03-2021

Date of acceptance: 23-03-2021

I. RIEMANNIAN SPHERE, CROSS RATIOS AND OCTONIANS

If on a Riemannian sphere three reference points $0, -1, \infty$ are chosen, the complex cross ratios of them with a complex variable z added are permuted as 4-tuple for getting the six cross ratios. Their normed coefficient matrices are the members of the D_3 symmetry of the nucleons quark triangle $id, \alpha\sigma_1, \alpha^2, \sigma_1$ (the first Pauli matrix), $\alpha^2\sigma_1, \alpha$ (rotation of order 3 by 120°). As for other forces, as symmetry act here the Moebius transformations MT with space S^2 , a complex closed 2-dimensional sphere by adding ∞ as stereographic point at infinity with the map $st: S^2 \rightarrow C$.

In most books, as reference points are chosen 1 instead of -1. With this change, the MT for the spherical θ written third real space z -coordinate is $(z-1)/z$. This is taken in normed form as transformation 2×2 -matrix G of order 6 with first row $(1 \ -1)$ and second row $(1 \ 0)$. An eigenvector

$e_0 = (-p_2, 1)$ of G is for a 60° rotation. It is used like a compass needle, but turns only discrete with the 6th roots of unity. In turning, it provides the area in between two adjacent rays by a color charge red r , green g , blue b , turquoise $c(r)$, magenta $c(g)$, yellow $c(b)$.

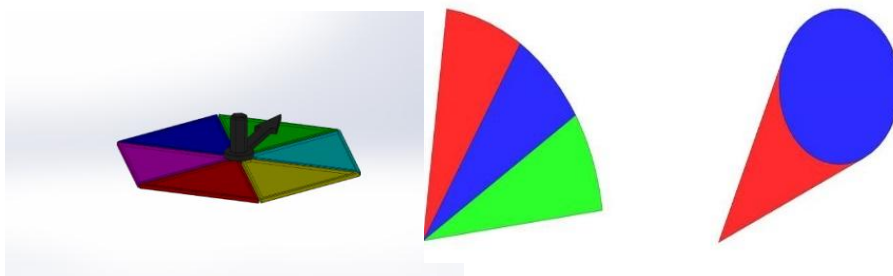


Figure 1 G-compass, rgb -graviton whirl cone

As a spin-like orthogonal base triple acts rgb , the neutral color charge of all baryons and nucleons. In indices of octonian coordinates they are listed as a Gleason operator frame GF, 126 for rgb -gravitons. Mostly this superposition is drawn as three overlapping circular disks, colored with one of the three $r, g, \text{ or } b$.

In a complex written 123456 octonian space they are the real parts r, g, b observable as color charge of a quark in a nucleons uud proton or ddu neutron. Not observable are the imaginary anticolors, except for the color charges in mesons or gluons. Gluons belong to the strong interaction

with symmetry $SU(3)$. The GF present the Copenhagen interpretation for the quantum measurements: *only one real value is observable in a measurement, the other two of the triple remain undetermined. The measured system can change its state in measurement.* Commuting operators can be measured simultaneously.

The octonian coordinates can have for e_0 radius r as measure, 123 are the space coordinates xyz , 4 is the time t coordinate, 5 as coordinate is for mass m with kg as measure and 6 is frequency f as inverse time interval with Hz as measure. 56 is a new energy plane added to spacetime as complex cross product $z_3 = (m,f) = z_1 \times z_2$ with the Einstein line $mc^2 = hf$. In the quaternionic 2×2 -matrix notation, complex spacetime coordinates are $z_1 = z + ict$, $z_2 = x + iy$. A eighth octonian coordinate 7 is introduced later on, measured in cd candela for light waves.

