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Dark matter existence or nonexistence

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Abstract
In this work, with a rigorous review of the physics and mathematics of stellar dynamics in large gravitational systems such as a galaxy or cluster of galaxies, it is evidenced that it is superfluous to introduce what could be called the postulate of the existence of Dark Matter. In other words, an analysis of the stellar dynamics in the galaxies is carried out, starting from fundamental equations, such as Gauss's Law for flow, in this case gravitational, and the Law of Universal Gravitation. Expressions are obtained for the speed of rotation of the stars and the force of gravity to which they are subjected, which differ from traditional expressions. In conclusion, not only is it proven unnecessary to introduce something inexplicable, such as Dark Matter, but also a recipe, with which to estimate the distribution of mass in a galaxy, from the curves of the rotational speeds of the stars, is proposed.
Keywords *Dark Matter· Galaxies· Galaxy cluster·Gravitational field· Gauss Law*

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I. Introduction

In the first half of the 1930s, Fritz Zwicky, a Swiss-American astronomer of Bulgarian origin, detected a supposed anomaly in the motions of several stars in various galaxies [1-2].

Experimentally, the rotation speeds of some stars were determined, whose magnitudes were excessive, according to the Newtonian prediction that was used to confirm the measurements obtained [1-2]. In a sense, this fact indicated that the speed with which a considerable number of stars rotate around the galactic centres it was too great for what could be justified by using the Newtonian model of the gravitational potential. These gravitational systems should be shattered by the "centrifugal" force that occurs with such high speeds, since the computed mass is not sufficient to maintain the observed dynamics [3]. However, the situation regarding these systems indicated that they were stable for extremely long periods of time. A system's gravitational dynamics, determined experimentally in an unexpected manner, did not change due to this fact. Many astronomers contemplated the reason for this stability, which occurred despite the large measured stellar rotation speeds [4]. Different explanations were proposed and between it was proposed thatperhaps Newton's force of gravity was such that, at the galactic level the gravitational field would have a mathematically different expression. Perhaps it was the distance, or perhaps the exponent of the distance, which appears squared, or the fact that the same mass could produce different influences in gravitational systems that are as large as a galaxy [5, 6]. For a long period of time, the discussion focussed on the missing mass that could have been left out of the calculations when aiming to justify the fact of the great speeds of stellar rotation [4]. In principle, this mass (if it exists) could not be seen, and supposedly it only manifests itself through gravitation. Over time, in aiming to solve the problem, a kind of postulate came into force, that affirms the existence of a strange and unknown matter, which could contribute in an important way to galactic stability [1-6]. Indeed, it would be a type of matter that could be considered as out of the ordinary, which cannot be seen, and it only manifests through the gravitational force [7, 8]. Due to these details and emphasising that it is simply not seen, the name Dark Matter was coined to designate it [3-6, 9, 10].

By the 1960s and 1980s, the American astronomer Vera Rubin took up the subject seriously and obtained refined measurements of the radial velocities of stars in various galaxies. She confirmed that the Newtonian potential that is used in the solar system and is applied to galactic systems, showed that the rotation speeds in various galaxies, particularly in the Andromeda galaxy (M31), were well above theoretical predictions [3-6].

However, despite all of the evidence on the missing matter, currently termed as Dark Matter, that has accumulated over the last 90 years, approximately, there still remains a certain amount of scepticism. This is likely the reason whyVera Rubin never received the Nobel Prize for her work in this area, despite the fact that an important part of the scientific community surrounding her during her life affirmed that she would be a well-deserving candidate [5, 6, 8].

In this work, the calculations that must be carried out to obtain the magnitudes which are considered in excess of the speeds of stellar rotation in various galaxies are mathematically explained, without introducing any foreign matter to compensate for the centrifugal force, which supposedly would disrupt the galactic systems and in this case, would also disrupt the galaxy clusters. As will be seen, it is possible to explain the reason for these high rotational speeds and it will be shown that the "new" gravitational dynamics, which is proposed in this writing, yields de facto the fast speeds determined in the observation. It will be seen that the gravitational field in many galactic systems, under this dynamic approach, turns out to be large enough to avoid breaking the stability of these gravitational systems.

