Cusum Control Chart For Type -I Censored Lomax Distribution Using Likelihood Ratio Statistic

M.Nandini[1,] CH.V. Aruna[2,] O. V. Raja Sekharam[3,] G. V. S. R. Anjaneyulu[4]

1 Research Scholar , Department of Statistics,
Acharya Nagarjuna University,
Guntur, Andhra Pradesh, India
2 Research Scholar , Department of Statistics,
Acharya Nagarjuna University,
Guntur, Andhra Pradesh, India
3 Guest faculty at Acharya Nagarjuna University,
Andhra Pradesh, India
4 Professor Department of Statistics, Acharya Nagarjuna University,
Guntur, Andhra Pradesh, India

Abstract
The objective of this paper is to describe the control chart for censored Lomax distribution with shape parameter(α)and scale parameter(λ). Many companies conduct life tests on their products to ascertain their quality. In some cases, the tests are performed under accelerated circumstances and very often under censoring. The Lomax distribution is widely used as a viable model for the lifetimes. In this paper, we develop likelihood ratio cumulative sum (CUSUM) control charts for Type I right-censored data to monitor the mean lifetime of a product.
Keywords: Lomax Parameters, Likelihood ratio,ARL, Type I right-censored data.

I.   INTRODUCTION

The primary focus of this research centers on monitoring changes in the scale parameter of the Lomax distribution while the shape parameter is kept fixed. A simulation study was conducted to assess the effect of sample size and censoring rate on the chart's average run length (ARL) to detect changes in the scale parameter. We use the simulation method to calculate the ARL of the chart:

The Lomax distribution, conditionally also called the Pareto Type II distribution, is a heavy-tailprobability distribution used in business, economics, actuarial science, queueing theory and Internet traffic modeling. It is named after K. S. Lomax. It is essentially a Pareto distribution that has been shifted so that its support begins at zero.

The Lomax distribution, also called "Pareto type II" distribution is a particular case of the generalized Pareto distribution (GPD). The Lomax distribution has been used in the literature in a number of ways. For example, it has been extensively used for reliability modelling and life testing; see, for example, Balkema and de Haan [1]. It also has been used as an alternative to the exponential distribution when the data are heavy tailed; see Bryson [2]. Ahsanullah [3] studied the record values of Lomax distribution. Balakrishnan and Ahsanullah [4] introduced some recurrence relations between the moments of record values from Lomax distribution. The order statistics from nonidentical right-truncated Lomax random variables have been studied by Childs et al. [5]. Also, the Lomax model has been studied, from a Bayesian point of view, by many authors; see, for example, Arnold et al. [6] and El-Din et al. [7]. Howlader and Hossain [8] presented Bayesian estimation of the survival function of the Lomax distribution. Ghitany et al. [9] considered Marshall-Olkin approach and extended Lomax distribution. Cramer and Schmiedt [10] considered progressively type-II censored competing risks data from Lomax distribution. The Lomax distribution has applications in economics, actuarial modelling, queuing problems and biological sciences; for details, we refer to Johnson et al. [11].

Censored Data

If we wish to develop a control chart for lifetime data, we should take into account that our data are likely to be skewed or non-normally distributed and censored. Many life processes are not normally distributed, and modeld by different distributions such as the Lomax distribution. Lifetime data usually take time to gather

and they are expensive to produce because many products take a long time until they fail. Because of that, we use censoring which restricts the ability to observe failure times. Escobar and Meeker (1998) discuss different types of censoring such as right censoring, left censoring and interval censoring. Right censoring is the most common type, and it is classified as type I and type II right censoring. It means that there are items that do not fail by the end of the test. Type I right censoring occurs when we fixed the time of censoring C and stop testing the units that did not fail whereas in Type II censoring we predetermine the number of units that should fail and we stop testing once such number of units is completed. Left censoring happens when units may fail before the end of the study. Interval censoring means that the actual failure time for units can not be determined exactly but we know it lies within an interval or between two censoring times C_1 and C_2. In this thesis we work with Type I right censoring only. Because is the most common type of censoring occurring in industry.

Quality Control and Control Charts

There are many ways to define quality. However, in simple terms, it means fitness for use (Montgomery, 2007). Statistical quality control (SQC) is a way that quality professionals de-scribe statistical tools to control quality (Awasthi and Chaudhary, 2009). It involves descriptive statistics, statistical process control (SPC), and acceptance sampling. Quality control has been implemented in the manufacturing process to compare products and assess customer opinions and preferences of any products.

