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Artificial Tobacco Drying Process: Representation Through A Mathematical Model

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Abstract
This paper aims at developing a mathematical model of the process of drying of tobacco using finite elements for the solution of the partial differential equations present in such a model. To choose the appropriate physical model, experiments were accomplished in a real greenhouse. Simulations were made for several diagram of distribution of leaves inside the greenhouse, and conditions of contact of the air in movement with the layer of the leaves were analyzed. For real greenhouses, the most appropriate diagram was found, which will allow increasing the quality of the drying of tobacco and reducing the expenses of energy for the studied issue.
Keywords: *Mathematical model, artificial drying, tobacco, air drainage*

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I. INTRODUCTION

Preoccupations concerning quality are present in all of the human life's aspects. More and more, people, companies or government institutions address their actions based on issues involving their products and services in search of quality. This way, tobacco houses and farmers try to invest in new technologies for the improvement of the quality of tobacco in Vale do Rio Pardo, Rio Grande do Sul, because the exports have increased from 30 million to 700 million dollars during the last 20 years, and Brazil has become one of the largest suppliers of tobacco in the world as well as the United States and Zimbabwe [6].

Aiming at obtaining larger quality of the product, many investments in the farmers' properties have occurred like, for example, the buying of agricultural machineries and the improvement of new greenhouses. With the new experiments and the acquired knowledge, it can be indicated an ideal time for planting and also new varieties of seeds that adapt in each area depending on each soil; which will facilitate the control of curses and diseases that may damage plantations. Several factors interfere in the harvest of tobacco: it should happen in the right time so that the product doesn't deteriorate in the farm; also, the influence of the climate is very important for the production of tobacco. In spite of this, the main issue regards the appropriate drying process [5].

In the search of new techniques for the drying of tobacco and its best income, one of the procedures consists of applying mathematical methods, trying to describe the dynamics of the process, establishing a simplified model of the reality [3, 2].

The several factors involved in the drying (time and temperature of drying, amount of leaves put in the greenhouse, flowing of the air and distribution of leaves inside the greenhouse) determine the expenses of energy (firewood and electricity) and the quality of the final product. Leaves arranged in different ways change the process of air drainage inside the greenhouse, forming the domains with insufficient or ample contact between the air and the layer of the leaves. The leaf drying process happens due to the difference of moisture content in the leaves and in the air, and it depends on the temperature and on the air speed.

Mathematically, this problem can be described by the system of partial differential equations of movement, of energy and of applied diffusion for the air and equations of energy and of diffusion applied in the leaves [3, 4]. At the moment, solving this problem is very complicated, because, besides mathematical and computational difficulties, there are many unknown factors, for instance, the coefficients of transfer of heat and of mass among the leaves and the air for several temperature values, speed and moisture content.

This work has as main purpose to develop a mathematical model of the process of drying of tobacco, aiming at obtaining a good quality of this product in Vale do Rio Pardo.

II. MATHEMATICAL MODEL

In this work, air drainage in artificial greenhouses has been investigated. In different points of the drainage, the vectorflow of the air can assume different values and directions, depending on the geometry of the system and the outline conditions [4, 1]. In order to analyze the air drainage process in the greenhouse separately (without any influence of transference of heat and mass with the leaves of tobacco), the study was limited to the isotherm air drainage. For the incompressible and stationary drainage, and also despising the volumetric forces, we have 2-D. Equation of Continuity:

$$div\ \ w = 0 \quad (1)$$

or

$$\frac{au}{ax} + \frac{av}{ay} = 0 \quad (2)$$

where: u = horizontal component of speed; v = vertical component of speed; x and y are the Cartesian Coordinates; w = vector of speed.

For the obtainment of an exact differential when ω is defined in terms of the coordinates of the field (and y) it is equal to a constant:

$$\psi = \psi(x, y) = const \quad (3)$$

This function's differential will be:

$$d\psi = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy = 0 \quad (4)$$

Using the Laplace equation regarding the function ψ and φ , defined as potential of speed, we have:

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0 \quad (5)$$

or if

$$\varphi = \mathrm{P} \ ; \ \nabla^2 \mathrm{P} = 0 \quad (6)$$

where: P = pressure; ∇ = Hamiltonian operator (gradient).