Existence or non-existence

From a philosophical perspective, it is sometimes much more difficult to prove the non-existence of something than to prove the actual existence itself. However, in the mathematics of physics, for this case, it is possible to establish a demonstration of non-existence by showing the superfluousness of introducing a postulate on the possible existence of Dark Matter, when the supposed anomaly is explained directly, with the proper considerations and calculations.

In the course of the last decades, a part of the international scientific community, for a long time, has aimed to demonstrate the existence of that missing mass in galaxies; the strange matter that would give stability to these gravitational systems [9-12]. At present, in various parts of the world, experiments are being carried out to find the evidence that justifies the so-called Dark Matter postulate; the unusual matter that only interacts gravitationally. With several experiments, in different parts of the world, the aim is to detect, among other things, particles that could represent this strange matter. To date, the efforts have been unsuccessful [13-20].

Given the affirmation of the existence of Dark Matter, some researchers calculate that this strange matter makes up approximately 25% of the total matter in the universe.

Once again, the philosophical character of the search appears, as it is important as to discover the nature of Dark Matter. But, when some tangible result does not appear, despite the efforts applied, there can be an unavoidable temptation to submit to denial avoid and immediately note that there is no such thing in existence. Even more so, some aspects of the scientific method are added to this, although a prohibition is not explicitly implied in the scientific method itself and, in fact, it is not prohibited, but it is recommended to not aim at explaining something new by introducing something that is inexplicable [21, 22]. That is, in this case, an attempt has been made to solve the problem of explaining the great speeds of rotation of the stars and this ended up having two problems in explaining what Dark Matter, the original problem and the present problem [19-22]. Traditionally, an appropriate approach to the scientific method consists of trying to explain the phenomenon with the simplest and most direct argumentation. This again, is connected with philosophical aspects and it is convenient here to quote the famous expression of William of Ockham (1287-1347, English Franciscan friar), commonly known as Ockham's Razor, which is "plurality should be avoided". Science also seeks to avoid magical thinking, since introducing something inexplicable makes use of the ethereal and, hence, the immeasurable. The alchemists sought to introduce a connection with something pure and divine when they expected the unexpected to happen over time, often with repetition [19-22]. Here, in this problem, one can see that if the cause of something can be explained in a simple way, then there is a high probability that this is the correct explanation [20]. Plurality does not only consist of having several explanations, because it also seeks to not multiply the problems that have to be solved. As will be seen, with a mathematical calculation, that is not only adequate, but also rigorous, it is shown that stellar rotation velocity curves of different forms can exist in galaxies, depending mainly on the distribution of mass in the volume. For this case, the radial function of the galactic mass distribution and all of these curves can be obtained by considering the gravitational force that is immersed in a massive medium other than a vacuum.

Newton and the mass distribution

In the Solar System, mass is practically restricted to the central part of the gravitational system, that is, for most problems, except for a few, with one likely being the anomaly of the NASA Pioneer spacecraft (the Pioneer effect), it is possible to consider that the entire mass of the system is in the sun [20]. This fact, as will be shown, makes a great difference compared to gravitational systems such as a galaxy where, normally, the totality of the mass is found to be distributed in practically the entire volume of the system. It can be said, with some approximation, that in a galaxy the stars move within a massive medium, which is the medium that is constituted by the other stars. To a large extent this is the reason why, although the distribution of mass also contributes, the resulting gravitational field within a galaxy can have variations with distances to the galactic centre that are very different from those in the solar system.