Most manufactured products are bound to malfunction or stop functioning altogether at some point (Dickinson et al., 2014). In other words, a certain glitch can prevent a product from functioning in a satisfactory fashion. Rates of failure are a common way to capture the reliability of a certain product. Reliability of a product can summarize and predict the lifetime of the product during which the product is meant to work efficiently and therefore meet certain standards (Escobar and Meeker, 1998). In addition, when certain glitches or errors are discovered early, this can lead to repairs and fixtures done more economically as opposed to the cost incurred as a result of rebuilding or checking the entire system. This is beneficial for companies to reduce warranty expenses.

Usually, companies conduct tests such as life tests on their products under certain conditions (Dickinson et al., 2014). Either they place items on a test until the products stop functioning, which is called an uncensored case, or conduct the tests for a specified time limit, which results in the censored case. In the latter case not all the products under test will stop functioning.

Since a manufacturer is interested in maintaining and improving the performance of the process, it is important to find where and when any change occurs to the process, and that may have a negative effect on the reliability of the product. Along with lifetime test, there are tools that used to determine if a manufacturing process is in control state. One of which is a control chart also known as Shewhart charts or process-behavior charts (Montgomery, 2007). This chart is considered to be the first of such tools, which was invented by Walter Shewhart while working for Bell Labs in 1942. Shewhart's chart can effectively detect sudden changes in a process, particularly if the changes are dramatic. However, it is inefficient in detecting subtle sustained changes in the mean of a process. Another powerful and useful tool in quality control is the cumulative sum (CUSUM) control chart, which is more effective than the Shewhart chart. CUSUM's control charts were first proposed by Page (1954) and have been studied by many authors afterward. This chart is a good candidate to use in the chemical and process industries where rational subgroups are frequently of size 1. In addition, it is effective in detecting sustained changes and variations in the mean. The exponentially weighted moving average (EWMA) control chart was introduced by Roberts (1959). This chart is also a good alternative to the Shewhart control chart when one is interested in detecting small shifts.

Any control chart contains a center line that represents the average value of the quality characteristic corresponding to the in-control state of the process (Montgomery, 2007). Also, there are two horizontal lines which are upper and lower control limits. They indicate the threshold, and they are chosen in a way that most of the data points fall between those limits when the process is in-control. We can assess whether a process it is in control or out-of-control by plotting a control chart of the data. If there is some points fall outside the control limits, we say the process is out-of-control. The process is then inspected to establish the cause for the out-of-control behaviour.

Likelihood Ratio Statistic

Hawkins and Olwell (1998) discussed in detail the development of CUSUM charts. They give examples of how to set the CUSUM chart for different distributions and the properties without censoring. They design CUSUM for any distribution using the score statistic

$Z_i = \ln\left(\frac{f(t_i/\theta_1)}{f(t_i/\theta_0)}\right)$

where $f(t_i/\theta_i)$ is the probability density function of the quality variable t, z_i is the score statistic (the log-likelihood ratio statistic) of the ith observation, θ_1 is our parameter of interest when the process is out-off-control, and θ_0 the parameter of interest when the process is in-control. If we want to detect an upward shift in

the process mean, the CUSUM chart will signal if the statistic $D_i^+ > h^+$, where $D_0^+ = 0$

$$D_i^+ = \max(0, D_{i-1}^+ + z_i)$$

where h^+ is the upper threshold (control limit) and i = 1, 2, ...

On the other hand, for downward shifts, the CUSUM chart will signal if the statistic $D_i^- < h^-$, where

$$D_0^- = 0$$

$D_i^- = \min(0, D_{i-1}^- + z_i)$

where h^- is the lower threshold and i = 1, 2, ... (Hawkins and Olwell, 1998).

Assuming that n items are independently tested at sampling period i yielding observations $t_i = (t_{i1}, ti2, ..., t_{in})$. We used the likelihood function for right censored data that Escobar and Meeker (1998) derived to calculate the score statistic z_i in equation (2.1), i.e.

$$L(\theta/t_i) = \prod_{j=1}^{n} f(t_{ij}/\theta)^{\delta_{ij}} \left[1 - F(t_{ij}/\theta)\right]^{1-\delta_{ij}},$$

$$\delta_{ij} = \begin{cases} 1 \; \text{if } t_{ij} \text{ is an exact} - \text{failure time}, \\ 0 \; \text{if } t_{ij} \text{ is a right} - \text{censored observation}, \end{cases}$$

where f is the pdf and F is cdf of a distribution.

