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Analysis of Fractional Goursat Problem by using Differential Transform Method.

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Abstract
In this study, we convert the classical Goursat problem into a fractional form in order to study its solution. The fractional Goursat problem can be solved in homogeneous, inhomogeneous, and linear cases applying the DTM (differential transform method) in its fractional form. To confirm the accuracy of our results, the acquired results converge with the exact solution and the work published in the literature.
Keywords*: Fractional Goursat problem, Differential Transform Method, Goursat problem, Analytical approximation.*

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I. Introduction

The hyperbolic partial differential equation known as the Goursat problem occurs in a number of science and engineering fields. Numerous numerical techniques, such as the Homotopy Analysis technique (HAM), the Runge-Kutta technique, and the finite difference method, the Adomian Decomposition Method (ADM), and with integral conditions, have been used to analyze the Goursat problem.
A classical formulation of the standard form of a homogeneous linear Goursat problem is provided.

$u_{xt}(x,t)$+A(x,t) u_x(x,t)+B(x,t) u_t(x,t)+C(x,t) u(x,t) = 0 (1)

$$u(x,0) = \varphi(x), \; u(0,t) = \psi(t), \tag{2}$$

$$\varphi(0) = \psi(0) = u(0,0). \tag{3}$$

This is called the problem of the Gorsat, after the French mathematician Edouard Goursat, who first analyzed the linear form of the problem in [1]."

In recent years, differential equation have seen advanced significant with applications spanning physics, chemistry, industrial mathematics, control theory, and fluid dynamics.. Fractional calculus is the extension of classical concept of differentiation and integration to arbitrary (non-integer) orders. Over the past two or three decades, the topic has become increasingly important [2–4]. Fractional derivatives have been used to mathematically characterize numerous phenomena in engineering and other areas [5–8]. When modeling real-world issues, these representations have produced positive outcomes [9–11]. Researchers have recently focused on studying fractional order partial differential equation solutions [12–14].

The idea of fractional calculus is nearly as old as conventional classical itself. The history of fractional calculus dates back to over three centuries, and the original question that gave rise to the name fractional calculus was: what does $\frac{d^n f}{dx^n}$ mean if n = $\frac{1}{2}$? This question first appeared in a correspondence between Gottfried Wilhelm Leibniz and the Marquis De L Hopital in 1695, when Leibniz was the first to mention the possibility of derivatives of arbitrary, non-integer orders [15].

Researchers have discovered that the differential transform method (DTM) is one of the best techniques. This approach stands out because it gives us analytical approximations, which are frequently precise solutions, in the form of a fast converging power series with tastefully constructed terms ([13, 16] and refer to the references therein).

Additionally, DTM minimizes the computation size and solves the equations directly and simply without the need of restrictive conditions, discretization, linearization, on perturbation techniques, Adomian's polynomial, or any additional r transformation [16–17].

The Differential Transform Method (DTM) has also been successfully used over recent years to solve various types of differential equations, such as fractional telegraph, cable, and Swift Hohenberg equations [18,19, 20]. It has been applied to linear, nonlinear, and system-type problems, and convergence results are established; additional work appears in [13, 16, 21, 22].
Here, we introduce the essential definitions of fractional calculus required for the formulation of the problem and implementation of the method.

II. Preliminary and basic definition

The generalization of the integral and derivative operators is important to fractional calculus. Riemann-Liouville, Caputo, and Grünwald-Letnikov fractional derivatives are the three most often used definitions in the literature. Every one of these definitions has unique benefits and works well for various kinds of problems.

The Riemann-Liouville definition of a fractional derivative of order α is expressed as [23]:

$$D_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_0^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau, \quad n-1 < \alpha < n, \qquad n \in N. \tag{4}$$

In this expression, α denotes the fractional order, n represents the smallest integer greater than α, $\Gamma(.)$ represents the Gamma function, and $\frac{d^n}{dt^n}$ represent the classical integer-order derivative. The Riemann-Liouville formulation is particularly mainly valuable in theoretical analysis because of its strong mathematical foundation.

The Caputo fractional derivative of order α is defined as follows [24]:

$$D^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_{\alpha}^{t} \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \tag{5}$$

Here $f^{(n)}(\tau)$ is nth-order classical derivative of f(t). This formulation is widely used in applied science because it incorporates of initial condition in a manner consistent with standard differential equation.

Moreover, the Grunwald-Letnikov definition provides a discrete form of fractional derivative and is given by [25]:

$$D^{\alpha} f(t) = \lim_{h \to 0} \frac{1}{h^{\alpha}} \sum_0^{\infty} (-1)^k \binom{\alpha}{k} f(t-kh) \tag{6}$$

Here h represents the step size and $\binom{\alpha}{k}$ *denotes the* generalized binomial coefficient, which is expressed as

$$\binom{\alpha}{k} = \frac{\Gamma(\alpha+1)}{\Gamma(k+1)\,\Gamma(\alpha-k+1)}. \tag{7}$$

This formulation is particular valuable in Numerical simulation and computational application.

The gamma function denoted Γ(z) play significant role in fractional calculus. It is defined as:

$$\Gamma(w) = \int_0^{\infty} t^{w-1} e^{-t} dt \tag{8}$$

$$\Gamma(w+1) = w\Gamma(w), \qquad w > 0. \tag{9}$$

Moreover, the Gamma function generalizes the classical factorial function, since for every n∈N,

$$\Gamma(n) = (n-1)!. \tag{10}$$

III. Definition of Differential Transform.

The basic definition of deferential transform method (DTM) is presented below.

Definition

Suppose u(x ,t) and v(x ,t) are analytic function that are continuously differentiable with respect to the variable t. Then the differential transform is defined as:

$$U_k(x) = \frac{1}{\Gamma(k\alpha+1)} \left[\frac{\partial^k}{\partial y^k} u(x,t)\right]_{t=0}, \tag{11}$$

And

$$V_k(x) = \frac{1}{\Gamma(k\alpha+1)} \left[\frac{\partial^k}{\partial y^k} v(x,t)\right]_{t=0} \tag{12}$$

Where the order of the fractional derivative of time is denoted by α while U_k(x) and V_k(x) represent the fully derived t-dimensional spectrum function of the transformation. In this article, lower case u(x, t) and v(x, t) represent the original functions and upper case U_k (x) and V_K(x) represent the transformed functions.

