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Fixed point theorem on fuzzy metric spaces with rational inequality and its applications

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Abstract*. In this paper, we prove a fixed point theorem in fuzzy metric spaces using rational inequality. Our result generalizes and extends the result of Vishal Gupta et al. [13]. We justify our result by a suitable example. Some applications are also submitted in support of our results.*
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I. INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh [1] in 1965. Kramosil and Michalek [2] introduced the concept of fuzzy metric space in 1975, which can be regarded as a generalization of the statistical metric space. Clearly this work plays an essential role for the construction of fixed point theory in fuzzy metric spaces. Subsequently, in 1988, M. Grabiec [4] defined G-complete fuzzy metric space and extended the complete fuzzy metric spaces. Following Grabiec's work, many authors introduce and generalize the different types of fuzzy contractive mappings and investigate some fixed point theorem in fuzzy metric space. In 1994, George and Veeramani [3] modified the notion of *M*complete fuzzy metric space with the help of continuous t-norms. A number of fixed point theorem have been obtained by various authors in fuzzy metric spaces by using the concept of implicit relations, compatible maps, weakly compatible maps, R-weakly compatible maps.

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Vishal Gupta et al. [13] proved some fixed point theorems in fuzzy metric spaces through rational inequality and gave some applications also in 2013.

In 2019, Lukman Zicky et al. [12] described the concept of a fuzzy metric is developed based on fuzzy concepts. This fuzzy metric is then applied to convergence problems and fixed point problems, it was concluded that some properties in ordinary metric still apply to fuzzy metric.

II. PRELIMINARIES

Now, we begin with some basic concepts.

Definition 2.1. *(Zadeh[1]). Let X be any set. A fuzzy set A in X is a function with domain X and values in* $[0,1]$.

Definition 2.2. *(Schweizer and Sklar[8]). A binary operation* $*: [0,1] \times [0,1] \to [0,1]$ *is called a continuous triangular norm (in short, continuous t-norm) if it satisfies the following conditions:*

(TN-1) $*$ *is commutative and associative.*

(TN-2) $*$ *is continuous.*

(TN-3) $a * 1 = a$ *for every* $a \in [0,1]$.

(TN-4) $a * b \le c * d$ *whenever* $a \le c$ *and* $b \le d$ *for all* $a,b,c,d \in [0,1]$.

Definition 2.3. *(George and Veeramani[3]). A fuzzy metric space is an ordered triple* $(X,M,*)$ *such that X is a nonempty set,* $*$ *defined a continuous t-norm and M is a fuzzy set on* $X \times X \times (0,\infty)$, *satisfying the following conditions, for all* $x,y,z \in X, s,t > 0$:

(FM-1) $M(x,y,t) > 0$,

(FM-2) $M(x, y, t) = 1$ *iff* $x = y$,

(FM-3) $M(x, y, t) = M(y,x,t)$,

(FM-4) $(M(x,y,t) * M(y,z,s)) \leq M(x,z,t+s)$,
(FM-5) $M(x,y,*) : (0,\infty) \to (0,1]$ *is left continuous.*
Definition 2.4. *(M. Grabiec [4]). Let* $(X,M,*)$ *is a fuzzy metric space then a sequence* $\{x_n\} \in X$ *is said to be convergent to a point* $x \in X$ *if* $\lim_{n\to\infty} M(x_n,x,t) = 1$ *for all* $t > 0$.
Definition 2.5. *(M. Grabiec [4]). Let* $(X,M,*)$ *is a fuzzy metric space then a sequence* $\{x_n\} \in X$ *is called to a Cauchy sequence if* $\lim_{n\to\infty} M(x_{n+p},x_n,t) = 1$ *for all* $t > 0$ *and* $p > 0$.
Definition 2.6. *(M.Grabiec [4]). Let* $(X,M,*)$ *is a fuzzy metric space then a fuzzy metric space in which every Cauchy sequence is convergent is called complete.*
Lemma 2.7. *(M.Grabiec [4]). For all,* $x,y \in X, M(x,y,.)$ *is non-decreasing.*
Lemma 2.8. *(S. N. Mishra et al. [11]). If there exist* $k \in (0,1)$ *such that* $M(x,y,kt) \geq M(x,y,t)$ *for all* $x,y \in X$ *and* $t \in (0,\infty)$, *then* $x = y$.
The objective of this work is to prove a fixed point theorem in fuzzy metric spaces using rational inequality. We furnish an example to validate our result. Some applications are also given.