Now, let's combine and apply the results from Hawkins and Olwell (1998) and Escobar and Meeker (1998) to the Lomax distribution with type I right censoring to design the CUSUM chart to detect a decrease or increase in the process mean. Because of the behaviour of the Lomax distribution, increasing or decreasing one or both of the parameters will result in an increase or a decrease in the mean. Suppose that a lifetime random variable T follows a Lomax distribution with scale and shape . The score statistic z_i of the ith observation to monitor a change in the scale parameter is given by:

$Z_i = \ln\left(\frac{L(\alpha, \lambda_1 | t_i)}{L(\alpha, \lambda_0 | t_i)}\right)$

$$= \ln\left[\frac{\prod_{j=1}^{n} f(t_{ij}|\alpha,\lambda_1)^{\delta_{ij}} \left[1 - F(t_{ij}|\alpha,\lambda_1)\right]^{1-\delta_{ij}}}{\prod_{j=1}^{n} f(t_{ij}|\alpha,\lambda_0)^{\delta_{ij}} \left[1 - F(t_{ij}|\alpha,\lambda_0)\right]^{1-\delta_{ij}}}\right]$$

$$= \sum_{j=i}^{n} \delta_{ij} \ln\left[\frac{f(t_{ij}|\alpha,\lambda_1)}{f(t_{ij}|\alpha,\lambda_0)}\right] + \sum_{j=1}^{n}(1 - \delta_{ij}) \ln\left[\frac{1-F(t_{ij}|\alpha,\lambda_1)}{1-F(t_{ij}|\alpha,\lambda_0)}\right]$$

$$= \sum_{j=1}^{n} \delta_{ij} \ln\left[\frac{\frac{\alpha}{\lambda_1}\left[1+\frac{t_{ij}}{\lambda_1}\right]^{-(\alpha+1)}}{\frac{\alpha}{\lambda_0}\left[1+\frac{t_{ij}}{\lambda_0}\right]^{-(\alpha+1)}}\right] + \sum_{j=1}^{n} \delta_{ij} \ln\left[\frac{1-\left(1-\left(1+\frac{t_{ij}}{\lambda_1}\right)^{-\alpha}\right)}{1-\left(1-\left(1+\frac{t_{ij}}{\lambda_1}\right)^{-\alpha}\right)}\right]$$

$$= \sum_{j=1}^{n} \delta_{ij} \ln\left[\frac{\alpha}{\lambda_1}\left[1+\frac{t_{ij}}{\lambda_1}\right]^{-(\alpha+1)} \cdot \frac{\lambda_0}{\alpha}\left[1+\frac{t_{ij}}{\lambda_0}\right]^{\alpha+1}\right] + \sum_{j=1}^{n}(1-\delta_{ij}) \ln\left[\left(1+\frac{t_{ij}}{\lambda_1}\right)^{-\alpha}\left(1+\frac{t_{ij}}{\lambda_0}\right)^{\alpha}\right]$$

$$= \sum_{j=1}^{n} \delta_{ij} \ln\left[\frac{\lambda_0}{\lambda_1}\left[1+\frac{t_{ij}}{\lambda_1}\right]^{-(\alpha+1)} \cdot \left[1+\frac{t_{ij}}{\lambda_0}\right]^{\alpha+1}\right] + \sum_{j=1}^{n}(1-\delta_{ij}) \ln\left[\left(1+\frac{t_{ij}}{\lambda_1}\right)^{-\alpha}\left(1+\frac{t_{ij}}{\lambda_0}\right)^{\alpha}\right]$$

