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# Modified Beavers and Joseph Condition in the Study of Flow through Composite Porous Layers

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***Abstract***

*Flow through a finite Brinkman porous layer underlain by a Darcy porous layer or a Forchheimer porous layer is analyzed in this work. Condition at the interface is the slip velocity condition of Beavers and Jospeh when a Darcy layer is involved, and a modified Beavers and Joseph condition when a Forchheimer porous layer is involved. Expressions for the interfacial velocity and shear stress at the interface are obtained together with a relationship between the slip coefficients for the different layers employed.*

***Keywords:*** *Composite Porous Layers; Slip Flow; Beavers and Joseph Modified Condition.*

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## I. INTRODUCTION

Prior to 1967, it has been customary to use a no-slip condition at the interface between a fluid and a porous surface (*cf.* Joseph & Tao [1]). In 1967, the experiments of Beavers and Joseph [2] initiated an interest in coupled parallel flow between a porous layer and free-space, and between composite porous layers. They observed that the mass flux through a free-space channel is greater than the predicted mass flux by Poiseuille flow when a no-slip condition is used, [2]. They provided an explanation in terms of a slip flow hypothesis at the interface and proposed the following empirical slip-flow condition that agreed well with their experiments:

$$\frac{\partial u}{\partial y} = \frac{\alpha}{\sqrt{k}}(u_i - u_D) \dots (1)$$

where $u_i$ is the tangential velocity component at the interface, $u_D$ is the uniform filtration (Darcy) velocity in a porous layer, $k$ is the permeability to the flow in the porous medium (assumed constant), and $\alpha$ is a slip coefficient whose value depends on the porous structure properties. Nield [3] provided the range of 0.01 to 5 for $\alpha$, and reported that in the experiments of Beavers and Joseph, the $\alpha$ values used were 0.78, 1.45, and 4.0 for Foametal having average pore sizes of 0.016, 0.034, and 0.045 inches, respectively, and 0.1 for Aloxite with average pore size of 0.013 or 0.027 inches.

While Beavers and Joseph [2] provided analysis of flow in a channel over a Darcy porous layer, many authors favoured the use of Brinkman's equation due to compatibility of differential orders between Brinkman's equation and the Navier-Stokes equations. Significance of Brinkman's equation has been discussed by Neale and Nader [4] who suggested velocity and shear stress continuity at the interface, and recovered Beavers and Joseph's results by defining the slip parameter, $\alpha$, in terms of the fluid viscosity coefficient, $\mu$, and the effective viscosity coefficient, $\mu_e$, as

$$\alpha^2 = \frac{\mu_e}{\mu} \dots (2)$$

In addition to the work of Neale and Nader, [4], many other investigations point to a general agreement that conditions at the interface must emphasize velocity continuity and shear stress continuity in order to facilitate the matching of flow in the channel with the flow through the porous medium. Rudraiah, [5], concluded that Brinkman's equation is a more appropriate model when the porous layer is of finite depth.

Modifications of the Beavers and Joseph condition have been proposed and appear in the many excellent reviews of flow through and over porous layers (*c.f.* [6-15] and the references therein). However, in the current work we are interested in an extension of the Beavers and Joseph condition to the problem of flow over a porous layer where Forchheimer's equation is valid.

Abu Zaytoon and Hamdan [16] considered flow in a channel underlain by a Forchheimer's porous layer. They provided the following modification to equation (1)

$$\frac{\partial u}{\partial y} = \frac{\beta}{\sqrt{k}}(u_i - v) \ldots (3)$$

where $v$ is the velocity in the Forchheimer's porous layer and $\beta$ is a slip coefficient associated with the Forchheimer-type porous microstructure.