Definition

The corresponding inverse differential transforms of U_k (x) and V_k (x) is given follow as:

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x) t^{k\alpha}, \tag{13}$$

and

$$v(x,t) = \sum_{k=0}^{\infty} V_k(x) t^{k\alpha}. \tag{14}$$

From (1) and (3) we get that

$$u(x,t) = \frac{1}{\Gamma(k\alpha+1)} \left[\frac{\partial^k}{\partial y^k} u(x,t)\right]_{t=0} t^{k\alpha} \tag{15}$$

and similarly from (2) and (4) we get

$$v(x,t) = \sum_{k=0}^{\infty} \frac{1}{\Gamma(k\alpha+1)} \left[\frac{\partial^k}{\partial y^k} v(x,t)\right]_{t=0} t^{k\alpha} \tag{16}$$

From the above definitions it is clear that the idea behind the differential transform approach arises from the power series expansion of a function. The basic mathematical operations involved in the reduced differential transform technique are summarized in Table 1.

Function	Transformation	Function	Transportation
z(x ,t)	$\frac{1}{\Gamma(k\alpha+1)}\left[\frac{\partial^k}{\partial y^k}u(x,t)\right]_{t=0}$	z(x,t)=u(x, t)v(x, t)	$Z_k(x)=\sum_{r=0}^{k} U_n V_{k-n}(x)$ $=\sum_{n=0}^{k} V_n U_{k-n}$
z(x,t)=u(x,t) ∓ v(x ,t)	$Z_k(x)=U_k(x)\pm V_k(x)$	$z(x,t)=\frac{\partial^n}{\partial y^n}u(x,t)$	$Z_k(x)$ $=(k+1)(k+2)\ldots(k+n)U_{k+n}(x)$
z(x,t)=$c.u(x,t)$	$Z_k(x)=c.U_k(x)$	$z(x,t)=\frac{\partial^{N\alpha}}{\partial t^{N\alpha}}u(x,t)$	$Z_k(x)$ $=\frac{\Gamma(k\alpha+N\alpha+1)}{\Gamma(k\alpha+1)}U_{k+N}(x)$
z(x ,t)=$x^m t^n$	$Z_k(x)=x^m\delta(k-n)$		
$z(x,t)=x^m y^n u(x,t)$	$Z_k(x)=x^m U_{k-n}(x)$		

IV. Implementation of Differential Transform Methods and Results

We applied DTM to the following linear homogeneous and non-homogeneous Goursat problems:

Problem 4.1 (Homogeneous Goursat Problem)

$$u_{xt}=u, \tag{17}$$

Subject to initial condition

$$u(x,0)=e^x, \qquad u(0,t)=e^t, \qquad u(0,0)=1. \tag{18}$$

$$D_x^\alpha D_t^\alpha u(x,t)=u(x,t), \quad 0<\alpha\leq 1 \tag{19}$$

To obtain the solution we apply differential transform method (DTM):

$$U_0:=e^x$$

$$U[k+1]:=\left(\frac{\Gamma(k\cdot\alpha+1)}{\Gamma(k\cdot\alpha+\alpha+1)}\right)\cdot int(U[k],x)=U_k \quad \text{for } k\geq 0 \tag{20}$$

Using the process described above, we get

$$U_1:=\frac{e^x}{\Gamma(1+\alpha)}$$

$$U_2:=\frac{e^x}{\Gamma(2\alpha+1)}$$

$$U_3:=\frac{e^x}{\Gamma(3\alpha+1)}$$

$$U_4:=\frac{e^x}{\Gamma(4\alpha+1)}$$

$$U_5:=\frac{e^x}{\Gamma(5\alpha+1)}, \quad \text{and so on....}$$

The inverse reduced transforms are finally applied, and we obtain

$$u(x,t) = \sum_{k=0}^{\infty} (U_k(x)) t^{k\alpha}. \tag{21}$$

$$u := e^{x} + \frac{e^{x} t^{\alpha}}{\Gamma(1+\alpha)} + \frac{e^{x} t^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{e^{x} t^{3\alpha}}{\Gamma(3\alpha+1)} + \dots$$

The graphical comparison of exact and 10th iteration of approximate solution is given as:

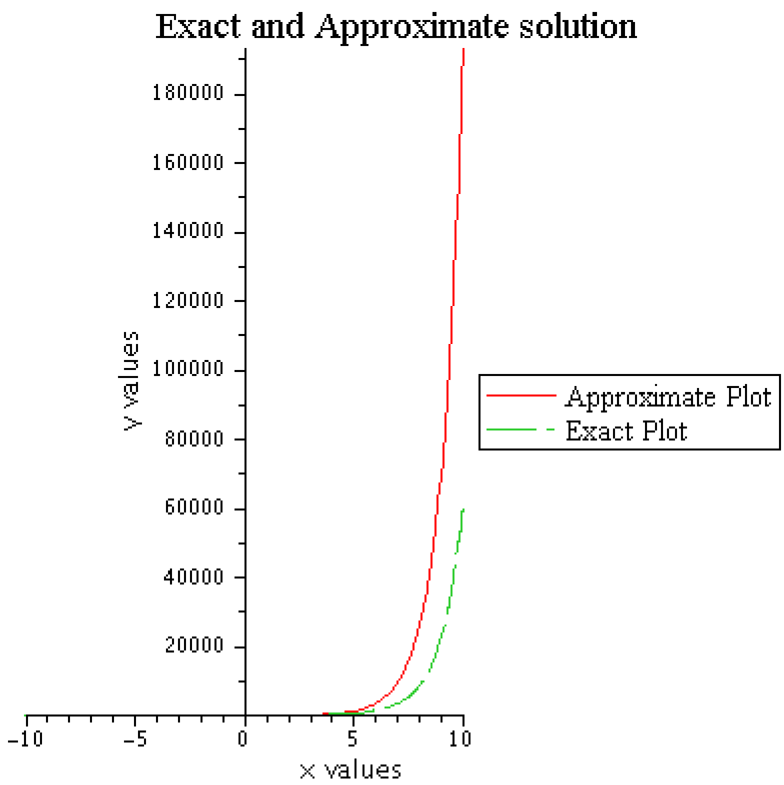

Fig4.1(a) Graphical of 10^{th} iteration by taking t=1 and α=0.25

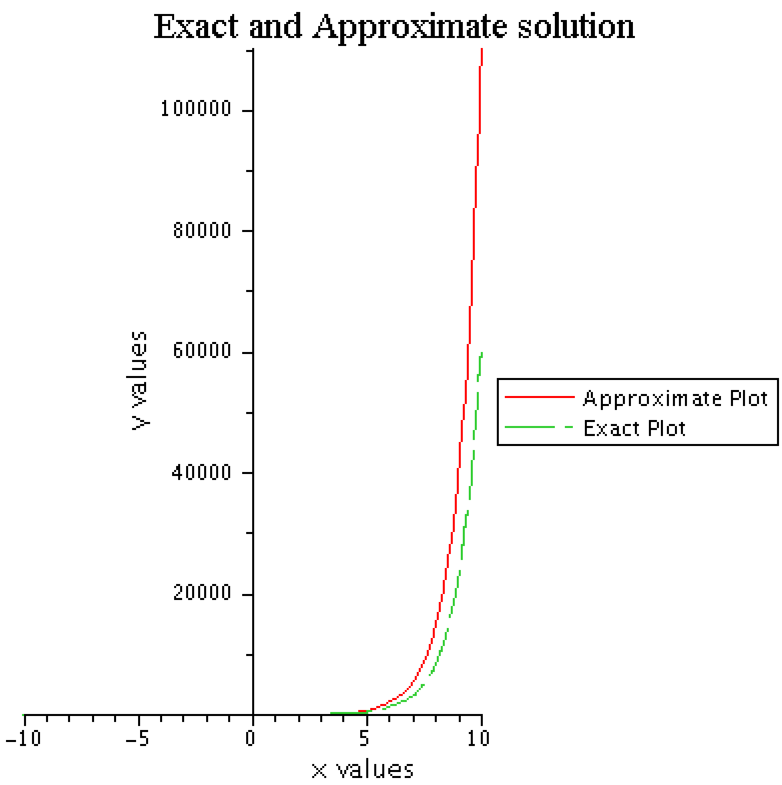

Fig4.2(b) Graphical of 10th iteration by taking t=1 and α=0.5

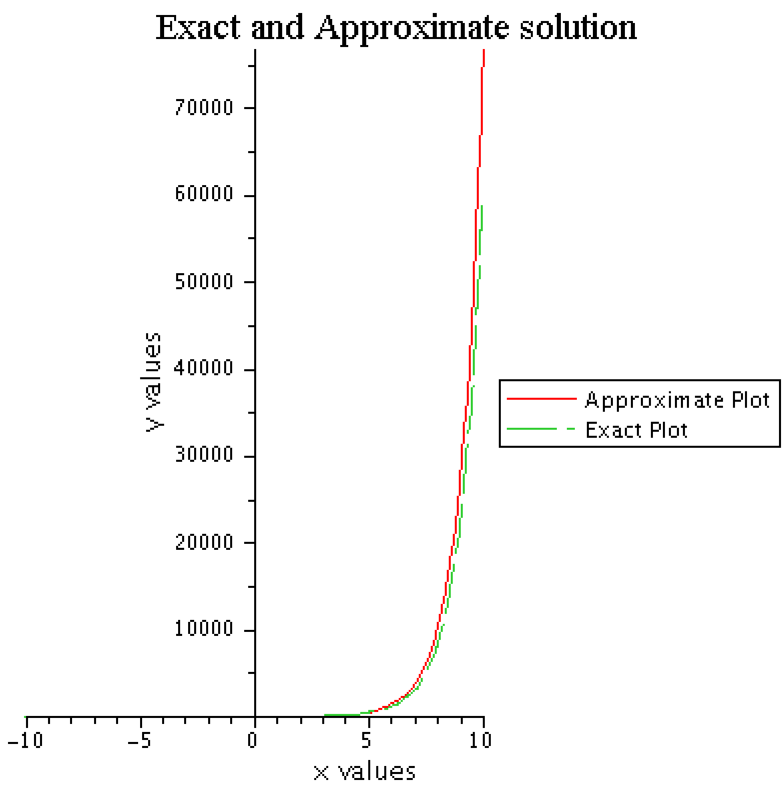

Fig4.3(c) Graphical of 10th iteration by taking t=1 and α=0.75

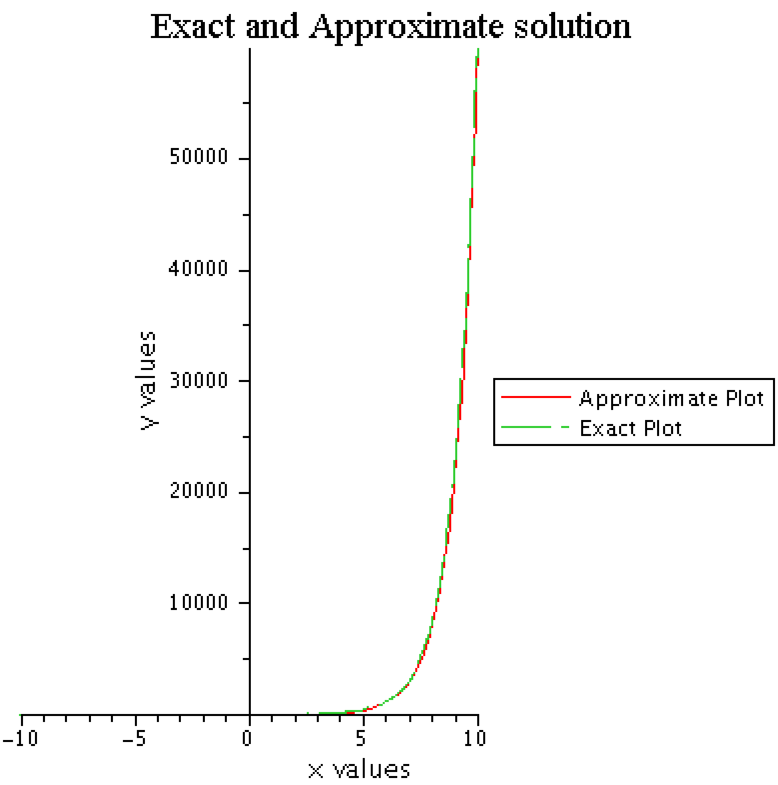

Fig4.3(c) Graphical of 10th iteration by taking t=1 and α=1

Problem 4.2 (Homogeneous Goursat Problem)

$$u_{xt} = -2u, \qquad (22)$$

With initial condition

$$U(x, 0)=e^{x}, \qquad u(0, t)= e^{t}, \qquad u(0, 0) = 1 \qquad (23)$$

Fractional Form of the Problem

$$D_x^{\alpha} D_t^{\alpha} f(x,t)= -2u(x ,t), \ \ 0 < \alpha \leq 1 \quad , \qquad (24)$$