$= \sum_{j=1}^{n} \delta_{ij} \ln\left(\frac{\lambda_0}{\lambda_1}\right) + \sum_{j=1}^{n} \delta_{ij}(-\alpha+1)\ln\left(1+\frac{t_{ij}}{\lambda_1}\right) + \sum_{j=1}^{n} \delta_{ij}(\alpha+1)\ln\left(1+\frac{t_{ij}}{\lambda_0}\right) + \sum_{j=1}^{n}(1-\delta_{ij})(-\alpha)\ln\left(1+\right.$

tijλ1+j=1n(1−δij)αln1+tijλ0

$Z_i = \sum_{j=1}^{n} \delta_{ij} \ln\left(\frac{\lambda_0}{\lambda_1}\right) - \alpha \sum_{j=1}^{n} \delta_{ij} \ln\left[\frac{1+\frac{t_{ij}}{\lambda_1}}{1+\frac{t_{ij}}{\lambda_0}}\right] + \sum_{j=1}^{n} \delta_{ij} \ln\left[\left(1+\frac{t_{ij}}{\lambda_1}\right)\left(1+\frac{t_{ij}}{\lambda_0}\right)\right] - \alpha \sum_{j=1}^{n} \ln\left[\frac{1+\frac{t_{ij}}{\lambda_1}}{1+\frac{t_{ij}}{\lambda_0}}\right] -$

$\alpha \sum_{j=1}^{n} \delta_{ij} \ln\left[\left(1+\frac{t_{ij}}{\lambda_1}\right)\left(1+\frac{t_{ij}}{\lambda_0}\right)\right]$

$$Z_i = \sum_{j=1}^{n} \delta_{ij} \ln\left(\frac{\lambda_0}{\lambda_1}\right) - 2\alpha \sum_{j=1}^{n} \delta_{ij} \ln\left(1 + \frac{t_{ij}}{\lambda_1}\right) + \sum_{j=1}^{n} \delta_{ij} \ln\left[\left(1 + \frac{t_{ij}}{\lambda_1}\right)\left(1 + \frac{t_{ij}}{\lambda_0}\right)\right] - \alpha \sum_{j=1}^{n} \ln\left[\frac{1 + \frac{t_{ij}}{\lambda_1}}{1 + \frac{t_{ij}}{\lambda_0}}\right]$$

$$Z_i = \sum_{j=1}^{n} \delta_{ij} \ln\left(\frac{\lambda_0}{(1-d)\lambda_0}\right) - 2\alpha \sum_{j=1}^{n} \delta_{ij} \ln\left(1 + \frac{t_{ij}}{(1-d)\lambda_0}\right) + \sum_{j=1}^{n} \delta_{ij} \ln\left[\left(1 + \frac{t_{ij}}{(1-d)\lambda_0}\right)\left(1 + \frac{t_{ij}}{\lambda_0}\right)\right] -$$
$$\alpha \sum_{j=1}^{n} \ln\left[\frac{1 + \frac{t_{ij}}{(1-d)\lambda_0}}{1 + \frac{t_{ij}}{\lambda_0}}\right]$$

Upper and Lower Chart Statistics

In the Lomax distribution, we fix the shape parameter and monitor changes in the scale parameter. We will define the in-control values of the parameters λ_0 and β, and assume that they are known. For the out-of-control situation the parameters are λ_1 and β, where $\lambda_1 = (1-d)\lambda_0$ and d × 100% is the percentage of the shift in λ_0. For a decrease in the process mean λ_0, 0 < d < 1; whereas for an increase in λ_0, d < 0. Using the later specification of the parameters, we can re-write the score statistic in equation

$$Z_i = \sum_{j=1}^{n} \delta_{ij} \ln\left(\frac{\lambda_0}{(1-d)\lambda_0}\right) - 2\alpha \sum_{j=1}^{n} \delta_{ij} \ln\left(1 + \frac{t_{ij}}{(1-d)\lambda_0}\right) + \sum_{j=1}^{n} \delta_{ij} \ln\left[\left(1 + \frac{t_{ij}}{(1-d)\lambda_0}\right)\left(1 + \frac{t_{ij}}{\lambda_0}\right)\right] -$$
$$\alpha \sum_{j=1}^{n} \ln\left[\frac{1 + \frac{t_{ij}}{(1-d)\lambda_0}}{1 + \frac{t_{ij}}{\lambda_0}}\right]$$

The lower chart statistic is used to detect a decrease in the process mean. It is defined as follows: for 0 < d < 1 and i = 1, 2, ...,

$$C_0^- = 0;$$

$C_i^- = \min(0, C_{i-1}^- - z_i)$

The chart signals a decrease in the mean when $C_i^- < h^-$

Similarly, the upper chart statistic is used to detect an increase in the process mean. It is defined as follows: for d < 0 and i = 1, 2, ...,

$C_0^+ = 0;$
$C_i^+ = \max(0, C_{i-1}^+ + z_i)$

The chart signals an increase in the mean when $C_i^+ > h^+$

In the above, h^- and h^+ are the control limits that give the desired average run length

Calculation of Average Run Length

Average Run Length by Simulation

Dickinson et al. (2014) assesses the performance of the CUSUM and EWMA charts of the likelihood-ratio to detect a decrease or increase in the process mean of Weibull distribution with type I right censoring by simulation. In this section, we use this simulation approach to evaluate the performance of the likelihood-ratio based CUSUM to monitor a decrease or increase in the process mean of the Lomax distribution with type I right censoring. The following subsections describe the simulation algorithm.