The SI rotor and eigenvectors

Instead of the color charged disks, for a quark triangle in a nucleon the color charges red, green are set on two sides of the triangle with one vectorial side e_0 having r as weight and at the other e_0 copy has g as weight. The third triangle vertex is the common initial point of the two vectors. When the red vector is conic clockwise cw rotated about the quark vertex with color charge r , the cone carries like a condenser plate the color charge r . Its conic whirl is similar understood as the magnetic field quantum as conic whirl for a magnetic field. In a 6-cyclic time sequence, set by the G -matrix, the first rotation generates a barycentrical coordinate of the nucleon as rotation axis. The gb -quarks exchange their place through a gluon exchange and b is cw rotated. For the color charged g quark the similar counterclockwise mpo rotation applies. The g -quark is kept fixed and its vector conic rotated which sets a second barycentrical coordinate for the nucleon triangle. A third barycentrical coordinate is generated by cw rotating the r -quark at its new position. This is repeated in sequence for the positions of r,g in six steps. At the intersection B of the barycentrical coordinates a Higgs boson can set a renormed mass of the nucleon such that its parts can move in an environment with a common group speed. The blue vector is rotated at each of the six steps and points at the triangles vertices (which carry a quarks mass) in opposite positions, - the triangle vertices are locally kept at rest.

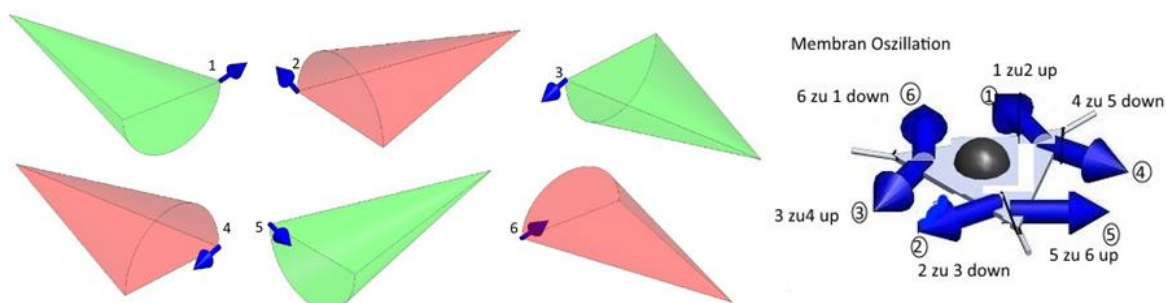


Figure 2 conic red-green rotations, blue rotation in six steps

The six color charge conic whirls are the field quanta for a color charge field. In the former mentioned G -compass version, the six color charge whirl cones are cut at one triangle side and set on two adjacent 6th roots rays of the compass (figure 1). In octonian coordinates and D_3 symmetries, they are in a factor group of the S_4 symmetry, permuting the four elements in the cross ratios. The coordinate 1 has r as color charge and σ_1 as symmetry, 2 has g and $\alpha\sigma_1$, 3 has $c(g)$ and α^2 , 4 has $c(b)$ and $\alpha^2\sigma_1$, 5 has id and $c(b)$, 6 has α and b .

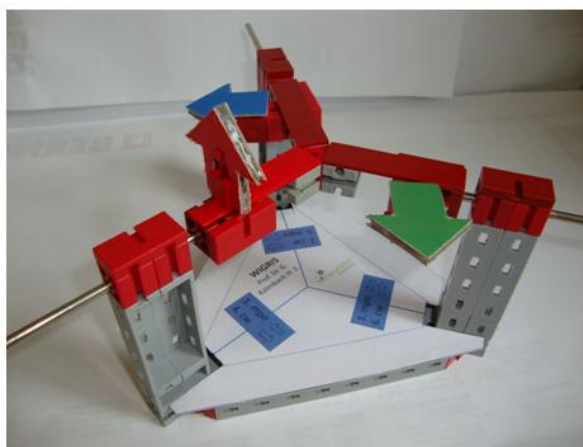
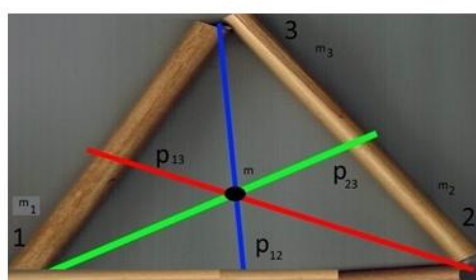