To obtain the solution we apply differential transform method (DTM):

$$U_0 := e^x$$

$$U[k+1] := \left(\frac{-2 \cdot \Gamma(k \cdot \alpha + 1)}{\Gamma(k \cdot \alpha + \alpha + 1)} \right) \cdot int(U[k], x) = U_k \ \text{ for } k \geq 0 \qquad (25)$$

By applying the above procedure, we obtain

$$U_1 := -\frac{2 e^x}{\Gamma(\alpha + 1)}$$

$$U_2 := \frac{4 e^x}{\Gamma(2\alpha + 1)}$$

$$U_3 := -\frac{8\,e^x}{\Gamma(3\,\alpha+1)}$$

$$U_4 := \frac{16\,e^x}{\Gamma(4\,\alpha+1)}$$

$$U_5 := -\frac{32\,e^x}{\Gamma(5\,\alpha+1)}$$, and so on

The inverse reduced transforms are finally applied, and we obtain

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x) t^{k\alpha}, \quad (26)$$

$$u := \frac{262144 e^x\, t^{18\,\alpha}}{\Gamma(18\,\alpha+1)} - \frac{2048\,e^x\, t^{11\,\alpha}}{\Gamma(11\,\alpha+1)} - \frac{8388608 e^x\, t^{23\,\alpha}}{\Gamma(23\,\alpha+1)} -$$

The graphical comparison of exact and 50th iteration of approximate solution is given as:

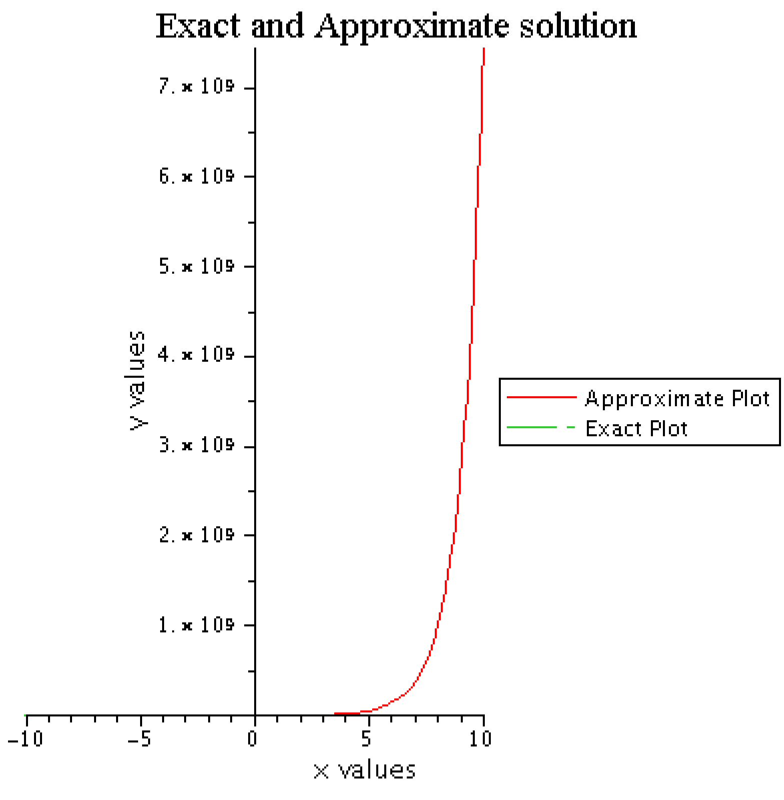

Fig4.5(a)Graphical of 50th iteration by taking t=1 and α=0.25

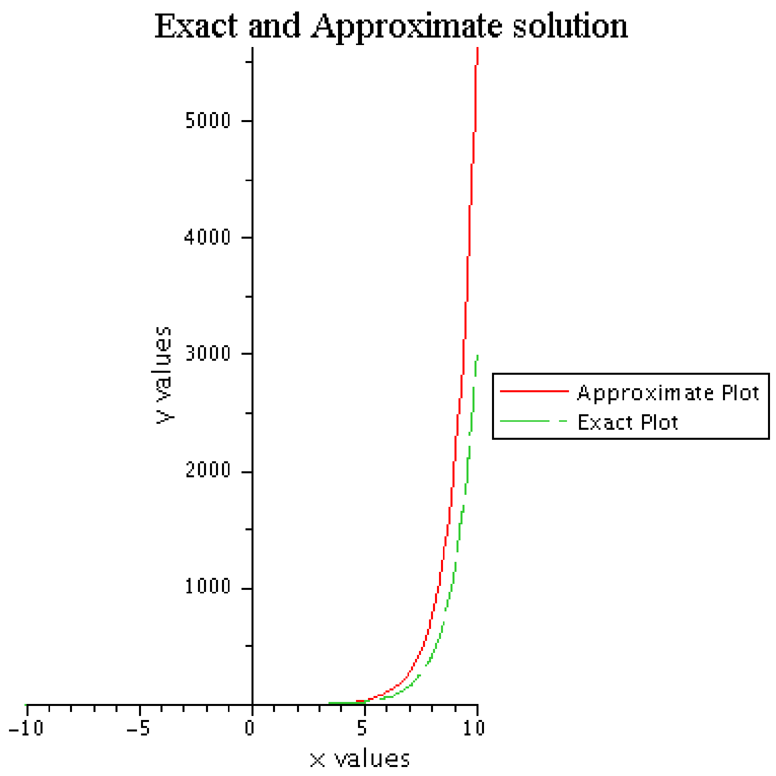

Fig4.6(b)Graphical of 50th iteration by taking t=1 and α=0.5

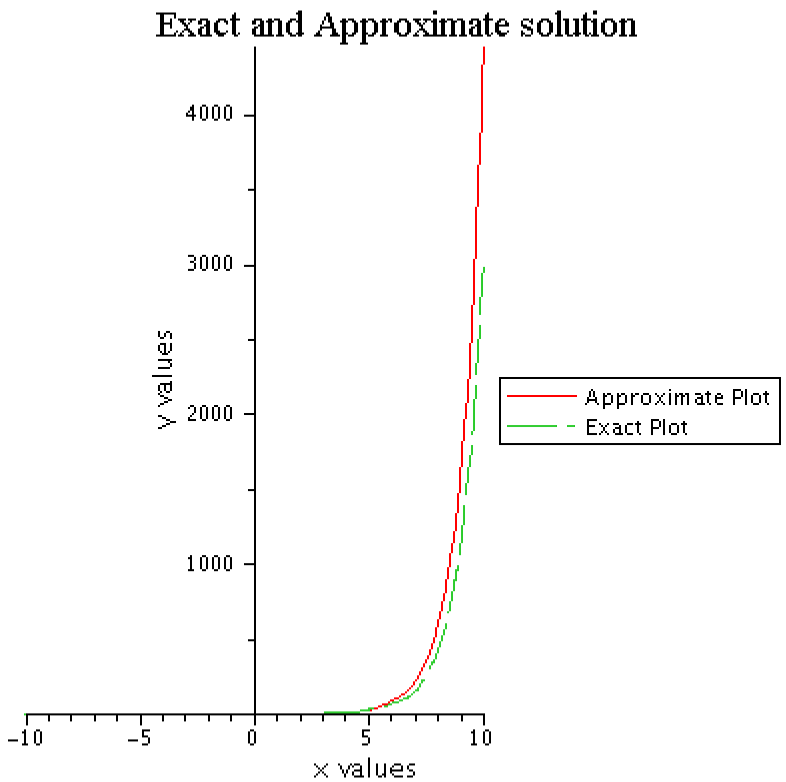

Fig4.7(c)Graphical of 50th iteration by taking t=1 and α=0.75

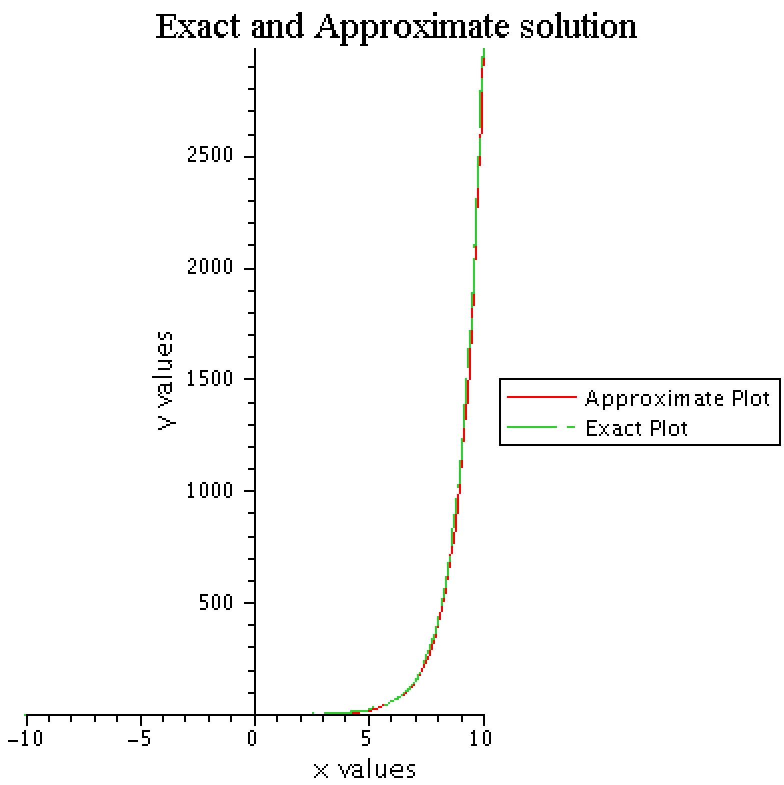

Fig4.8(a)Graphical of 50th iteration by taking t=1 and α=1

Problem 4.3 (Inhomogeneous Goursat Problem)

$$u_{xt} = u - t, \tag{27}$$

With initial condition

$$U(x, 0) = e^{x}, \quad u(0, t) = t + e^{t}, \quad u(0, 0) = 1 \tag{12}$$

Fractional Form of the Problem

$$D_x^{\alpha} D_t^{\alpha} f(x,t) = u(x,t) - \delta(k), \ \ 0 < \alpha \leq 1 \tag{28}$$

Where

$$\delta(k) := \begin{cases} 1 & k = 0 \\ 0 & k \neq 0 \end{cases}$$

To obtain the solution we apply differential transform method (DTM):

$$U_0 := t + e^{x} \tag{29}$$

$$U[k+1] := \left(\frac{\Gamma(k\cdot\alpha+1)}{\Gamma(k\cdot\alpha+\alpha+1)}\right)\cdot int\left(U[k] - \delta(k), x\right) = U_k \ \text{ for } k \geq 0 \tag{30}$$

By applying the above procedure, we obtain

$$U_1 := \frac{t\,x + e^x - x}{\Gamma(1+\alpha)}$$

$$U_2 := \frac{1}{2}\,\frac{x^2\,t - x^2 + 2\,e^x}{\Gamma(2\,\alpha + 1)}$$

$$U_3 := \frac{1}{6}\,\frac{x^3\,t - x^3 + 6\,e^x}{\Gamma(3\,\alpha + 1)}$$

$$U_4 := \frac{1}{24}\,\frac{x^4\,t - x^4 + 24\,e^x}{\Gamma(4\,\alpha + 1)}$$

$$U_5 := \frac{1}{120}\,\frac{120\,e^x + x^5\,t - x^5}{\Gamma(5\,\alpha + 1)}$$ and so on...