Monitoring a Decrease in the Mean of Lifetimes

The following steps are used to calculate the in-control average run length ARL_0:

(1) The desired ARL_0 is 370 which is the standard and commonly used.

(2) Initialize the in-control parameters and the out-of-control parameters:

Table 1: In-control and out-of-control parameters to monitor a decrease in the process mean

Process stage	Sample size n	Shift Size d	Scale parameter	Shape parameter
In-control	n = 3; 5; 10	0.15, 0.20, 0.30, 0.35	$\lambda_0 = 1$	α_0 =0:5;1;3
Out-of-control	n = 3; 5; 10	0.15, 0.20, 0.30, 0.35	$\lambda_1 = (1-d)\lambda_0$	α_1 = 0.5;1;3

(3) Generate a random variable T from a Lomax (α_0, λ_0) with type I right censoring.

(a) Generate a random sample size n from Lomax (α_0, λ_0)

(b) Initialize the censoring rate pc which is the probability of an observation being censored and it can be calculated from the following equation:

$$pc = P(T > C) \Rightarrow pc = 1 - P(T \leq C)$$

$\Rightarrow \text{pc} = 1 - F(C; \alpha_0, \lambda_0)$,

where C is predetermined the censoring time.

(c) Predetermined the censoring time C which can be calculated using equation

$$C = F^{-1}(1 - \text{pc}; \alpha_0, \lambda_0),$$

which is the cdf of Lomax (α_0, λ_0) random variable. We calculate C that corresponds to $100 \times \text{pc} = 10\%, 30\%$, $50\%, 80\%$.

(d) Generate $2 \times n$ matrix where all the entries of the first row are C and the second row has entries of the random sample. Take the minimum value between each rows entry to get the censored data.

(4) Initialize the control limits h^- which is the mid point of h_1^- and h_2^-, where $h_1^- > h_2^-$.

(5) Calculate the CUSUM chart statistics C_i^- from equation

(6) Stop when $C_i^- < h^-$ and count the number of samples required

(7) Repeat the process 50, 000 times and take the average of the counts to calculate ARL_0.

To evaluate the out-of-control ARL, we repeat the same steps above except we generate a random sample size n = 3, 5, 10 from Lomax (α_1, λ_1)

Monitoring an Increase in the Mean of Lifetimes

We evaluate the upper chart statistic to detect an upward change in the process mean by calculating ARL0 and ARL1. We use the same steps for monitoring a decrease in the of lifetimes and we specify the following parameters:

• Initialize the in-control parameters and the out-of-control parameters

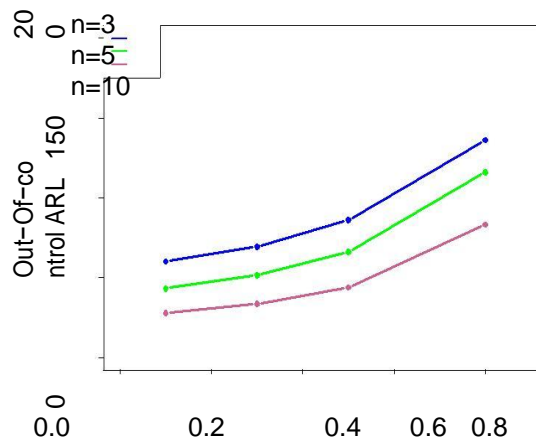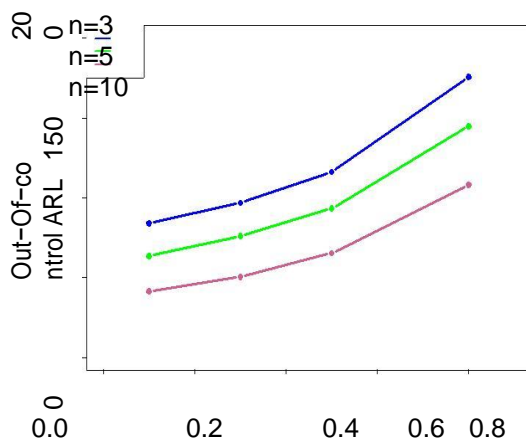Table 2: In-control and out-of-control parameters to monitor an increase in the process mean.