Both conditions (2) and (3) emphasize that the shear stress at the interface with a porous layer is proportional to the difference between the interfacial velocity and the velocity in the porous medium. Beavers and Joseph [2] used condition (1) to obtain the following expression at the interface between a channel and a Darcy porous layer:

$$u_i = -\frac{k}{2\mu}\left[\frac{\sigma^2 + 2\alpha\sigma}{1 + \alpha\sigma}\right]p_x \ldots (4)$$

where $\sigma = \frac{h}{\sqrt{k}}$ and $p_x = \frac{dp}{dx} < 0$ is the common driving pressure gradient.

Abu Zaytoon and Hamdan [16] used condition (2) to obtain the following expression at the interface between a channel and a Forchheimer porous layer:

$$u_i = -\frac{k}{2\mu}\left[\frac{\sigma^2 + 2\alpha\sigma\xi}{1 + \alpha\sigma}\right]p_x \ldots (5)$$

wherein $h$ is the channel width, $C_f$ is the Forchheimer coefficient,$\rho$ is the fluid density and

$$\xi = \frac{\mu}{2\rho C_f k\sqrt{k}p_x}\{\mu - \sqrt{[\mu^2 - 4\rho C_f k\sqrt{k}p_x]} \ldots (6)$$

The above analysis and interfacial conditions relate to the Navier-Stokes flow over a porous layer. Of interest is to study the effects of conditions (1) and (3) when flow is through a finite porous layer underlain by a semi-infinite Darcy or a Forchheimer porous layer and bounded above by a solid, impermeable wall on which the no-slip condition is imposed. As per Rudraiah's suggestion, [5], Brinkman's equation can be used to govern the flow in a finite porous layer, and it viscous shear term in suitable for the no-slip condition on the upper wall. In both flow situations, we obtain expressions for the interfacial velocity and investigate the relationship between the slip parameters $\alpha$ and $\beta$. We present our analysis for the general case of different constant permeabilities in the composite layers and the specific case of equal permeabilities.

## II. PROBLEM ANALYSIS

Fluid flow through a porous medium in the presence of an impermeable solid wall can be described by the following continuity and Brinkman's equations, written here for the steady flow of an incompressible viscous fluid through a constant permeability medium:

$$\nabla \cdot \vec{u} = 0 \ldots (7)$$

$$\rho(\vec{u} \cdot \nabla)\vec{u} = -\nabla p + \mu_e \nabla^2 \vec{u} - \frac{\mu}{\eta}\vec{u} \ldots (8)$$

wherein $\rho$ is the fluid density, $p$ is the pressure, $\mu$ is the fluid base viscosity, $\eta$ is the Brinkman layer permeability, $\mu_e = \frac{\mu}{\varphi}$ is the effective viscosity (that is, viscosity of fluid saturating the porous medium), $\varphi$ is the porosity, $\vec{u}$ is the fluid velocity field.

For uni-directional flow, the tangential velocity component is $u = u(y)$, and equations (7) and (8) reduce to

$$u_{yy} - \frac{u}{\eta\vartheta} = \frac{p_x}{\mu_e} \ldots (9)$$

$$\vartheta = \frac{\mu_e}{\mu}$$

Equation (9) is valid in the finite porous layer $0 \leq y \leq h$ while Darcy's law and Forchheimer's equation are valid in the semi-infinite lower layer, $-\infty < y \leq 0$. The interface between the layers is assumed to be sharp with location at $y = 0$, as shown in **Fig. 1**.

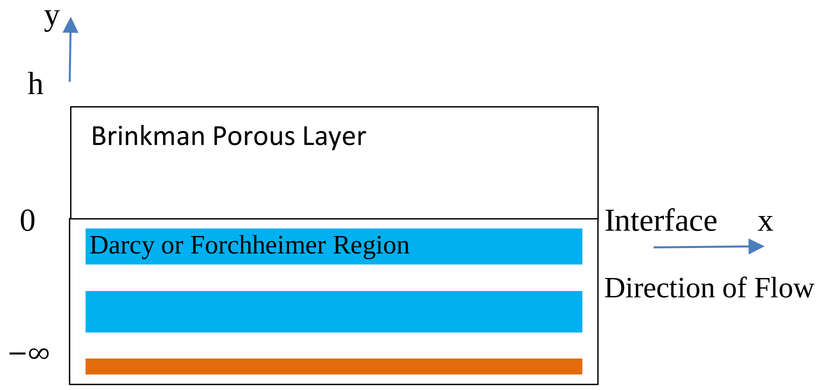


**Fig. 1. Schematic Sketch**

Darcy's law, for uni-directioal flow, is written as:

$$u_D = -\frac{kp_x}{\mu} \dots (10)$$

and Forchheimer's equation takes the form:

$$\frac{\rho C_f}{\sqrt{k}} v|v| + \frac{\mu}{k} v + p_x = 0 \dots (11)$$

where in equations (10) and (11), $u_D$ is the Darcy seepage tangential velocity, $v$ is the Forchheimer tangential velocity, $C_f$ is the Forchheimer drag coefficient.

We are thus required to solve (9) subject to $u(h) = 0$ and $u(0) = u_i$, where $u_i$ is the velocity at the interface $y = 0$ that can be determined with the help of Beavers and Joseph [2] condition (1) when the lower layer is a Darcy layer and condition (3) when the lower layer is a Forchheimer porous layer. We will first assume that permeabilities $k$ and $\eta$ are different.

***Case 1: Flow through a finite porous layer underlain by a semi-infinite Darcy porous layer***

Solution to (9) is given by

$$u = c_1 cosh \frac{y}{\sqrt{\vartheta\eta}} + c_2 sinh \frac{y}{\sqrt{\vartheta\eta}} - \frac{\eta p_x}{\mu} \dots (12)$$

with

$$u_y = \frac{c_1}{\sqrt{\vartheta\eta}} sinh \frac{y}{\sqrt{\vartheta\eta}} + \frac{c_2}{\sqrt{\vartheta\eta}} cosh \frac{y}{\sqrt{\vartheta\eta}} \dots (13)$$

where $c_1$ and $c_2$ are arbitrary constants.

Using $u(h) = 0$ and condition (1) in (12) and (13), we obtain

$$c_2 = \alpha \sqrt{\frac{\vartheta\eta}{k}} \left( u_i + \frac{kp_x}{\mu} \right) \dots (14)$$

$$c_1 = \frac{\eta p_x}{\mu} sech \frac{\sigma}{\sqrt{\vartheta}} - \alpha \sqrt{\frac{\vartheta\eta}{k}} \left( u_i + \frac{kp_x}{\mu} \right) tanh \frac{\sigma}{\sqrt{\vartheta}} \dots (15)$$

and the following velocity distribution in the Brinkman's layer

$$u = \left[ \frac{\eta p_x}{\mu} sech \frac{\sigma}{\sqrt{\vartheta}} - \alpha\sqrt{\vartheta\eta} \left( \frac{u_i}{\sqrt{k}} + \frac{\sqrt{k} p_x}{\mu} \right) tanh \frac{\sigma}{\sqrt{\vartheta}} \right] cosh \frac{y}{\sqrt{\vartheta\eta}}$$
$$+ \left( \alpha \sqrt{\frac{\vartheta\eta}{k}} u_i + \alpha\sqrt{\vartheta\eta k} \frac{p_x}{\mu} \right) sinh \frac{y}{\sqrt{\vartheta\eta}} - \frac{\eta p_x}{\mu} \dots (16)$$

Using $u(0) = u_i$ in (16) and solving for $u_i$, we obtain:

$$u_i = \frac{\eta p_x}{\mu} \frac{\left(1 - cosh \frac{\sigma}{\sqrt{\vartheta}}\right) - \alpha \sqrt{\frac{\vartheta k}{\eta}} sinh \frac{\sigma}{\sqrt{\vartheta}}}{cosh \frac{\sigma}{\sqrt{\vartheta}} + (\alpha\sqrt{\vartheta\eta k}) sinh \frac{\sigma}{\sqrt{\vartheta}}} \dots (17)$$