Figure 3 barycentrical coordinates, model SI rotor (models are from the Tool bag [7])

The grouping of the S_4 factor classes is complemented in the same order by the quality of an energy $EM(pot)$ electrical potential, $E(heat)$ temperature, $E(rot)$ angular momentum and rotational energy, $E(magn)$ magnetic energy, $E(pot)$ gravitational potential, $E(kin)$ kinetic energy and frequency f in

$E = hf$, h the Planck constant. For the classes, S_4 is factored by the CPT Klein-group $Z_2 \times Z_2$ as commutative version of the Pauli $SU(2)$ generators. The group $D_3 = S_4/\cong$ is presenting the six classes. It has as representation the SI rotor dynamics described above for the generation of barycentric coordinates. In tangential metrical form du , each coordinate u is for integrations of functions $f(u)$ in $\int f(u)du$. 1 has r radius, integrating a force $f(r) = b/r^2$ to a potential $-b/r$. 2 has a volume V function integration in space coordinates for entropy inside a volume $\int f(x,y,z)dV$, 3 and 6 are dt -integrated from $E(\text{rot})$, $E(\text{kin})$ to angular speed and linear speed, 4 is area A integrated to induction $B = \int \Phi dA$ for a magnetic field (strength) crossing transversal an electrical currents loop. B is a rotational momentum and real cross product of the two electromagnetic EM energies.

The e_0 G-compass needle has an $(-p_2, 1)$ eigenvector of G representation for the color charge energy. The same is repeated for the other 5 cross ratio energies with color charges attached. The id matrix for scalings and the EM charge 1 has the eigenvector $(1,0)$, 2 heat $(0,1)$ or $(0,i)$, 4 magnetic energy $(i,0)$, 5 mass $(1,1)$, 6 frequency $(1,-p_1)$. In figure 3 right are shown 1 red as vector $E(\text{pot})$ 5, 2 green as vector $E(\text{rot})$ 3 and 6 blue $E(\text{kin})$ as vector for the SI rotors 6-cycle dynamics as observables. The vector presentation of forces is taken as in physics. The symmetries for them are the cross ratios.

Spaces of SU(3) for spacetime, deuteron and atomic kernels, gravity with mass

Above was mentioned the complex cross product extension from quaternionic spacetime 2×2 -matrices to z_3 . The new coordinate was given by the complex cross ratios D_3 matrix elements for mass and frequency. Another matrix presentation is due to $SU(3)$ and the GellMann projection 3×3 -matrices which arise by inserting a row and a column of coefficients 0 in the three Pauli matrices. In this notation there are then in 3×3 -matrix projections form the spaces with first row $(z_1 z_2 0)$, $(z_1 0 z_3)$, $(0 z_2 z_3)$.

The first space is projected down by *rgb*-gravitons (using the extended Pauli matrices) to real xyz -space and time is added for this projection, extending the gravitons GF to a 4-dimensional subspace Boolean block. This $xyzt$ -space 1234 is taken as a Hilbert space R^4 with the Euclidean metric. In complex extended C^4 form it has a Hermitean metric, but the same subspace lattice structure L as union of Boolean blocks 2^4 [1]. They present a maximal set of commuting projection operators. The second space (z_1, z_3) is for an inner spacetime 3456 of nucleons and atomic kernels. They are projected into spacetime as bubbles and have a complex closure to CP^2 , described in the reference articles and book [2]. The SI rotor describes their inner dynamics.