Process stage	Sample size n	Shift Size d	Scale parameter	Shape parameter
In-control	n = 3; 5; 10	0.15, 0.20, 0.30, 0.35	$\lambda_0 = 1$	$\alpha_0 = 0.5; 1; 3$
Out-of-control	n = 3; 5; 10	0.15, 0.20, 0.30, 0.35	$\lambda_1 = (1 + d)_0$	$\alpha_1 = 0.5; 1; 3$

• The control limit $h^+ = (h_1^+ + h_2^+)/2$, where h^+, h_1^+ and $h_2^+ > 0$ and $h_1^+ > h_2^+$.

• Calculate the CUSUM chart statistics C_i^+ and average 50, 000 replications of the simulation to calculate ARL_0 and ARL_1.

Whether we are looking to detect a decrease or an increase in process mean, we must check if ARL_0 is close to the desired 370. If $|\text{ARL}_0 - 370| < 5$, then ARL_0 is close enough and we return the threshold. Otherwise, we repeat the simulation steps and find the threshold that gives the desired ARL_0. To find out the threshold that gives $|\text{ARL}_0 - 370| < 5$, we use the partition search method (Dickinson et al., 2014). The following steps describe how the partition method was designed:

1. Initialized h_1, h_2 that take values between 0 and 30, where $h_1 > h_2$.

2. Calculate the midpoint h = $(h_1 + h_2)/2$.

3. Obtain ARL_0 for h and check if the ARL0 $\approx$ 370. If ARL0 $\approx$ 370, stop.

4. If $\text{ARL}_0 > 370$, then take $h_2 = h$ and update the midpoint h.

5. If $\text{ARL}_0 \leq 370$, then take $h_1 = h$ and update the midpoint h.

6. Calculate the ARL_0 using the update threshold.

7. Repeat the above steps to find out the threshold h that gives $\text{ARL}_0 \approx 370$.

Censoring rate

Figure 1: ARL_1 for the lower CUSUM chart with = 3, and (a) d = 0:15, (b) d = 0:20,

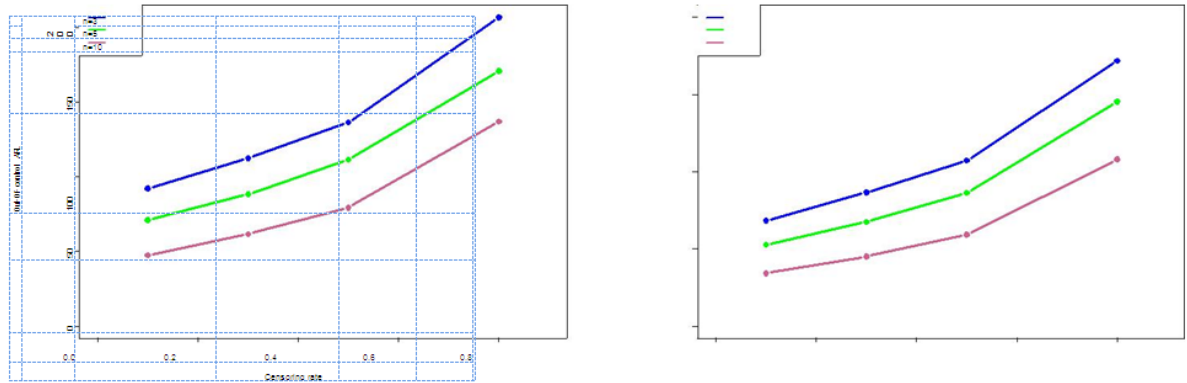
Figure 2: ARL_1 for upper CUSUM chart with = 0:5, and (a) d = 0.15, (b) d = 0.20,

The results of simulations

Table: 1

Lower CUSUM chart for a decrease in the scale parameter, in-control and out-of-control ARL and $\alpha = \alpha_0 = 0.5$; 1; 3 using simulation method