Equation (6) describes the distribution of the pressures for case 2-D. Equation (5) is a Laplace equation of the speed potential. The relative pressure is given by:

$$p = \frac{P - P_a}{P_{\max} - P_a} \quad (7)$$

where: $p = p(x,y)$= relative pressure; P = pressure in a given point; P_a = atmospheric pressure; $P_{máx}$= maximum pressure of the air in the entrance of the greenhouse, with the following border conditions:

$$p = p_s \quad (8)$$

for S_1 and p_e for S_2

$\vec{n} \cdot gradP = 0$ for S_3 (9)

where:

$$\frac{\partial p}{\partial x} = \frac{\partial P}{\partial x} \frac{L}{P_{m\grave{a}x} - P_a}; \qquad (10)$$
$$\frac{\partial p}{\partial y} = \frac{\partial P}{\partial y} \frac{L}{P_{m\grave{a}} - P_a};$$

$p_s = 0$ for the free side of the leaves; $p_e = 1$ for the entrances of air with maximum pressure; L = typical size of the greenhouse; S_1 = surface of exit of the air in the greenhouse; S_2 = surface of entrance of the air in the greenhouse; S_3 = surface of contact of the air with the leaves and with the walls of the greenhouse; ***n*** = normal unitary vector to the surface.

The problem given by Equations (5), (8) and (9) is equivalent to the problem of minimization of functional for function of two variables x and y.

$$\Phi\left[z(x, y)\right] = \iiint_V F\left(x, y, z, z_x, z_y\right) dV ; \qquad (11)$$

and applying the Srogradski-Gauss equation:

$$\iiint_V div\, \vec{w}\, dv = \iint_S \vec{n} \cdot \vec{w}\, ds \quad (12)$$

Changing z for p, the problem is given in Equations (5), (8) and (9). This way, Equation (5), with the conditions present in (8) and (9), results in what is called Equation of Ostrogradski for the functional:

$\Phi = \Phi\left[p(x, y)\right]$ (13)

Transforming the functional in:

$$\Phi = \mathrm{p}(\mathrm{x}, \mathrm{y}) = \iiint_{\mathrm{v}} \frac{1}{2}\left[\left(\frac{\partial \mathrm{p}}{\partial \mathrm{x}}\right)^2 + \left(\frac{\partial \mathrm{p}}{\partial \mathrm{y}}^2\right)\right] \mathrm{dV} \quad (14)$$

Dividing the above integral in finite elements, we have:

$$\Phi\left[P(x, y)\right] = \sum_{e=1}^{E} \iiint_{v^{(e)}} \frac{1}{2}\left\{P^{(e)}\right\}^T \left[B^{(e)}\right]^T \left[D^{(e)}\right]\left[B^{(e)}\right]\left\{P^{(e)}\right\} dV \quad (15)$$

where: *e*- is the index of the finite element; and *E*- is the total number of elements.

The function to integrate doesn't depend on the coordinates x and y for the division of the chosen integration domain. Really, $\left\{P^{(e)}\right\}$ is determined by values in the knots we have considered constant, $\left[B^{(e)}\right]$ is the matrix determined by the coordinates of the vertexes of element "e" and matrix $\left[D^{(e)}\right]$ is constant for the given element. Therefore, we have:

$$\Phi\left[P(x, y)\right] = \sum_{e=1}^{E} \frac{1}{2}\left\{P^{(e)}\right\}^T \left[B^{(e)}\right]^T \left[D^{(e)}\right]^T \left[B^{(e)}\right]\left\{P^{(e)}\right\} \cdot \iiint_{V^{(e)}} dV \quad (16)$$

Considering the thickness of the material being studied in normal direction to the integration domain equal to 1, we have:

$$\iiint_{v^{(e)}} dV = A^{(e)} \cdot 1 = A^{(e)} \quad (17)$$

where $A^{(e)}$ is the area of the finite element "e". Regarding the analytical geometry for the area of the triangle, we have:

$$A^{(e)} = \frac{1}{2}\begin{vmatrix} 1 & x_i^{(e)} & y_i^{(e)} \\ 1 & x_j^{(e)} & y_j^{(e)} \\ 1 & x_m^{(e)} & y_m^{(e)} \end{vmatrix} = \frac{1}{2}\det C \quad (18)$$

where $\left(x_i^{(e)}\ y_i^{(e)}\right),\left(x_j^{(e)}\ x_j^{(e)}\right),\left(x_m^{(e)}\ x_m^{(e)}\right)$ are the coordinates of the vertexes of the element "e". Therefore, Equation (17) can be written in this way:

$$\Phi\left[P(x,y)\right] = \frac{1}{4}\sum_{e=1}^{E} \det C^{(e)}\left\{P^{(e)}\right\}^T\left[B^{(e)}\right]^T\left[D^{(e)}\right]\left[B^{(e)}\right]\left\{P^{(e)}\right\} (19)$$