The inverse reduced transforms are finally applied, and we obtain

$$u(x,t) = \sum_{k=0}^{\infty} (U_k(x))t^{k\alpha}, \tag{31}$$

$$u := t + e^x + \frac{(t\,x + e^x - x)\,t^{\alpha}}{\Gamma(1+\alpha)} + \frac{1}{2}\,\frac{(x^2\,t - x^2 + 2\,e^x)\,t^{2\,\alpha}}{\Gamma(2\,\alpha + 1)} + \frac{1}{6}\,\frac{(x^3\,t - x^3 + 6\,e^x)\,t^{3\,\alpha}}{\Gamma(3\,\alpha + 1)} +$$

The graphical comparison of exact and 50th iteration of approximate solution is given as:

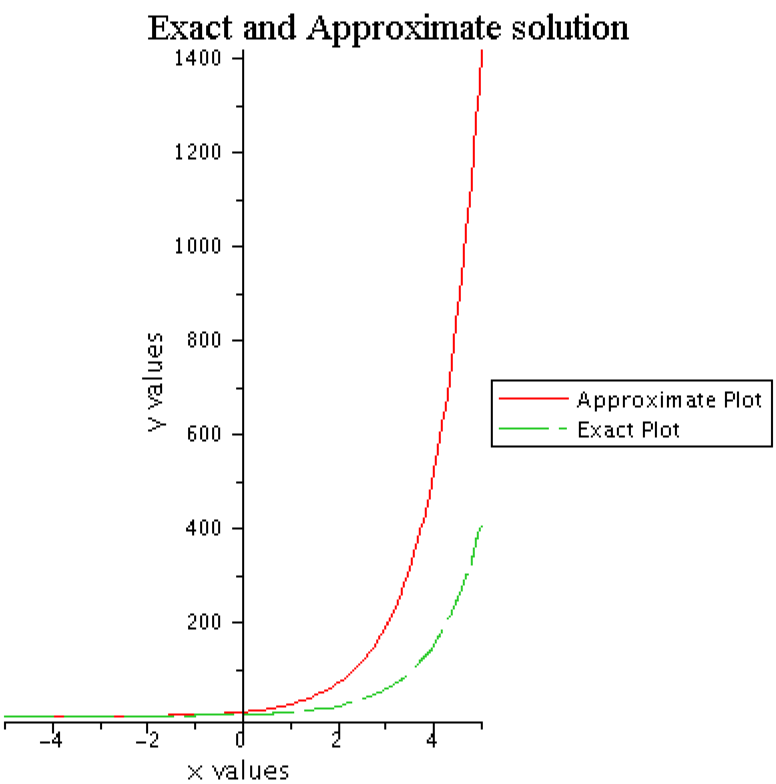

Fig4.9(a)Graphical of 50th iteration by taking t=1 and α=0.25

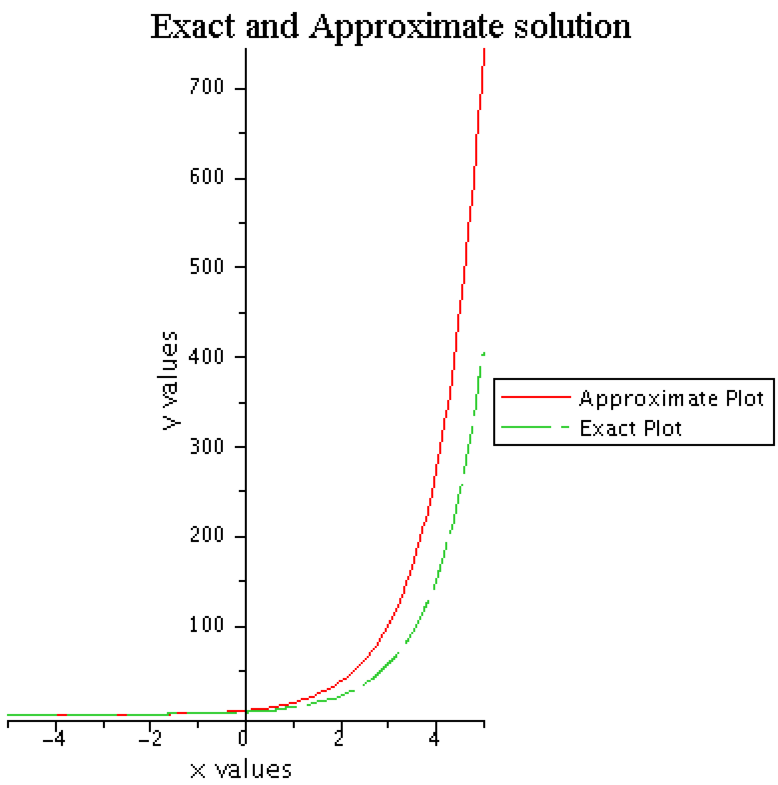

Fig4.10(b)Graphical of 50th iteration by taking t=1 and α=0.5

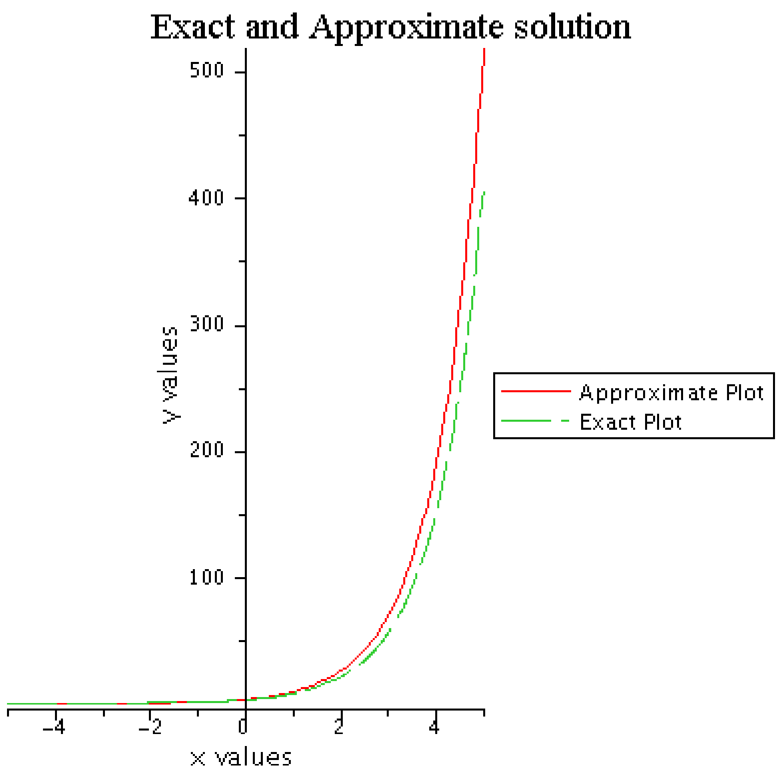

Fig4.11(c)Graphical of 50th iteration by taking t=1 and α=0.75

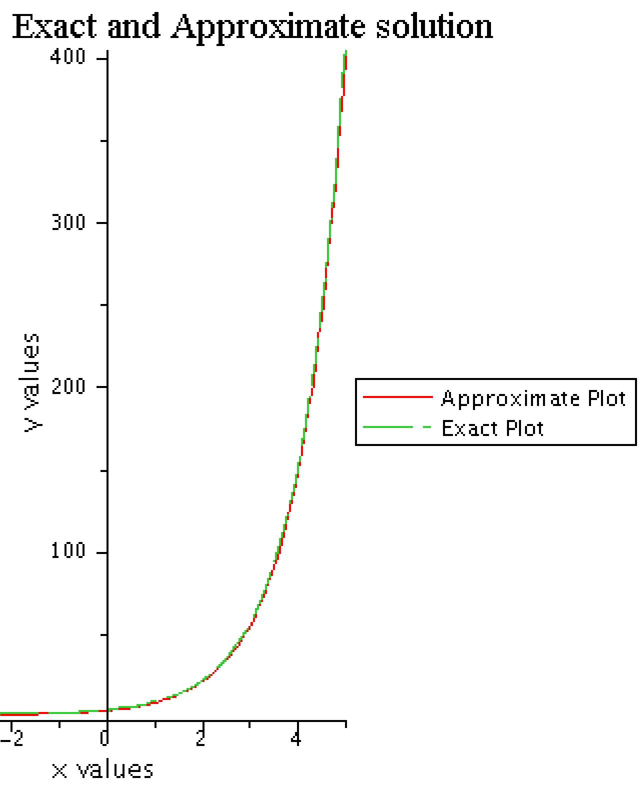

Fig4.12(d)Graphical of 50th iteration by taking t=1 and α=1

Problem 4.4 (Inhomogeneous Goursat Problem)

$$u_{xt} = u, + 4xt - x^2t^2, \tag{32}$$

With initial condition

$$U(x, 0) = e^x, \qquad u(0, t) = t + e^t, \; u(0, 0) = 1, \tag{33}$$

Fractional Form of the Problem

$$D_x^{\alpha} D_x^{t} f(x,t) = U_k + 4x\delta(k) - x^2\delta(k-1), \qquad 0 < \alpha \le 1 \tag{34}$$

Where

$$\delta(k) := \begin{cases} 1 & k = 0 \\ 0 & k \neq 0 \end{cases}$$