α_0	Censoring Proportion	Sample Size	Shift Size	Control Limit	ARL_0	ARL_1
0.5	0.10	3	0.15	-2.0695	372.718	83.116
0.5	0.10	5	0.15	-2.0596	374.944	63.178
0.5	0.10	3	0.35	-3.9831	374.021	26.660
0.5	0.10	5	0.35	-3.9931	370.087	18.270
0.5	0.50	3	0.15	-1.7978	374.448	118.000
0.5	0.50	5	0.15	-1.9819	370.302	92.948
0.5	0.50	3	0.35	-2.9036	374.530	39.426
0.5	0.50	5	0.35	-3.2157	372.823	28.142
1	0.10	3	0.15	-2.5801	372.773	54.960
1	0.10	5	0.15	-2.9199	370.684	40.002
1	0.10	3	0.35	-3.8843	371.592	15.519
1	0.10	5	0.35	-4.1698	374.759	10.594
1	0.50	3	0.15	-2.2300	372.676	72.544
1	0.50	5	0.15	-2.5224	371.602	53.702
1	0.50	3	0.35	-3.5786	371.443	20.040
1	0.50	5	0.35	-3.8289	371.894	13.814
3	0.10	3	0.15	-3.3789	374.679	26.472
3	0.10	5	0.15	-3.6764	371.075	18.374
3	0.10	3	0.35	-4.3931	373.890	6.585
3	0.10	5	0.35	-4.4908	374.524	4.383
3	0.50	3	0.15	-3.1161	374.003	31.692
3	0.50	5	0.15	-3.4185	371.367	22.158
3	0.50	3	0.35	-4.2536	373.728	7.252
3	0.50	5	0.35	-4.3675	374.087	4.858

Table :2

Upper CUSUM chart for an increase in the scale parameter, in-control and out-of-control ARL and $\alpha = \alpha_0 = 0.5; 1; 3$ using simulation method

α_0	Censoring Proportion	Sample Size	Shift Size	Control Limit	ARL_0	ARL_1
0.5	0.10	3	0.15	1.8208	370.153	91.128
0.5	0.10	5	0.15	2.1672	372.887	70.714
0.5	0.10	3	0.35	2.8151	374.483	39.147
0.5	0.10	5	0.35	322149	370.111	27.840
0.5	0.50	3	0.15	1.3848	373.470	138.443
0.5	0.50	5	0.15	1.6780	374.716	112.258
0.5	0.50	3	0.35	2.2717	373.973	68.998
0.5	0.50	5	0.35	2.5774	370.632	50.803
1	0.10	3	0.15	2.3242	347.982	62.533
1	0.10	5	0.15	2.7692	373.937	46.063
1	0.10	3	0.35	3.2858	372.431	23.577
1	0.10	5	0.35	3.5780	370.322	16.454
1	0.50	3	0.15	1.9789	374.898	91.010
1	0.50	5	0.15	2.2454	370.931	67.921
1	0.50	3	0.35	2.9218	372.780	38.650
1	0.50	5	0.35	3.2672	374.547	27.654
3	0.10	3	0.15	3.0758	374.987	30.823
3	0.10	5	0.15	3.3977	374.582	21.935
3	0.10	3	0.35	3.9310	370.125	10.336
3	0.10	5	0.35	4.1565	374.093	7.089
3	0.50	3	0.15	2.8189	374.152	42.648
3	0.50	5	0.15	3.1551	373.633	30.594
3	0.50	3	0.35	3.7500	370.613	16.237
3	0.50	5	0.35	4.0305	371.909	11.155

II.   RESULTS AND CONCLUSIONS

From the Tables and the Figures, we can see that:

1)   The shape parameter, sample size, censoring rate, and the shift size have an impact on the chart performance and ARL_1.

2)   By increasing the sample size the chart signals quickly and we get a lower ARL_1. For instance, from Table 1 and Figure 1 for = 0.5, n = 3, pc = 10%, and d = 0:15 the chart signals on average after 83.991 samples whereas if we use the same parameters and use n = 10 the chart signals after 41.389.

3)   There is a negative correlation between censoring and the sensitivity of the chart. The chart detects the change fast if we decrease the censoring.

4)   The chart signals quickly if we increase the shape parameter. For example, when we look at Figure 2 with = 0.5, it takes 80 samples or less on an average for the chart to signal, whereas from Figure 2with = 3, it takes only 40 samples or less on an average for the chat to signal.

The scale parameter (i.e. the shift size) seems to have more impact than other parameters. With large negative or positive shift to detect, the chart signals and perform better than with small shift.

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