It is possible to elaborate on this point. There is a well-known conceptualisation of Newton's law of gravitation. Aristotle, among others, considered that things in heaven were perfect, whereas those of the earthly realm were imperfect [21-24]. That is, towards a very precise differentiation of what could exist in the cosmos, in the interstellar medium (perfection), with respect to the earthly and imperfect nature of things. It is likely that this type of argument led Isaac Newton to name the law of gravitation as the Law of Universal Gravitation [20-24]. In order to emphasise this point, on the one hand, this law should actually be the same in heaven as it is on Earth. On the other hand, thinking about the fall of the apple and the attraction of the moon to the earth, both of which are subject to the same law, the problem of attraction could be generalised by making a universal law, which would be valid both for the sphere of perfection (as in the heavenly) as well as the imperfect earthly realm [20-24].

It is very likely that something in this scenario has also influenced Vera Rubin to claim that the law of gravity, as applied in the Solar System, should also be valid for a gravitational system such as a galaxy [3, 4]. In a relatively recent work from 2006, Vera Rubin stated verbatim that “High school students learn that in gravitationally bound systems a planet moves in a closed orbit, such that $\mathrm{MG} = v^2/\mathrm{r}$ where M is the mass of the Sun, G is the gravitational constant and v and r are the speed and its distance from the Sun. In M31 (Andromeda), the same relationship between mass, speed, and distance is valid” [3]. This argument by Vera Rubin will soon be shown to be wrong.

From the relationship mentioned that was by Vera Rubin, which is precisely that whichgives the Coulomb potential for the Solar System, it can be seen that the rotation speed decreases as one over the square root of r [3-6]. In fact, it can be seen as a curve that decreases when the distance increases, that is, the mobile is considered farthest from the centre of force

$$v \propto \frac{1}{\sqrt{r}} \tag{1}$$

In contrast, the speeds that Vera Rubin determined experimentally, particularly in Andromeda, provided a curve of rotational speeds where the speed turns out to be a constant. The measured velocity of several stars was practically constant [3-6]

$$v = cte \tag{2}$$

Unfortunately for Vera Rubin and several researchers who were aiming to find evidence of the existence of Dark Matter, it is possible to demonstrate with a mathematical procedure using Gauss’s Law for the flow of a field, in this case the gravitational field, to mathematically justify the existence of these high speeds of stellar rotation in galaxies and also the galactic rotation within galaxy clusters.

Gravitational field radial dependence

Below are two examples (1 and 2, Fig. 1) in which the gravitational field is calculated for two different distributions of the field source, in this case the mass. It will be seen how and in what way the gravitational field values can vary by changing the mass distribution, that is, a) the case of the concentrated source and b) when it is dispersed.

In a first case, example 1), the value of the gravitational field is compared when the mass is not distributed, but is concentrated in the centre of gravity, with a situation of distributed mass (see Fig 1). Here, the point at which it is analysed, point P, is outside the distribution.