If $\eta = k$ then

$$u_i = \left[\frac{1}{cosh\frac{\sigma}{\sqrt{\vartheta}} + \alpha\sqrt{\vartheta}sinh\frac{\sigma}{\sqrt{\vartheta}}} - 1\right]\frac{kp_x}{\mu} \ldots (18)$$

Since $\vartheta = \frac{\mu_e}{\mu}$ and, by (2), $\alpha^2 = \frac{\mu_e}{\mu}$, it follows that $\alpha^2 = \vartheta$ or $\alpha = \sqrt{\vartheta}$, and (18) can be written as:

$$u_i = \left[\frac{1}{cosh\frac{\sigma}{\sqrt{\vartheta}} + \vartheta sinh\frac{\sigma}{\sqrt{\vartheta}}} - 1\right]\frac{kp_x}{\mu} \ldots (19)$$

By comparison, when the flow is through free-space over a Darcy porous layer, velocity at the interface is given by equation (4).

***Case 2: Flow through a finite porous layer underlain by a semi-infinite Forchheimer porous layer***

In this case, flow is governed by the Forchheimer's equation (11). We note that if $C_f = 0$, equation (11) reduces to Darcy's equation (10).

We are required to solve (9) and (11) subject to $u(h) = 0$ and condition (3) at $y = 0$. Solution to (9) is given by (12) with $u_y$ given by (13).

When all the parameters are constant, equation (11) is an algebraic equation in the velocity, written in the following form:

$$v^2 + \frac{\mu}{\rho C_f\sqrt{k}}v + \frac{\sqrt{k}\,p_x}{\rho C_f} = 0 \ldots (20)$$

Solving (20) and choosing the positive root so that the velocity is non-negative, yields:

$$v = -\frac{\mu}{2\rho C_f\sqrt{k}} + \sqrt{\frac{\mu^2}{4\rho^2 C_f^2 k} - \frac{\sqrt{k}\,p_x}{\rho C_f}} \ldots (21)$$

Using $u(h) = 0$ in (9) and using (21) in (13) when $y = 0$, and solving for the arbitrary constants $c_1$ and $c_2$ appearing in (9), we get:

$$c_2 = \frac{\beta\sqrt{\vartheta\eta}}{\sqrt{k}}\left[u_i + \frac{\mu}{2\rho C_f\sqrt{k}} - \sqrt{\left(\frac{\mu}{2\rho C_f\sqrt{k}}\right)^2 - \frac{\sqrt{k}p_x}{\rho C_f}}\right] \ldots (22)$$

$$c_1 = \frac{\eta p_x}{\mu}sech\frac{h}{\sqrt{\vartheta\eta}} - \beta\sqrt{\vartheta\eta/k}tanh\frac{h}{\sqrt{\vartheta\eta}}\,u_i - \beta\left[\frac{\sqrt{\vartheta\eta}\,\mu}{2\rho C_f k} - \sqrt{\vartheta\eta/k}\sqrt{\left(\frac{\mu}{2\rho C_f\sqrt{k}}\right)^2 - \frac{\sqrt{k}p_x}{\rho C_f}}\right]tanh\frac{h}{\sqrt{\vartheta\eta}} \ldots (23)$$