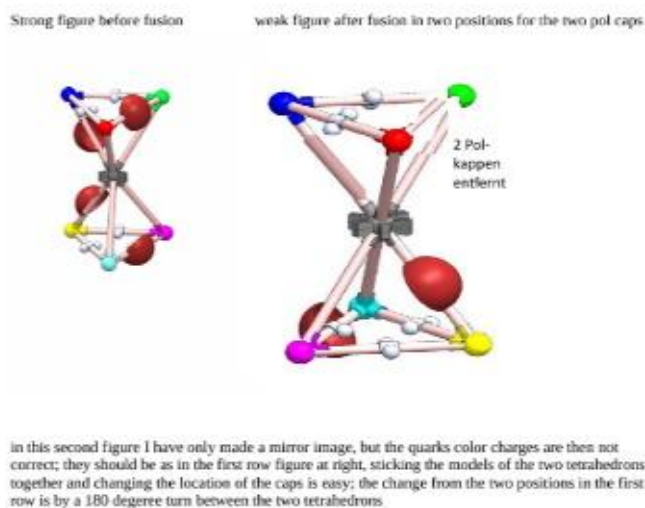


Figure 4 two nucleons are shown for fusion, at left two protons, at right as deuteron and upper neutron with one proton having emitted a positron (two red caps at left) and a neutrino

The third space 1256 is for gravity with the spin-like 126 GF for *rgb*-gravitons and with Higgs mass 5 added. - From the cross ratio symmetries used, the $c(g)$, $c(b)$ 34 cross ratios are missing. As GF 126 generates a nucleon tetrahedron with the S_4 symmetry where this GF has its center in the center of a sphere S^2 as boundary of the former deuteron CP^2 space (figure 4). Higgs has set at left a common barycenter for two protons and one uu -quark pair is weak decaying (figure 4) one u -quark to a d -quark, $uud \rightarrow udd$ for the neutron at right.

The xyz-space coordinates are presented at right as rays on the coordinate axes, 6 on -z, 2 on +y, 1 on +x. The Heisenberg uncertainties add on their opposite rays 4 on +z, 3 on -y, 5 on -x. This way 123456 as octonian color charge space with the complex cross ratios as generators is now 126 or xyt-space 3-dimensional, with mass 5 added it is 4-dimensional for gravity with mass as charge and *rgb*-gravitons as field quantum. In deuteron as nucleon Cooper pair, 126 is for a proton and 345 for a neutron tetrahedron, but they can exchange places through a weak isospin I_3 exchange.

This projection of 1256 into CP^2 3456 makes atomic kernels AK and deuteron to a common gravity, $SU(3)$ SI space 123456, in octonians real 6- or complex 3-dimensional. A rescaling of the quarks mass sum m_0 is described in [3]. This is necessary for getting a common group $v < c$ of the AK with which it moves on a world line as matter wave in its spacetime environment 1234. Mass is special relativistic rescaled, also differentiated as $m(v)$ function of v , and inner frequencies, speeds f are added to m_0 , transformed by $mc^2 = hf$ into kg.

The Gravity-Mass Space

The gravity-mass space 1256 is responsible for many kinds of projections and projective shapes with different kinds of metrics. Also the quadric metrical approach is generalized as in catastrophe theory where higher degree polynomials describe potentials [5]. They allow sudden changes of a systems states, jumps on different potential levels. As example, the cusp is shown in figure 6. Bifurcations are also observed in their control spaces. For CP^2 the elliptic umbilic can be used for a quark-gluon plasma 123456 inside with its potential $a(x^3 - 3xy^2) + b(x^2 + y^2) + v_y y + v_x x$, v_u flows in the u-coordinate direction, b a vorticity, $a \neq 0$ constants, for the 6 roll mill (figure 5). For the weak decays (Heegard decompositions for leptons), $b = 0$ which is a 4 roll mill 1456, 23 rolls are deleted.