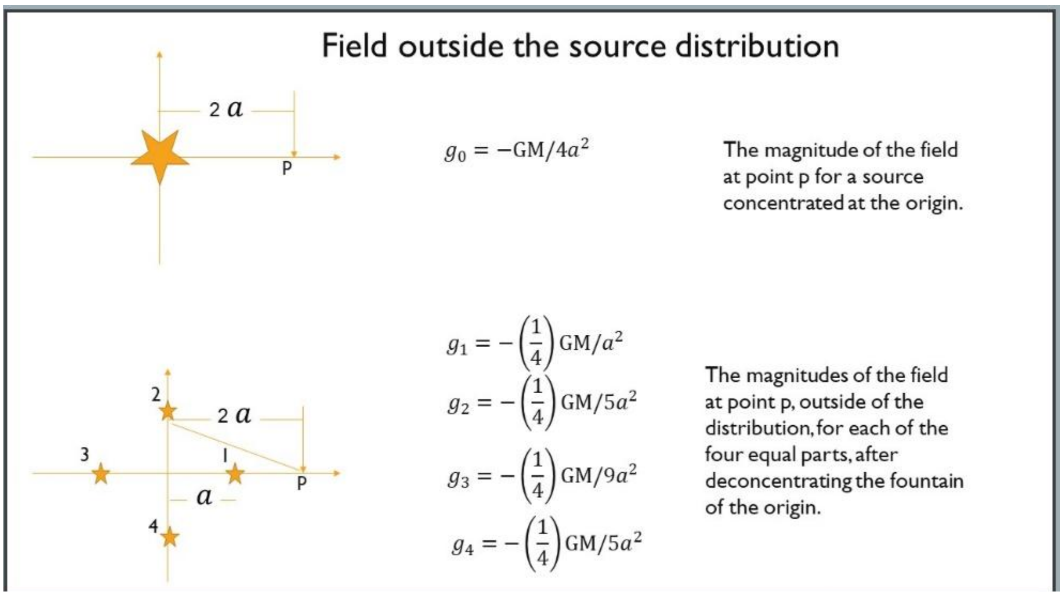

Fig. 1 Example 1). The gravitational field is exhibited and the cases of concentrated sources and discretised sources are compared. It is possible to appreciate that, in fact, the field acquires different values when the mass is discretised here, for a point outside the distribution

In a second case example 2) a variant is introduced, in addition to first calculating the field as in a), then the field is calculated at a test point P, which is now within the distribution (Fig. 2).

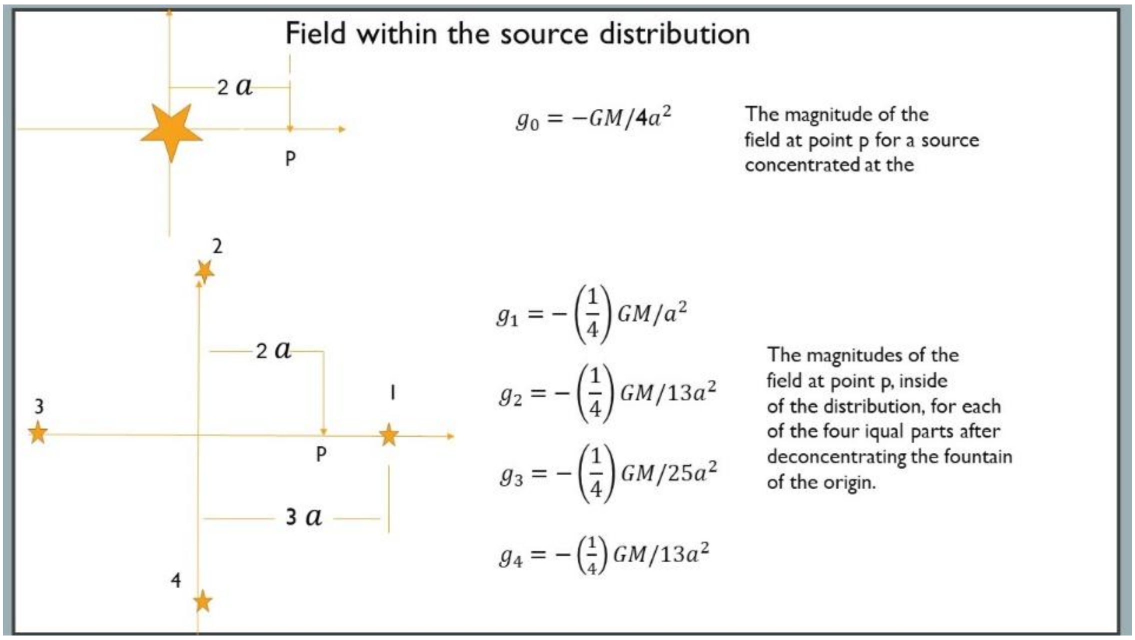

Fig. 2 Example 2). The gravitational field is displayed and the cases of concentrated and discretised sources are compared. It is possible to appreciate that, in fact, the field acquires different values when the mass is discretised here, for a point within the distribution

With these two examples, it is possible to observe a way in which the distribution of the mass significantly affects the final result, that is, how the value of the gravitational field actually changes depending on the distribution of the sources. These two examples are somewhat crude and at the same time simplistic, but they illustrate the idea that the distribution of mass is decisive. In a galaxy, as will be seen, things turn out with elegance and forcefulness and there the mass generally has a certain disseminated, discretised distribution.