Velocity profile in the Brinkman layer thus takes the form:

$$\begin{aligned} u = &\left\{\frac{\eta p_x}{\mu}sech\frac{h}{\sqrt{\vartheta\eta}} - \beta\sqrt{\vartheta\eta/k}\,tanh\frac{h}{\sqrt{\vartheta\eta}}\,u_i \right. \\ &\left. - \beta\sqrt{\vartheta\eta/k}\left[\frac{\mu}{2\rho C_f\sqrt{k}} - \sqrt{\left(\frac{\mu}{2\rho C_f\sqrt{k}}\right)^2 - \frac{\sqrt{k}p_x}{\rho C_f}}\right]tanh\frac{h}{\sqrt{\vartheta\eta}}\right\}cosh\frac{y}{\sqrt{\vartheta\eta}} \\ &+ \beta\sqrt{\vartheta\eta/k}\left[u_i + \frac{\mu}{2\rho C_f\sqrt{k}} - \sqrt{\left(\frac{\mu}{2\rho C_f\sqrt{k}}\right)^2 - \frac{\sqrt{k}p_x}{\rho C_f}}\right]sinh\frac{y}{\sqrt{\vartheta\eta}} - \frac{\eta p_x}{\mu} \ldots (24) \end{aligned}$$

Evaluating (24) at $y = 0$ and solving for $u_i$, we obtain the following expression for the interfacial velocity:

$$u_i = \frac{\left\{\frac{\eta p_x}{\mu} - \beta\sqrt{\vartheta\eta/k}\left[\frac{\mu}{2\rho C_f\sqrt{k}} - \sqrt{\left(\frac{\mu}{2\rho C_f\sqrt{k}}\right)^2 - \frac{\sqrt{k}p_x}{\rho C_f}}\right]sinh\frac{h}{\sqrt{\vartheta\eta}}\right\}}{\left[cosh\frac{h}{\sqrt{\vartheta\eta}} + \beta\sqrt{\vartheta\eta/k}sinh\frac{h}{\sqrt{\vartheta\eta}}\right]} \ldots (25)$$

If $\eta = k$ then (25) can be written in the form:

$$u_i = \frac{\left\{\frac{kp_x}{\mu} - \beta\sqrt{\vartheta}\xi sinh\frac{\sigma}{\sqrt{\vartheta}}\right\}}{\left[cosh\frac{\sigma}{\sqrt{\vartheta}} + \beta\sqrt{\vartheta}sinh\frac{\sigma}{\sqrt{\vartheta}}\right]} \ldots (26)$$

where $\xi$ is as given by (6).

By comparison, when the flow is through free-space over a Forchheimer porous layer, velocity at the interface is given by equation (5).
Equating the interfacial velocities in (18) and (26), we obtain the following expression for $\beta$ in terms of $\alpha$:

$$\beta = \frac{sinh\frac{\sigma}{\sqrt{\vartheta}}\left\{\left[1-\left(1-\frac{\mu}{kp_x}\xi\right)cosh\frac{\sigma}{\sqrt{\vartheta}}\right]-\alpha\left[\sqrt{\vartheta}-\frac{\sigma\mu}{kp_x}\xi\right]sinh\frac{\sigma}{\sqrt{\vartheta}}\right\}}{\frac{1}{\sqrt{\vartheta}}cosh^2\frac{\sigma}{\sqrt{\vartheta}}+\alpha\, sinh\frac{\sigma}{\sqrt{\vartheta}}(1+cosh\frac{\sigma}{\sqrt{\vartheta}})} \ldots (27)$$

If $= \sqrt{\vartheta}$ , as per equation (2), then (27) takes the form:

$$\beta = \frac{sinh\frac{\sigma}{\sqrt{\vartheta}}\left\{\left[1-\left(1-\frac{\mu}{kp_x}\xi\right)cosh\frac{\sigma}{\sqrt{\vartheta}}\right]-\left[\vartheta-\frac{\sigma\mu\sqrt{\vartheta}}{kp_x}\xi\right]sinh\frac{\sigma}{\sqrt{\vartheta}}\right\}}{\frac{1}{\sqrt{\vartheta}}cosh^2\frac{\sigma}{\sqrt{\vartheta}}+\sqrt{\vartheta}sinh\frac{\sigma}{\sqrt{\vartheta}}(1+cosh\frac{\sigma}{\sqrt{\vartheta}})} \ldots (28)$$