III. MAIN RESULT

In this section, we establish fixed point theorem in fuzzy metric spaces.
Theorem 3.1. *Let* $(X,M,*)$ *be a complete fuzzy metric space and* $f : X \to X$ *be a mapping satisfying.*

$$M(x,y,t) = 1 \tag{3.1}$$

and

$$M(fx,fy,kt) \geq \lambda(x,y,t). \tag{3.2}$$

Where

$$\begin{aligned}\lambda(x,y,t) &= \alpha(min\{\frac{M(y,fy,t)[1+M(x,fx,t)]}{[1+M(x,y,t)]}, M(x,y,t)\}) \\ &+ (1-\alpha)(min\{\frac{M(x,fx,t)[1+M(y,fy,t)]}{[1+M(x,fx,t)]}, \\ &M(x,fx,t), M(y,fy,t)\}),\end{aligned} \tag{3.3}$$

for all $x,y \in X, \alpha \in [0,1]$ *and* $k \in (0,1)$, *then* f *has a unique fixed point.*
Proof: Let $x \in X$ be any arbitrary point in X. Let us consider a sequence $\{x_n\}$ in X such that $fx_n = x_{n+1}$ for all $n \in \mathbb{N}$. Firstly we will show that $\{x_n\}$ is a Cauchy sequence. Let $x = x_{n-1}$ and $y = x_n$ put in (3.2), we get

$$M(x_{n-1},x_n,kt) = M(fx_{n-1},fx_n,kt) \geq \lambda(x_{n-1},x_n,t). \tag{3.4}$$

Since

$$\begin{aligned}\lambda(x_{n-1},x_n,t) &= \alpha(min\{\frac{M(x_n,fx_n,t)[1+M(x_{n-1},fx_{n-1},t)]}{[1+M(x_{n-1},x_n,t)]}, M(x_{n-1},x_n,t)\}) \\ &+ (1-\alpha)(1-\alpha)(min\{\frac{M(x_{n-1},fx_{n-1},t)[1+M(x_n,fx_n,t)]}{[1+M(x_{n-1},fx_{n-1},t)]}, \\ &M(x_{n-1},fx_{n-1},t), M(x_n,x_{n+1},t)\}) \\ &= \alpha(min\{M(x_n,x_{n+1},t), M(x_{n-1},x_n,t)\}) \\ &+ (1-\alpha)(min\{\frac{M(x_{n-1},x_n,t)[1+M(x_n,x_{n+1},t)]}{[1+M(x_{n-1},x_n,t)]}, \\ &M(x_{n-1},x_n,t), M(x_n,x_{n+1},t)\}).\end{aligned} \tag{3.5}$$

We shall consider the following cases:
Case(i): If

$$\begin{aligned}&min\{\frac{M(x_{n-1},x_n,t)[1+M(x_n,x_{n+1},t)]}{[1+M(x_{n-1},x_n,t)]}, M(x_{n-1},x_n,t) \\ &M(x_n,x_{n+1},t)\} = \frac{M(x_{n-1},x_n,t)[1+M(x_n,x_{n+1},t)]}{[1+M(x_{n-1},x_n,t)]},\end{aligned}$$

then

$$min\{M(x_n,x_{n+1},t), M(x_{n-1},x_n,t)\} \geq \frac{M(x_{n-1},x_n,t)[1+M(x_n,x_{n+1},t)]}{[1+M(x_{n-1},x_n,t)]}. \tag{3.6}$$

We shall embellish that this case is not possible. For this reason, we consider the following sub-cases:
Case$(i)_a$. Suppose that, $min\{M(x_n,x_{n+1},t), M(x_{n-1},x_n,t)\} = M(x_n,x_{n+1},t)$, that is,