Function (16) can be considered a function of several variables P_u (u=1, 2, 3,.... n), where n is the number of knots. To minimize this function, (16) is derived in relation to P_u and a system of linear equations is obtained:

$$\begin{cases} \dfrac{\partial \Phi}{\partial P_1} = 0 \\ \dfrac{\partial \Phi}{\partial P_2} = 0 \\ \dots\dots\dots\ \dots \\ \dfrac{\partial \Phi}{\partial P_n} = 0 \end{cases} \quad (20)$$

In order to derive the products of the matrixes, the following formula can be used:

$$\frac{\partial\left(\{P\}^T[G]\{P\}\right)}{\partial\{P\}} = 2[G]\{P\}\ (21)$$

where [G] is a symmetric matrix. So, deriving (16), the following expression is obtained:

$$\frac{\partial \Phi}{\partial\left\{P^{(e)}\right\}} = \sum_{e=1}^{E}\frac{\det C^{(e)}}{2}\left[B^{(e)}\right]^T\left[D^{(e)}\right]\left[B^{(e)}\right]\left\{P^{(e)}\right\} = 0 \quad (22)$$

or

$$\sum \frac{1}{2\det C^{(e)}}\left(\begin{bmatrix} C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33}\end{bmatrix}^T\begin{bmatrix} M_{xx} & 0 \\ 0 & M_{yy}\end{bmatrix}\begin{bmatrix} C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33}\end{bmatrix}\right)\left\{P^{(e)}\right\} = 0\ (23)$$

To obtain the non-trivial solution, it is necessary to transform the system, choosing the values of P_u in the borders of S_1 and S_2. For example, for S_1 (layer of the leaf), all of the value of P_u are equal to zero. For the entrance of the air with maximum pressure, the values of P_u are equal to 1. The system can be written in the following way:

$$[M]\{P\}=b \quad (24)$$

where b is the load vector that depends on the border conditions; $[M]$ is the rigidity matrix; $\{P\}$ is the vector of unknown. The determinant of matrix $[M]$ is always different from zero.

III. RESULTS

It is necessary six or seven days to dry the tobacco, and it depends on temperature, climate and texture of the leaf. To fill an oven with tobacco, 236 bundles are necessary, (There are approximately 500 to 600 leaves per bundle). A bundle weighs 22,50kg, and 5196kg of tobacco are needed to fill up a greenhouse.

During the drying process, the sighs of the oven are closed, because moisture cannot go beyond established limits. Because of this, it is not necessary to open the sighs of the oven, which, in the case of artificial ovens, it is automatic, due to the existence of moisture sensors. Moisture and temperature grow up in function of the time of drying of the tobacco leaves, and moisture decreases after 13 days, for the baixeira(lower) leaf; and after 23 days, for the meeira (medium) leaf. Considering the results of the measurements of the temperature parameters and air moisture, we could conclude the following:

i) Moisture and temperature variations occur very slowly inside the greenhouse;

ii) The level of speeds is very small inside the greenhouse. It means that the presence of re-circulation areas can be despised.

Based on these conclusions, air drainage inside the greenhouse can be considered stationary, isotherm and irrotational. Several simulations of air drainage were accomplished, and they are presented considering two diagrams. The first one consists of obtaining the leaves of tobacco in the superior part of the greenhouse.

In Diagram 1(Figure1), air goes directly from the entrance to the exit with small contact with the leaves. This way, it doesn't occur a good drying, and leaves will present a terrible quality. In Diagram 2 (Figure 2), leaves are put in the inferior and superior extremity of the greenhouse.

Diagram 2 (Figure 2) presents the best result for drying, because the contact of air among the leaves increased. So, distribution of placement of leaves inside the greenhouse can be done so that time of permanence of air increases, what can intensify the drying process or reduce expenses of energy during such a process.