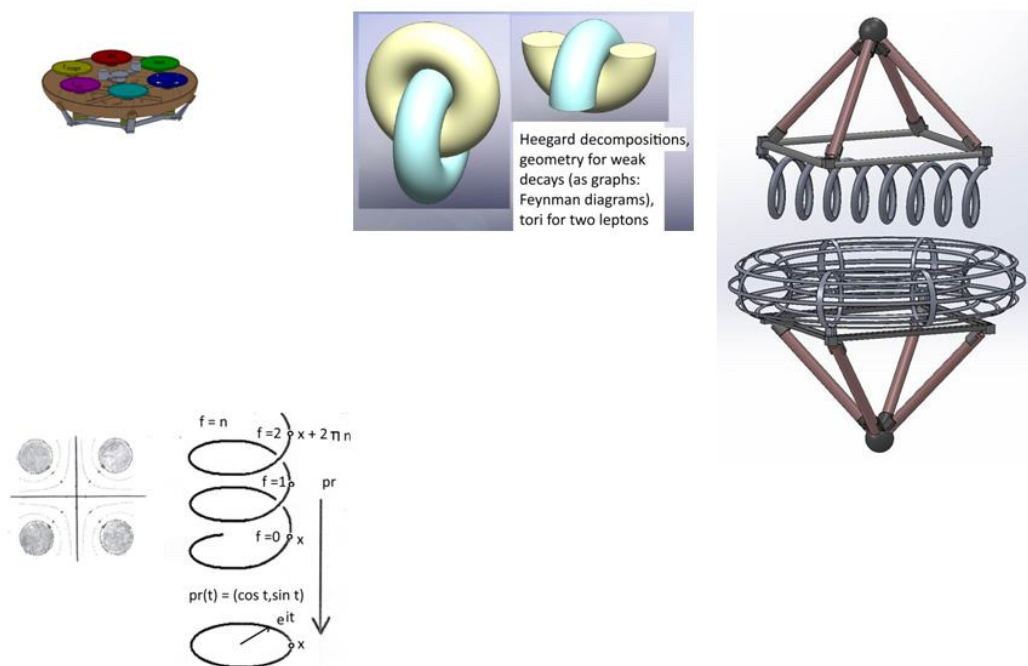


Figure 5 6 roll mill as inner flow for deuteron, below: 4 roll mill as inner flow for leptons, and Heegard decompositions of the 3-dimensional Hopf sphere S^3 by splitting two twisted tori (upper miccle) into two leptons with own 4-dimensional coordinates, at right a neutral spindle shaped lepton above an EM charged toroidal lepton; the Heegard decomposition (lower line) is also into two photon cylinders (beside the 4 roll mill) with helix lines for their in time expanding frequency location on the surface

As catastrophe for 1256 is suggested the parabolic umbilic. It has an umbilic point for the Minkowski double cone. A (force) eigenvector (1,2) for this metric belongs to the metrical scaling M 2x2-matrix with first row (-1 1) and second row (0 1). For the Zeeman gravitational machine 4 hyperbolic umbilics are in superposition and generate the 4 cusps in motion ([5], figure 6). They can present the bifurcations of spacetime coordinates into octonian coordinates 15, 46 the Heisenberg bifurcations and new 27, 03 for the G-compass 0 and the electromagnetic interaction 7 with a rolled $U(1)$ (cylindrical expanded figure 5) Kaluza-Klein coordinate. It is stereographic projected down to the tangent line 7 of the circle.

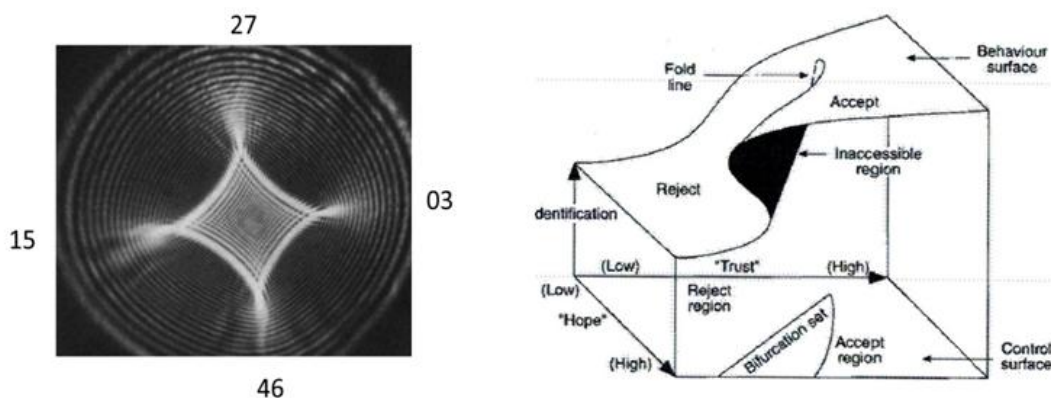


Figure 6 4 cusps in superposition, at right on the manifold with three potential levels (upper accept, lower reject and black a cubics inaccessible potential level); bifurcations are in a projection on the lower control surface