To obtain the solution we apply differential transform method (DTM):

$$U_0 := e^x \tag{35}$$

$$U[k+1] := \left(\frac{\Gamma(k\cdot\alpha+1)}{\Gamma(k\cdot\alpha+\alpha+1)}\right)\cdot int\big(U[k] + 4\,x\,\delta(k) - x \text{^}2\,\delta(k-1), x\big) = U\_k \;\; \textbf{for}\; k \ge 0 \tag{36}$$

By applying the above procedure we obtain

$$U_1 := \frac{e^x + 2x^2}{\Gamma(1+\alpha)}$$

$$U_2 := \frac{\Gamma(1+\alpha)\left(\frac{\frac{2}{3}x^3 + e^x}{\Gamma(1+\alpha)} - \frac{1}{3}x^3\right)}{\Gamma(2\alpha+1)}$$

$$U_3 := -\frac{1}{12}\,\frac{x^4\,\Gamma(1+\alpha) - 2x^4 - 12e^x}{\Gamma(3\alpha+1)}$$

$$U_4 := -\frac{1}{60}\,\frac{x^5\,\Gamma(1+\alpha) - 2x^5 - 60e^x}{\Gamma(4\alpha+1)}$$

$$U_5 := \frac{1}{360}\,\frac{-x^6\,\Gamma(1+\alpha) + 2x^6 + 360e^x}{\Gamma(5\alpha+1)}$$ and so on

The inverse reduced transforms are finally applied, and we obtain

$$u(x,t) = \sum_{k=0}^{\infty} (U_k(x))t^{k\alpha}, \tag{37}$$

$$u := e^x + \frac{1}{2520}\,\frac{1}{\Gamma(19\alpha+1)}\left(\left(\frac{1}{241359326208000}x^{20} - \frac{1}{482718652416000}x^{20}\,\Gamma(1+\alpha) + 2520e^x\right)t^{19\alpha}\right) + \frac{1}{60}\,\frac{\left(2x^5 - x^5\,\Gamma(1+\alpha) + 60e^x\right)t^{4\alpha}}{\Gamma(4\alpha+1)} +$$

The graphical comparison of exact and 50th iteration of approximate solution is given as:

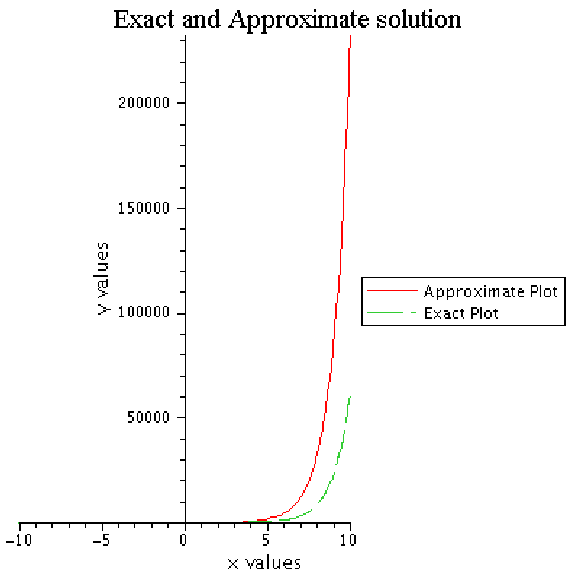

Fig4.13(a)Graphical of 50th iteration by taking t=1 and α=0.25

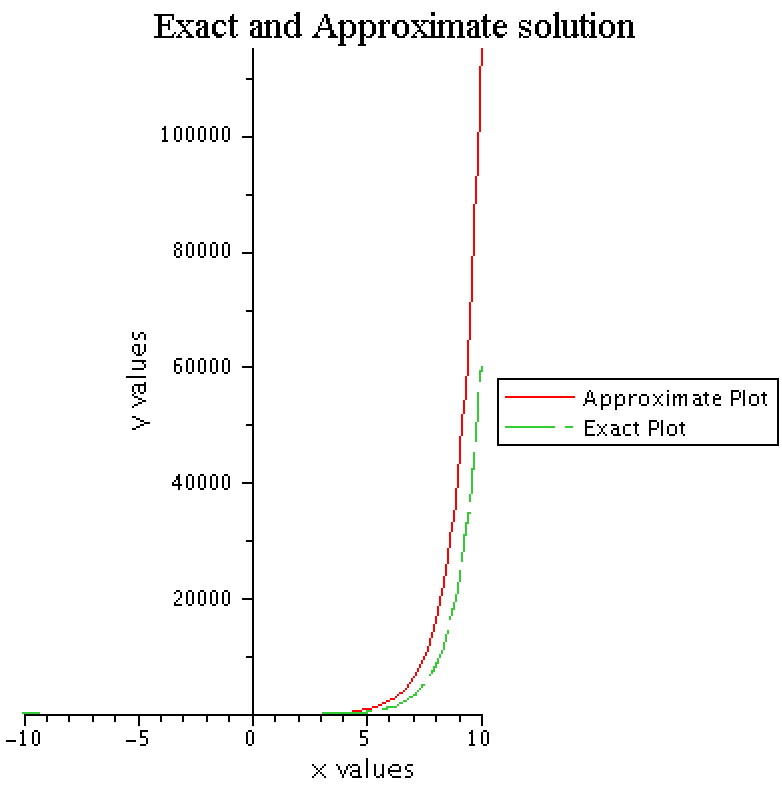

Fig4.14(b)Graphical of 50th iteration by taking t=1 and α=0.5

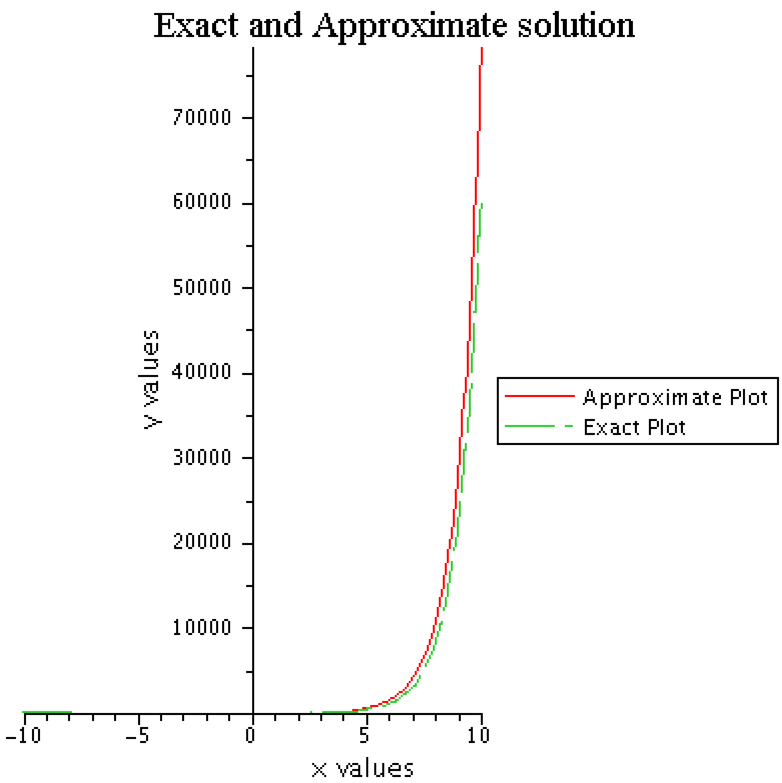

Fig4.15(c)Graphical of 50th iteration by taking t=1 and α=0.75

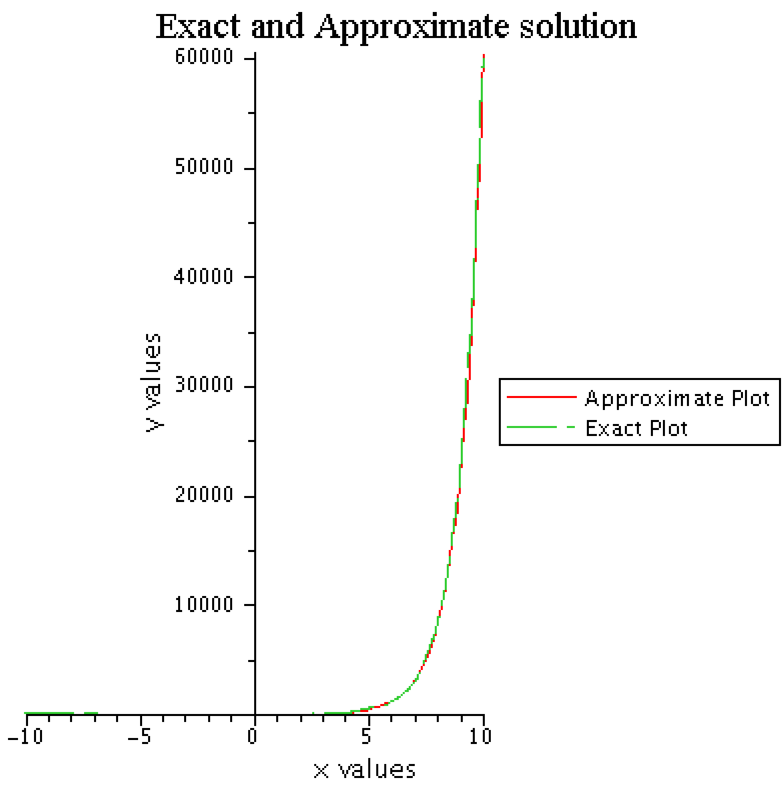

Fig4.16(d)Graphical of 50th iteration by taking t=1 and α=1

4.5 Conclusion:

In this thesis, we studied the Goursat problem by changing it from the classical form to a fractional form. To solve it, we used the Differential Transform Method (DTM) in its fractional version. Our goal was to test how accurate, reliable, and effective this method is when dealing with both linear and nonlinear types of Goursat problems.

We began by explaining the basic rules and tools of fractional calculus that are needed for this method. After building the necessary foundation, we applied fractional DTM to different types of Goursat problems — including simple (homogeneous), more complex (inhomogeneous), linear, and nonlinear problems. When we compared our results to exact solutions already available in the research, we found a very close match. This shows that our method works well and produces trustworthy results.

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