In the Solar System, an example of reality that can also illustrate this influence of the distribution of mass on the gravitational force,is the discovery of the planet Neptune. Already, since the end of the 18th century, several researchers of the cosmos had noticed certain irregularities in the orbit of the planet Uranus [20-24]. At the time, it was also suspected that Newton's theory could be false, which as an event that fortunately did not prosper [20-24]. It is said that in two years of intense work, Urbain Le Verrier was able to predict the position of the new planet. On the night of September 23-24, 1846, the German astronomer Johann Gottfried Galle observed a small bluish spot where Le Verrier had claimed the new planet would be found, which was called Neptune [20-24]. That is, the presence of another mass that would ultimately modify the distribution of the sources, indicated that the gravitational field on Uranus should be different from what has already been calculated and would account for the anomalies [20-24].

A different celestial Dynamic

When a massive body rotates under the influence of a central force, this is a force that should be called a centripetal force, although, normally without consistency it is said to be under the influence of a "centrifugal" force [25, 26]. In fact, it is stated that it is subjected to a certain force, among other things due to its speed

$$f = mv^2/r \tag{3}$$

where m is the mass that rotates at a distance r from the centre of forces and moves with rotational speed v.
In the case of the Solar System, the gravitational force has the traditional expression and is normally written as

$$f = GMm/r^2 \tag{4}$$

where G is the gravitational constant, M is the source of the gravitational field, and m is the mass that rotates at the distance r from the centre of rotation.

Normally, to study the dynamics of the system, the gravitational force is equated with the centrifugal force Eqs. (3) and (4) and the expression of the speed that occurs is obtained, under the influence of the traditional Newtonian potential, like that used by Vera Rubin [3-6]

$$v = \sqrt{GM/r} \tag{5}$$

This turns out to be part of the dynamics of the Solar System that appears relatively simple and can be limited to saying that all planets must obey the law of speeds of Eq. (5). However, due to measurements, this speed law seems to be unfulfilled at the level of galactic gravitational systems, and the rotation speed seems to have another expression [3-6]. This of course is not due to the perfection of the celestial gravitational systems nor is it due to the aforementioned imperfection of the earthly realm, but rather it could be considered as a kind of tidal force.

The stellar dynamics of galactic systems and also of galaxies in galaxy clusters is different and has its reason for being so [20-26].

Let us see how this happens.Gauss's law for the flow produced by a force field is much more familiar for the case of the electric field [27, 28], Fig. 3. In fact, one of Maxwell's equations is called Gauss's Law and in its integral form it is written as [27, 28]

$$\oint_S \bar{E} \cdot d\bar{s} = q/\varepsilon \qquad (6)$$

where S represents a closed surface, in this case Gaussian $\bar{E}$ represents the electric field, $d\bar{s}$represents a differential vector perpendicular to the outside of the Gaussian surface, q represents the source or sources of the field, in this case electric field. Here ε is the permittivity of the medium [27, 28].

In order for this expression for Gaussian field fluxes to be used in the case of the gravitational field, that is, using $\bar{g}$instead of $\bar{E}$ it is necessary to change q / 4πε and replace it by GM. With this change, Eq. (4) becomes [27, 28]

$$\oint_S \bar{g} \cdot d\bar{s} = -4\pi GM \qquad (7)$$

Since G is the gravitational constant before, $\bar{g}$represents the gravitational field and the sources of the field are represented by M.