$M(x_{n-1},x_n,t) \geq M(x_n,x_{n+1},t)$, (3.7)
furthermore, from (3.6), we observe that

$$M(x_n,x_{n+1},t) \geq \frac{M(x_{n-1},x_n,t)[1+M(x_n,x_{n+1},t)]}{[1+M(x_{n-1},x_n,t)]}, \tag{3.8}$$

by a simple calculation, we derive from the above that $M(x_n,x_{n+1},t) \geq M(x_{n-1},x_n,t)$. Which contradicts the assumption (3.7).
Case$(i)_b$. Assume that, $min\{M(x_n,x_{n+1},t),M(x_{n-1},x_n,t)\} = M(x_n,x_{n+1},t)$, that is,
$M(x_{n-1},x_n,t) \leq M(x_n,x_{n+1},t)$, (3.9)
then again from (3.6), we have

$$M(x_{n-1},x_n,t) \geq \frac{M(x_{n-1},x_n,t)[1+M(x_n,x_{n+1},t)]}{[1+M(x_{n-1},x_n,t)]}, \tag{3.10}$$

by a simple calculation, we derive from the above that $M(x_n,x_{n+1},t) \leq M(x_{n-1},x_n,t)$. Which contradicts the assumption (3.9). Hence case (i) does not occur.
Case(ii). If $min\{M(x_n,x_{n+1},t),M(x_{n-1},x_n,t)\} = M(x_n,x_{n+1},t)$, from (3.5)
$\lambda(x_{n-1},x_n,t) = \alpha M(x_n,x_{n+1},t) + (1-\alpha)(M(x_n,x_{n+1},t) = M(x_n,x_{n+1},t)$, thus we have

$$M(x_n,x_{n+1},kt) \geq M(x_n,x_{n+1},t).$$

Since by Lemma 2.8, this implies that $x_n = x_{n+1}$, therefore sequence $\{x_n\}$ is a Cauchy sequence.
Case(iii). If $min\{M(x_n,x_{n+1},t),M(x_{n-1},x_n,t)\} = M(x_{n-1},x_n,t)$, from (3.5),
$\lambda(x_{n-1},x_n,t) = \alpha M(x_{n-1},x_n,t)+(1-\alpha)M(x_{n-1},x_n,t) = M(x_{n-1},x_n,t)$. Now by simple induction, for all n, and $t>0$,

$$M(x_n,x_{n+1},kt) \geq M(x,x_1,\frac{t}{k^{n-1}}), \tag{3.11}$$

now for any positive integer s we have

$$\begin{aligned}M(x_n,x_{n+s},t) &\geq M(x_n,x_{n+1},\frac{t}{s}) * M(x_{n+1},x_{n+2},\frac{t}{s}) * \cdots * M(x_{n+p-1},x_{n+p},\frac{t}{s})\\ &\geq M(x,x_1,\frac{t}{sk^n}) * M(x,x_1,\frac{t}{sk^n}) * \cdots * M(x,x_1,\frac{t}{sk^n}),\end{aligned}$$

taking limit $n \to \infty$ and using (3.1), we get

$$lim_{n\to\infty}M(x_n,x_{n+s},t) = 1. \tag{3.12}$$

This implies that sequence $\{x_n\}$ is a Cauchy sequence. Since $(X,M,*)$ is a complete fuzzy metric space. Then sequence $\{x_n\}$ is a convergent in it. Let $\{x_n\}$ convergence to
$u \in (X,M,*)$.

$$\begin{aligned}M(u,fu,t) &\geq M(u,x_{n+1},t) * M(x_{n+1},fu,t)\\ &\geq M(fx_n,fu,t) * M(u,fx_{n+1},t)\\ &\geq \lambda(x_n,u,\frac{t}{2k}) * M(u,x_{n+1},t),\end{aligned} \tag{3.13}$$

now

$$\begin{aligned}\lambda(x_n,u,\frac{t}{2k}) = \alpha(min\{&\frac{M(u,fu,\frac{t}{2k})[1+M(x_n,fx_n,\frac{t}{2k})]}{[1+M(x_n,u,\frac{t}{2k})]}, M(x_n,u,\frac{t}{2k})\})\\ &+(1-\alpha)(min\{\frac{M(x_n,fx_n,\frac{t}{2k})[1+M(u,fu,\frac{t}{2k})]}{[1+M(x_n,fx_n,\frac{t}{2k})]}\\ &M(x_n,fx_n,\frac{t}{2k}), M(u,fu,\frac{t}{2k})\}),\end{aligned} \tag{3.14}$$