The potentials g are in these presentations treated parametrized g_t and the catastrophes have 1-4 parameters and 1 or 2 variables, the umbilics have 2 variables and 3 parameters for the elliptic and hyperbolic cases and 4 parameters for the parabolic umbilic. The fold (figure 7) has 1 variable and 1 parameter. It serves for the gravities expansion/contraction pendulum motion where the change is on the right fold points end.

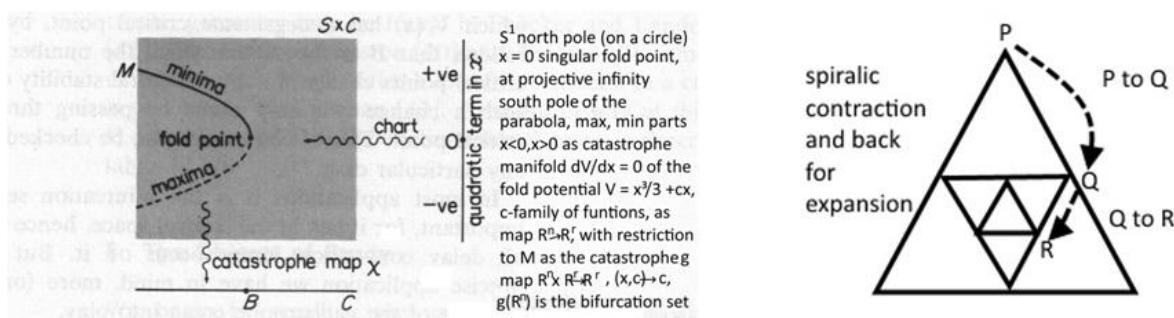


Figure 7 fold catastrophe, at right the nucleons quarks, as dogs chasing one another head to tail for contraction of the nucleon triangles area and (pendulum) reversely the expansion; this map for gravity is spiralic presented

The pendulum action of rgb -gravitons is in superposition with the six SI rotors dynamics (figure 8). The triangle is contracted between the integrations of EM(pot) from large to middle size, of E(magn) form middle to small size, it remains small for E(pot) integration, for E(rot) it expands to the middle size, for E(kin) to the large size and for E(heat) it remains large.

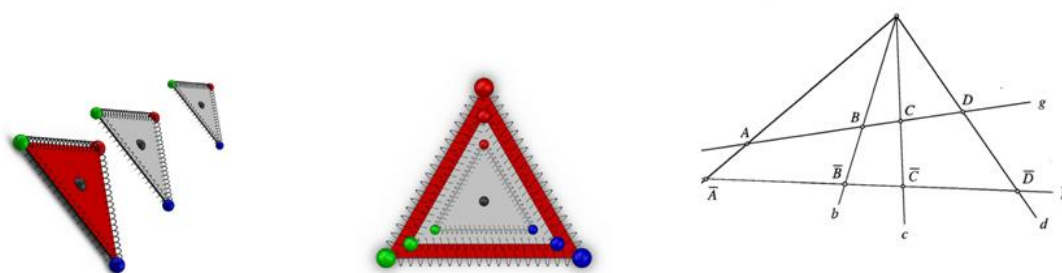


Figure 8 rgb -graviton area change, perspective view at left, top view at right; at right central projection for rescaling distances with proportions preserved

The proportions for the nucleon triangles area can be taken from the basic spins degenerate D_3 orbit as scaled $1/2:1:2$.