Because, in the case of the gravitational field, so far there are no negative masses, it is known that the gravitational force is only attractive, that is the reason for the negative sign [27, 29].

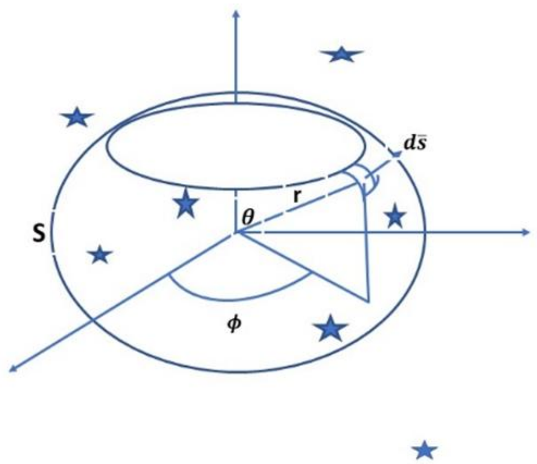

Fig. 3 An arbitrary Gaussian surface s is shown for calculating the gravitational field flux. Here,$d\bar{s}$ represents a vector perpendicular to the surface for the differential area

One of the important factors with respect to this last expression is precisely the mass M, which is the amount of matter contained within the Gaussian surface, since the mass that is outside will produce a net flow through the surface equal to zero. A key point in this argument is precisely the calculation of mass. Not only the value of M, but also the expression in function of r that in the most general case it can have, will be the most important aspect for the analysis that is going to be developed.

There is an important difference between the Solar System, where the mass is practically concentrated in the Sun and when the mass has a complex distribution, as in a galaxy, occupying a volume that sometimes goes beyond the visible part of the galaxy.

In the most general case, when a mass distribution is represented by a density ρ, the expression for the mass is written as

$$M = \oint_S \rho \, dV \qquad (8)$$

Here, ρ represents the mass density (mass per unit volume), dV represents the differential element of volume, in this case, the Gaussian volume. The rest of the representations is as before.

Normally, the integral is performed on a Gaussian surface that has adequate symmetry, in order to simplify the calculations. When the distribution of the sources is symmetrical, for example spherical, the Gaussian surface can be a sphere. Due to symmetry, on the Gaussian surface the field is considered to be constant and the integral turns out to be the value of the field multiplied by the surface of the sphere.

Here, we present a first example. To simplify the calculations, suppose that a spherical galaxy with a distribution of stars that is represented by a density ρ (r). Also, for simplicity, suppose that the density of stars decays as

$$\rho \propto 1/r \tag{9}$$

With this density and the differential element of volume in spherical coordinates

$$dV = r^2 \sin\theta \, dr \, d\theta \, d\varphi \tag{10}$$

Substituted intoEq. (8), we have the expression for the mass which is

$$M = \oint_S \frac{cte}{r} r^2 \sin\theta \, dr \, d\theta \, d\varphi \quad \propto r^2 \tag{11}$$

By integrating, then we find that the mass M in Eq. (7) varies as r^2. Also, when integrating the left member of Eq. (7), the gravitational field remains constant on the Gaussian surface, and we have

$$g \, 4\pi \, r^2 = -Gcte \, r^2 \tag{12}$$

It is found from this expression that the gravitational field for this mass distribution within this spherical galaxy is a constant

$$g = cte \tag{13}$$

Equating the gravitational force with the centrifugal force

$$gm = cte \, m = mv^2/r \tag{14}$$

Therefore, in this galaxy the stars obey a different dynamic than that which the planets in the Solar System obey. Here, the stars are subject to a constant force, which does not depend on the distance to the galactic centre, within the Gaussian volume, and the stars move with a rotational speed

$$v = \sqrt{cte \, r} \tag{15}$$

Here we present a second example. Now, to compare with the measurements of Vera Rubin, with relative ease, it can be shown that if instead of having a stellar density as in expression (9) we have

$$\rho \propto 1/r^2 \tag{16}$$

The gravitational field obtained for the interior of the galactic volume is

$$g \propto r \tag{17}$$

Therefore, the speed of stellar rotation will be

$$v = cte \tag{18}$$

Using the star's density of expression (16), and the curve of rotation speeds that was determined experimentally by Vera Rubin, Eq. (2) is reproduced with a very good approximation [3-6].