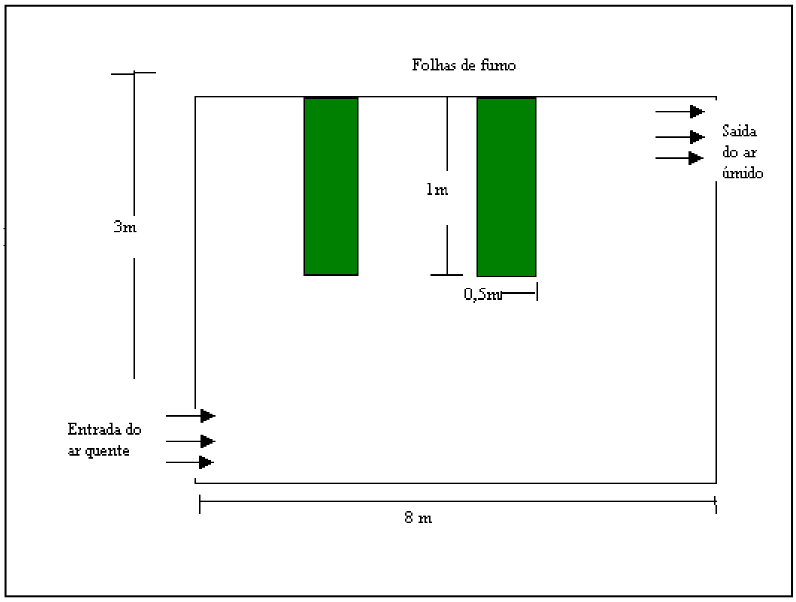

Figure. 1: Representation of the drying of leaves of tobacco – Diagram 1

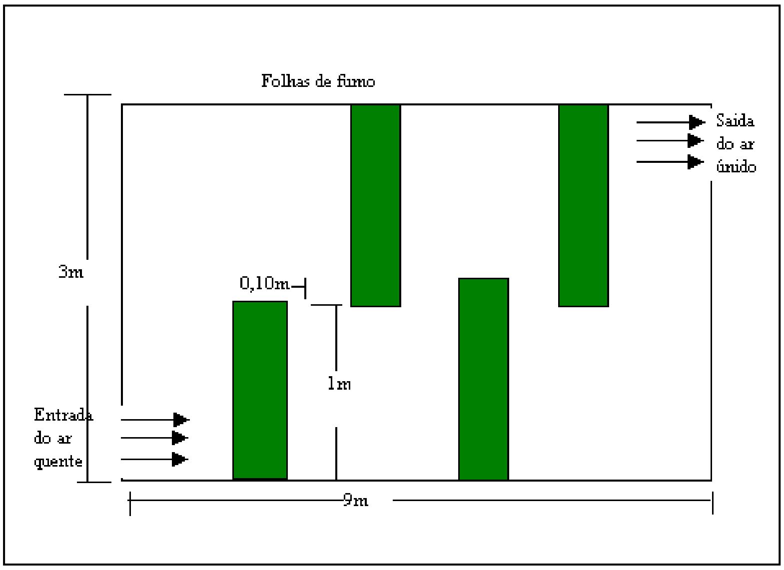

Figure 2: Representation of the drying of leaves of tobacco -Diagram - 2

IV. CONCLUSION

At first, air drainage inside the greenhouses for several placement diagrams of leaves has been studied. To choose the appropriate physical model, experiments were accomplished in real greenhouses, for obtention of good quality product. Variations per time of parameters in the greenhouse (speed, temperature and concentration of steam of water) should be very limited. Analysis of the results allowed considering the drainage in the greenhouse as an ideal two-dimensional stationary isotherm drainage.

A mathematical model, which describes the air drainage in the greenhouse of tobacco, was adapted, and a computational applicative was elaborated, in which the technique of the finite elements was used for the solution of partial differential equations present in the model (for pressure and draughtfunction), making possible the construction of the graphs of the isobars and of the current lines, which could be used for simulation of air drainage in greenhouses and it could be adapted for silos and grocery stores with different geometries and air distribution systems.

Simulations for several diagrams of distribution of leaves inside the greenhouse were done, and the conditions of contact of the air in movement with surfaces of the leaves were analyzed. For real greenhouses, the most appropriate diagram was proposed, what will allow the increase of the quality of drying of tobacco and the reduction of expenses of energy for the studied issue.

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