Central and stereographic projections are one kind of maps used by rgb -gravitons. Projective normings is another kind. If 56 is projective extended to $[5,6,w]$ as P^2 , the normal forms appear as orbits for systems P in motion: circle, ellipse for Kepler's central rotations of P about a huge barycenter B (for instance of a sun with P

as planet), where Einstein corrections are described later on, parabolas, hyperbolas for escape orbits of P from B where Higgs common barycenter is annihilated. Other forms are two intersecting lines, a line ($mc^2 = hf$ for instance), a point (B for a barycenter). Metrical important is that projective to a point a dual line is associated and a quadric, consisting of those points incident with their dual line. These are 3-dimensional extended to P^3 which is not repeated here, also not the projective closures of P^n .

Another orthogonal norming due to gravity is observed for two hitting galaxies with spiralic orbits.

This is presented as raytracing. The Schwarzschild radius R_s of a mass system Q depends on mass which can be rescaled when Higgs sets a common barycenter for several mass systems. In the following descriptions R_s is used, keeping in mind that for different cases it is rescaled. In a central projection cpr with tip T as (stereographic) point on the vertical space z-axis, Q is sitting below on a vertical x-axis at the origin. It measures its distance to another small mass system P in rotation as r and use raytracing by setting the distance $|QT| = r$. Gravity lifts P from the x-axis up vertical to the height R_s of the Schwarzschild radius of Q when connected by the second central projection ray with T. The distance measure for P is subtracting R_s in $|PT| = r - R_s$ on the z-axis. In a projective plane, the coordinates are then for $[r-R_s, r, w]$ and setting $w = 0$ the projective norming $[(r-R_s)/r, 1, 0]$ generates the scaling factor for the time differential dt in the general relativistic Schwarzschild metric $dt' = \cos \beta \cdot dt$ with $\sin^2 \beta = R_s/r$. For the rescaling of Minkowski metric, the differentials area is preserved in $dr' dt' = dr \cdot dt$, dr is stretched by the $\cos \beta$ division.

For the former mentioned Kepler orbit of a planet P rotating about Q as sun it means that (as in the pendulum contraction case of two hitting galaxies) there is a spiralic expanding acceleration for the speed of P. It is discrete drawn between two spirals rays where P would move according to the Kepler computation with its speed to the lower ray g position P' . The cpr map uses accelerating on the upper ray for the larger P location an orthogonal projection with length R_s down to P' . This is repeated for a full revolution where the Kepler ellipse larger diagonal is shifted by a periodic angle φ_0 to the nearest distance of P to Q (figure 9, the rosette orbit).

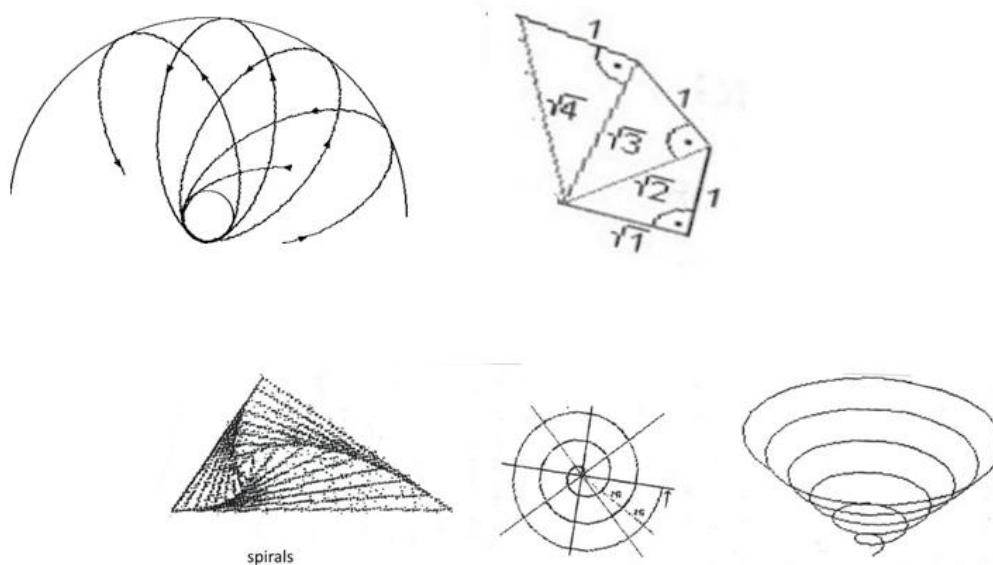


Figure 9 rosette orbit of planets, discrete spiralic rays with R_s orthogonal projections, below three kinds of twisted, 2- or 3-dimensional spirals