on making $n \to \infty$ and by (3.1) and (3.14),

$$\lambda(u,u,\frac{t}{2k}) = \alpha(min\{\frac{M(u,fu,\frac{t}{2k})[1+M(x_n,fx_n,\frac{t}{2k})]}{[1+M(x_n,u,\frac{t}{2k})]}, M(x_n,u,\frac{t}{2k}\})$$
$$+ (1-\alpha)(min\{\frac{M(x_n,fx_n,\frac{t}{2k})[1+M(u,fu,\frac{t}{2k})]}{[1+M(x_n,fx_n,\frac{t}{2k})]},$$
$$M(x_n,fx_n,\frac{t}{2k}), M(u,fu,\frac{t}{2k})\})$$
$$= \alpha(min\{M(u,fu,\frac{t}{2k}),1)\}) + (1-\alpha)(min\{M(u,fu,\frac{t}{2k}),1)\})$$
$$= (\alpha \ +(1-\alpha))(min\{M(u,fu,\frac{t}{2k}),1\})$$
$$= min\{M(u,fu,\frac{t}{2k}),1\}. \tag{3.15}$$

If $M(u,fu,\frac{t}{2k}) \geq 1$, then $\lambda(u,u,\frac{t}{2k}) = 1$. Therefore, by equation (3.15) and using definition 2.3, then we get *u* is a fixed point of*f*.
If $M(u,fu,\frac{t}{2k}) \leq 1$,then $\lambda(u,u,\frac{t}{2k}) = M(u,fu,\frac{t}{2k})$. Hence from equation (3.15), we have

$$M(u,fu,t) \geq M(u,fu,\frac{t}{2k}) * M(u,x_{n+1},t), \tag{3.16}$$

as $n \to \infty$ in (3.16) and using (3.1) and Lemma 2.8, we get $fu = u$.
For uniqueness, Let $u,v \in X$ are two fixed point of f, then $fu = u$ and $fv = v$. Consider

$$1 \geq M(v,u,t) = M(fu,fv,t) \geq \lambda(v,u,\frac{t}{k}), \tag{3.16}$$

where

$$\lambda(v,u,\frac{t}{k}) = \alpha(min\{\frac{M(u,fu,\frac{t}{k})[1+M(v,fv,\frac{t}{k})}{1+M(v,u,\frac{t}{k}}, M(v,u,\frac{t}{k})\})$$
$$+ (1-\alpha)(min\{\frac{M(v,fv,\frac{t}{k})[1+M(u,fu,\frac{t}{k})}{1+M(v,fv,\frac{t}{k}}, M(v,fv,\frac{t}{k}), M(u,fu,\frac{t}{k})\})$$
$$= \alpha(min\{\frac{M(u,u,\frac{t}{k})[1+M(v,v,\frac{t}{k})}{1+M(v,u,\frac{t}{k})}, M(v,u,\frac{t}{k})\})$$
$$+ (1-\alpha)(min\{\frac{M(v,v,\frac{t}{k})[1+M(u,u,\frac{t}{k})}{1+M(v,v,\frac{t}{k})}, M(v,v,\frac{t}{k}), M(u,u,\frac{t}{k})\})$$
$$= \alpha(min\{\frac{1(1+1)}{1+M(v,u,\frac{t}{k})}, M(v,u,\frac{t}{k})\}) + (1-\alpha)(min\{\frac{1[1+1]}{1+1},1,1\})$$
$$= \alpha(min\{\frac{2}{1+M(v,u,\frac{t}{k})}, M(v,u,\frac{t}{k})\}) + (1-\alpha)(min\{\frac{2}{2},1,1\})$$
$$= \alpha(min\{\frac{2}{1+1},1\}) + (1-\alpha)(min\{\frac{2}{2},1,1\})$$
$$= \alpha(min\{1,1\}) + (1-\alpha)(min\{1,1,1\})$$
$$= \alpha(1) + (1-\alpha)(1)$$
$$= 1. \tag{3.17}$$

Therefore by using (3.17), $1 \geq M(v,u,t) \geq 1$, this implies that $M(v,u,t) = 1$, we have $v = u$. There exists unique fixed point of*f*. This completes the proof of the theorem.
Remark 3.2. *For* $\alpha = 1$, *we get theorem 4.1 of Gupta et al. [13].*
Remark 3.3. *For* $\alpha = 0$, *we get the following corollary.*
Corollary 3.4. *Let* $(X,M,*)$ *be a complete fuzzy metric space and* $f: X \to X$ *be a mapping satisfying* $M(x,y,t) = 1$ *and* $M(fx,fy,kt) \geq \lambda(x,y,t)$,
where

$$\lambda(x,y,t) = min\{\frac{M(x,fx,t)[1+M(y,fy,t)]}{[1+M(x,fx,t)]}, M(x,fx,t), M(y,fy,t)\}$$
,

for all x,y ∈ X,α ∈ [0,1],k ∈ (0,1). Then f has a unique fixed point.