In this way, through this procedure, different stellar rotational velocity curves can be obtained according to the mass distribution of the galaxy, that is, according to ρ. It is important that, looking at the problem in reverse, by obtaining the stellar rotation curves it is possible to know, to some extent, the mass distribution in a galaxy.

Numerical calculations

To estimate the velocities of a discrete distribution of particles of mass m, subject to a gravitational interaction, we calculate the gravitational field to which a test particle m_0 is subjected due to the gravitational influence of the n particles of the distribution. Superimposing the gravitational field of the n particles and assuming that they all have the same mass m, we have:

$$\bar{g} = \sum_{i=1}^{n} \bar{g}_i = Gm \sum_{i=1}^{n} \frac{\bar{r}-\bar{r}_i}{|\bar{r}-\bar{r}_i|^3} \tag{19}$$

So that

$$g_\mu = Gm \sum_{i=1}^{n} \frac{\mu-\mu_i}{|\bar{r}-\bar{r}_i|^3} \qquad \mu = x, y, z \tag{20}$$

They are the scalar components of the gravitational field at the place where the test mass m_0 is located. From here we get $g = \sqrt{\sum g_\mu^2}$ and hence from Eq. 14 we have:

$$v = \sqrt{rg} \tag{21}$$

We simulate a discrete distribution of $n = 79$ particles, arbitrarily located in a spheroidal volume of radius one unit (see Fig.4). We place the test mass m_0 at different points along a radius that makes an angle $\phi = 67.5^{\circ}$ on the x-y plane (Fig. 4 b) to calculate its velocity.

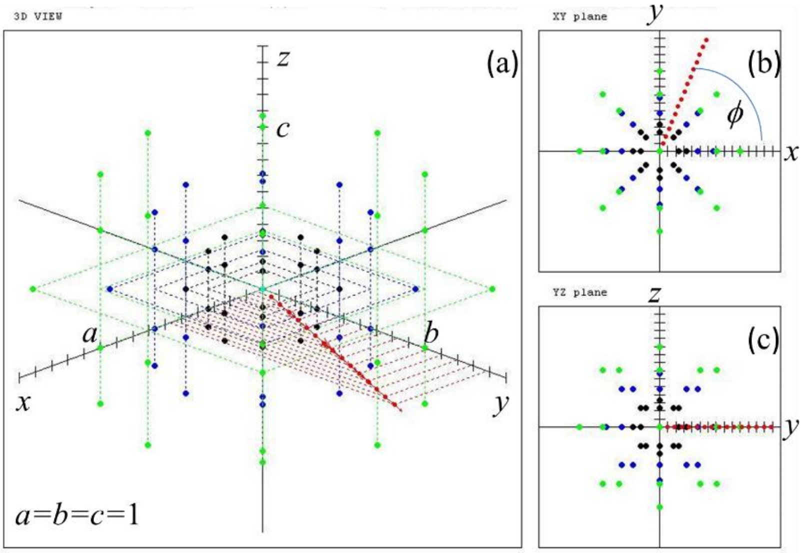

Fig. 4 In (a) the discrete distribution of n = 79 particles for the spherical case, that is, when the semi-axes in x, y and z are a = b = c = 1.0, located in three layers of radii r = 1/3, 2/3 and 1.0 is shown. Their angular positions are shown in (b) and (c)

The results are shown in Fig. 5, where the velocities that m_0 would have when placed at the mentioned points have been graphed. The velocity profiles show the same behaviour for a spherical distribution, where a = b = c = 1.0 as for a disk with semi-axes a = b = 1.0 and c = 0.1, where the profiles are almost flat from r ≅ 0.4, in contrast with the velocity profile for a Newtonian potential. This shows that the gravitational field produced by a discrete mass distribution is very different from the field produced by an equivalent mass located in the centre of the distribution.