The other two projective normings of the Schwarzschild metric are for double lensing and redshift of EMI waves ψ , light where no speed acceleration can occur. It uses R_s for shifting the transversal cylinders $U(1)$ section by turning the plane in a $\cos \beta$ angle (instead of the former lifting of a P' by a central projections raytracing - by R_s to P). The ψ wave has as new world line a cpr projection of its cylindrical axis moved to the new cylinders axis and through $mc^2 = hf$ (from the huge star it passes by) it gets an additional frequency added which makes also for the redshift the larger wave length observed. In the redshift case, gravity which did send out the light acts accelerating, but not on the speed of ψ .

Looking at astronomic distance measures where far away stars or galaxies distances are measured by light send out, the Hubble equation needs a revision since the ψ world line is R_s broken, not linear. From the broken added $n \cdot R_s$ spiralic shifts the linear measured distances between galaxies are smaller. The expanding speed in the Hubble equation for the universe has to be revised. In case a gravity pendulum action happens when the expansion has a catastrophes maximum threshold, it can also be reversed to a contraction in time of the

universes space. There can be cases for other superpositions of 1256 with 1234 spacetime. Mentioned are mathematical inversions. Speed of light c is an upper bound for matter speeds $v < c$ in the universe and is used for dark energy inversion v' with $v'v = c^2$, - for dark energy speeds inside a pinched torus. A new Heisenberg uncertainty is introduced for c and the 01367 subspace of octonians. For astronomy is also mentioned the Schwarzschild radius R_s of dark matter inversion for radii r of mass systems in the universe to radii r' inside of the R_s bubble with $r'r = R_s^2$. The bubble can have a surface as Horn torus with a singularity where the inner of a torus is shrunk (figure 10).

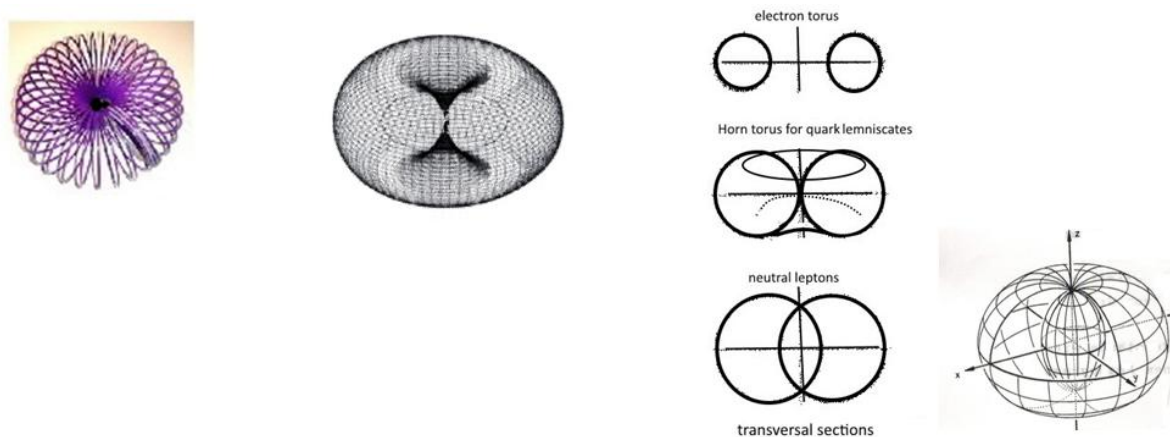


Figure 10 Horn torus, pinched torus, spindle torus for neutral leptons

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