Example 3.5. *Let (X,M,∗) be a complete fuzzy metric space, let* $X = \{1, \frac{1}{2}, \frac{1}{3}\}$. M *is a fuzzy set on X × X × (0,∞) defined as* $M(x,y,t) = \frac{min\{x,y\}+1}{max\{x,y\}+1}$ *and* $f(1) = f(\frac{1}{2}) = f(\frac{1}{3}) = \frac{1}{2}$*, for all x,y ∈ X,α ∈ [0,1],k ∈ (0,1). Then f has a unique fixed point.*

Solution. Let $k \in (0,1)$ and $t \in (0,\infty)$, given $M(x,y,t) = \frac{min\{x,y\}+1}{max\{x,y\}+1} = \frac{min\{x,y\}+\frac{t}{t}}{max\{x,y\}+\frac{t}{t}}$.

$$M(1,\frac{1}{2},t) = M(\frac{1}{2},1,t) = \frac{3}{4},$$
$$M(1,\frac{1}{3},t) = M(\frac{1}{3},1,t) = \frac{2}{3},$$
$$M(\frac{1}{2},\frac{1}{3},t) = M(\frac{1}{3},\frac{1}{2},t) = \frac{8}{9},$$

$$M(1,1,t) = M(\frac{1}{2},\frac{1}{2},t) = M(\frac{1}{3},\frac{1}{3},t)$$
= 1,

and if $t = 1, s = 1$, then we observe that

1.$M(1,\frac{1}{2},t) * M(\frac{1}{2},\frac{1}{3},s) \leq M(1,\frac{1}{3},t+s)$ implies $\frac{3}{4} * \frac{8}{9} \leq \frac{7}{9}$,

2.$M(\frac{1}{2},\frac{1}{3},t) * M(\frac{1}{3},1,s) \leq M(\frac{1}{2},1,t+s)$ implies $\frac{8}{9} * \frac{2}{3} \leq \frac{5}{6}$,

3.$M(\frac{1}{3},1,t) * M(1,\frac{1}{2},s) \leq M(\frac{1}{3},\frac{1}{2},t+s)$ implies $\frac{2}{3} * \frac{3}{4} \leq \frac{14}{15}$.

We will consider the following cases:

(i) For $x = 1, y = \frac{1}{2}$. We have

$$\begin{aligned} M(f1,f\frac{1}{2},kt) &\geq \alpha(min\{\frac{M(\frac{1}{2},f\frac{1}{2},t)[1+M(1,f1,t)]}{[1+M(1,\frac{1}{2},t)]}, M(1,\frac{1}{2},t)\}) \\ &+ (1-\alpha)(min\{\frac{M(1,f1,t)[1+M(\frac{1}{2},f\frac{1}{2},t)]}{[1+M(1,f1,t)]}, M(1,f1,t), M(\frac{1}{2},f\frac{1}{2},t)\}) \\ &\geq \alpha(min\{1,\frac{3}{4}\}) + (1-\alpha)(min\{\frac{6}{7},\frac{3}{4},1\}) \\ &\geq 1, \forall\alpha \in [0,1]. \end{aligned}$$

(ii) For $x = 1, y = \frac{1}{3}$. We have

$$\begin{aligned} M(f1,f\frac{1}{3},kt) &\geq \alpha(min\{\frac{M(\frac{1}{3},f\frac{1}{3},t)[1+M(1,f1,t)]}{[1+M(1,\frac{1}{3},t)]}, M(1,\frac{1}{3},t)\}) \\ &+ (1-\alpha)(min\{\frac{M(1,f1,t)[1+M(\frac{1}{3},f\frac{1}{3},t)]}{[1+M(1,f1,t)]}, M(1,f1,t), M(\frac{1}{3},f\frac{1}{3},t)\}) \\ &\geq \alpha(min\{\frac{168}{180},\frac{2}{3}\}) + (1-\alpha)(min\{\frac{204}{252},\frac{3}{4},\frac{8}{9}\}) \\ &\geq \alpha\frac{2}{3} + (1-\alpha)\frac{3}{4}, \forall\alpha \in [0,1]. \end{aligned}$$