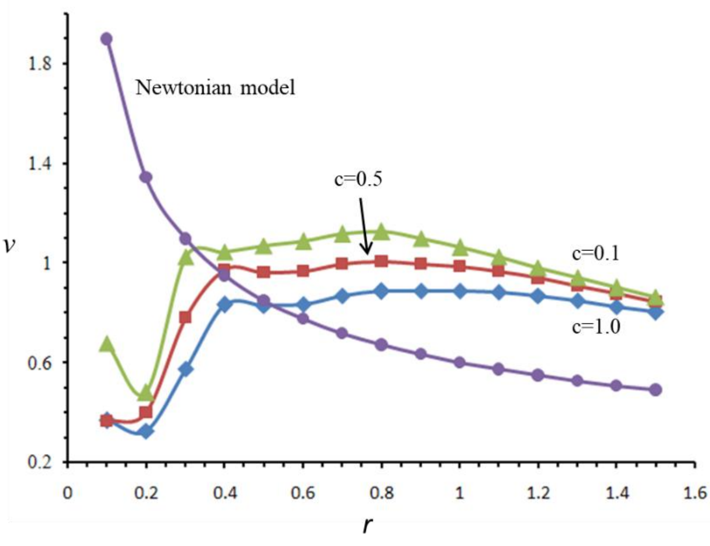

Fig. 5 The velocity profiles, obtained along a radius, for a discrete spheroidal distribution of particles are shown. Spheroids consist of 79 particles located in three layers of radii 1/3, 2/3 and 1.0 on a normalized scale, and whose vertical semi-axis is c = 1.0, c = 0.5 and c = 0.1 in each case. In contrast to the velocity distribution for the Newtonian model, it is observed that these maintain an almost constant profile for r > 0.4

II. Comments and conclusions

Once the postulate of the existence of Dark Matter was established, the search also began to determine what that strange substance could be, that matter of ethereal consistency that cannot be seen, and that in the end could not be found despite the time and effort spent. After almost 90 years of waiting, to try to define what Dark

Matter is constituted of, with results that in the best of cases were false alarms, the temptation is very great to say that this thing does not really exist.

Although the reasons may have a justification, to propose this postulate, these reasons lose their support when a physic-mathematical reasoning is found with which the reason for the supposedly high rotation speeds of stars in galaxies and galaxies in galaxy clusters is directly explained.

Here, a mathematical analysis has been shown, with which the rotation speeds that can be in a galaxy are calculated. Therefore, it is now possible to consider the introduction of something so elusive and undetectableand, in fact, can now be considered out of place. Ultimately, it can be said that was an error postulating the existence of Dark Matter, which in the end results the realisation of an unnecessary venture.

Given the affirmation of the existence of Dark Matter, some researchers calculate that this strange matter makes up approximately 25% of the total matter in the universe. We consider that this percentage has to be modified.

As a corollary, it is established that as a thing which is apart from the stellar rotation speeds, it is also possible to explain to a large extent the mass distribution in a galaxy, from the determination of the curve of the rotation speeds of the stars.

In order to ensure and confirm that the analytical calculations made here are correct, the numerical calculation of an approximate distribution of what a galaxy could be was also carried out. The result of the numerical calculation is also strongly outlined towards the result determined experimentally by Vera Rubin.

Finally, returning to the philosophical questions mentioned at the beginning of this paper, it is possible to affirm with physical property, the non-existence of Dark Matter. This was a resource that should not have been used at any time. This led to a fruitless search for almost 90 years for something that is not only inexplicable but ultimately also seemingly non-existent.

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