## III. DIMENSIONLESS FORM

We can express the velocity at the interface and the relationship between $\alpha$ and β in dimensionless form by introducing the following dimensionless variables, wherein $h$ is takes as the characteristic length and $u_c$ is the characteristic velocity:

$$(X,Y) = \frac{(x,y)}{h};\;\; (U,V) = \frac{(u,v)}{u_c};\;\; K = \frac{k}{h^2};\;\; p^* = \frac{p}{\rho(u_c)^2};\;\; Re = \frac{\rho u_c h}{\mu} \ldots (29)$$

Note that in dimensionless form, dimensionless $\sigma$ is $\frac{1}{\sqrt{K}}$ , $P$ is the dimensionless pressure gradient and dimensionless $\xi$ is $\xi^* = \frac{2}{1+\sqrt{1-4K\sqrt{K}C_f Re^2 P}}$.
Equation (18) takes the following dimensionless form:

$$u_i = \left[\frac{1}{cosh\frac{1}{\sqrt{K\vartheta}}+\alpha\sqrt{\vartheta}sinh\frac{1}{\sqrt{K\vartheta}}}-1\right]ReKP \ldots (30)$$

Equation (26) takes the following dimensionless form:

$$U_i = \frac{\left\{ReKP-\beta\sqrt{\vartheta}\xi^* sinh\frac{1}{\sqrt{K\vartheta}}\right\}}{\left[cosh\frac{1}{\sqrt{K\vartheta}}+\beta\sqrt{\vartheta}sinh\frac{1}{\sqrt{K\vartheta}}\right]} \ldots (31)$$

Equation (27) takes the following dimensionless form:

$$\beta = \frac{sinh\frac{1}{\sqrt{K\vartheta}}\left\{\left[1-\left(1-\frac{1}{ReKP}\xi^*\right)cosh\frac{1}{\sqrt{K\vartheta}}\right]-\alpha\left[\sqrt{\vartheta}-\frac{1}{ReK\sqrt{KP}}\xi^*\right]sinh\frac{1}{\sqrt{K\vartheta}}\right\}}{\frac{1}{\sqrt{\vartheta}}cosh^2\frac{1}{\sqrt{K\vartheta}}+\alpha(1+cosh\frac{1}{\sqrt{K\vartheta}})sinh\frac{1}{\sqrt{K\vartheta}}} \ldots (32)$$

If $\alpha = \sqrt{\vartheta}$ then (33) becomes:

$$\beta = \frac{sinh\frac{1}{\sqrt{K\vartheta}}\left\{\left[1-\left(1-\frac{1}{ReKP}\xi^*\right)cosh\frac{1}{\sqrt{K\vartheta}}\right]-\left[\vartheta-\frac{\sqrt{\vartheta}}{ReK\sqrt{KP}}\xi^*\right]sinh\frac{1}{\sqrt{K\vartheta}}\right\}}{\frac{1}{\sqrt{\vartheta}}cosh^2\frac{1}{\sqrt{K\vartheta}}+\sqrt{\vartheta}(1+cosh\frac{1}{\sqrt{K\vartheta}})sinh\frac{1}{\sqrt{K\vartheta}}} \ldots (33)$$

## IV. CONCLUSION

In this work, we considered the flow through a finite porous layer underlain by a semi-infinite porous layer. Flow through the finite layer is governed by Brinkman's equation, while flow through the semi-infinite lower layer is governed by either Darcy's law or Forchheimer equation. At the interface between the layers we used the Beavers and Joseph slip condition when the lower layer is of the Darcy type, and a modified form of Beavers and Joseph condition when the lower layer is of the Forchheimer type. In both cases we obtained expressions for the velocity at the interface as a function of flow parameters and derived an expression for the relationship between the slip parameters, $\alpha$ when using the Beavers and Joseph condition and $\beta$ when using the modified Beavers and Joseph condition.

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