(iii) For $x = \frac{1}{2}, y = \frac{1}{3}$. We have

$$M(f\frac{1}{2},f\frac{1}{3},kt)\geq \alpha(min\{\frac{M(\frac{1}{3},f\frac{1}{3},t)[1+M(\frac{1}{2},f\frac{1}{2},t)]}{[1+M(\frac{1}{2},\frac{1}{3},t)]},M(\frac{1}{2},\frac{1}{3},t)\})$$
$$+(1-\alpha)(min\{\frac{M(\frac{1}{2},f\frac{1}{2},t)[1+M(\frac{1}{3},f\frac{1}{3},t)]}{[1+M(\frac{1}{2},f\frac{1}{2},t)]},M(\frac{1}{2},f\frac{1}{2},t),M(\frac{1}{3},f\frac{1}{3},t)\})$$
$$\geq \alpha(\frac{16}{17},\frac{8}{9})+(1-\alpha)(\frac{17}{18},1,\frac{8}{9})$$
$$\geq 1,\forall\alpha\in[0,1].$$

Thus, all conditions of theorem 3.1 are satisfied. Hence *f* has a unique fixed point.

IV. APPLICATION

In this section, we give applications related to our result. Let us define ψ : $[0,\infty)$ → $[0,\infty)$ as $\psi(t)=\int_0^t\varphi(t)dt$ for all $t>0$, be a non-decreasing and continuous function, for each $\epsilon>0,\varphi(\epsilon)>0$. Which shows that $\phi(t)=0$ iff $t=0$.

Theorem 4.1. *Let* $(X,M,*)$ *be a complete fuzzy metric space and* $f{:}X\to X$ *be a mapping satisfying* $M(x,y,t)=1$ *and*

$$\int_0^{M(fx,fy,t)}\varphi(t)dt\geq\int_0^{\lambda(x,y,t)}\varphi(t)dt$$

where

$$\lambda(x,y,t)=\alpha(min\{\frac{M(y,fy,t)[1+M(x,fx,t)]}{[1+M(x,y,t)]},M(x,y,t)\})$$
$$+(1-\alpha)(min\{\frac{M(x,fx,t)[1+M(y,fy,t)]}{[1+M(x,fx,t)]},M(x,fx,t),M(y,fy,t)\}).\quad(4.1)$$

for all $x,y\in X,\alpha\in[0,1],k\in(0,1)$ *and* $\phi(t)\in\psi$. *Then f has a unique fixed point.*

Proof. By taking $\phi(t)=1$ and applying Theorem 3.1, we obtain the result.

Remark 4.2. *For* $\alpha=1$, *we get the following corollary.*

Corollary 4.3. *Let* $(X,M,*)$ *be a complete fuzzy metric space and* $f: X\to X$ *be a mapping satisfying* $M(x,y,t)=1$ *and*

$$\int_0^{M(fx,fy,t)}\varphi(t)dt\geq\int_0^{\lambda(x,y,t)}\varphi(t)dt$$

where

$$\lambda(x,y,t)=min\{\frac{M(y,fy,t)[1+M(x,fx,t)]}{[1+M(x,y,t)]},M(x,y,t)\}$$

for all $x,y\in X,\alpha\in[0,1],k\in(0,1)$. *Then f has a unique fixed point.*

Remark 4.4. *For* $\alpha=0$, *we get the following corollary.*

Corollary 4.5. *Let* $(X,M,*)$ *be a complete fuzzy metric space and* $f{:}X\to X$ *be a mapping satisfying* $M(x,y,t)=1$ *and*

$$\int_0^{M(fx,fy,t)}\varphi(t)dt\geq\int_0^{\lambda(x,y,t)}\varphi(t)dt$$

where

$$\lambda(x,y,t)=min\{\frac{M(x,fx,t)[1+M(y,fy,t)]}{[1+M(x,fx,t)]},M(x,fx,t),M(y,fy,t)\},$$

for all $x,y\in X,\alpha\in[0,1],k\in(0,1)$. *Then f has a unique fixed point.*

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