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Line Mycielskian Graphs and Colorings

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Abstract
One of the fascinating concept in graph theory is graph coloring. It is assigning colors to the vertices of graphs such that adjacent vertices have distinct colors and minimum number of colors needed to color the graph is called chromatic number. In this article, the gamma chromatic number and rainbow neighbourhood number of line Mycielskian graph are computed, Also the equitable chromatic number of line Mycielskian of some graphs are calculated.

Keywords: *Rainbow neighbourhood, Gamma coloring, Equitable Coloring, Line Mycielskian graph.*

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I. INTRODUCTION

Let G be the graph with n vertices and m edges, degree of the vertex $d(u)$ is defined as the number of edges that are incident to it and the minimum degree among the vertices is denoted by $\delta(G)$ while $\Delta(G)$ is maximum degree. Neighbourhood of the vertex $v \in V(G)$ is the collection of all vertices that are adjacent to vertex v including the vertex v in that vertex set is referred as closed neighbourhood and is denoted by $N[v]$. The line graph $L(G)$ of graph G is the graph with vertex set same as the edge set of G and two vertices in $L(G)$ are adjacent if the corresponding edges in G have a vertex in common [2]. Graph coloring is an exciting branch of graph theory with its applications in many different fields. It is a way to assign colors to the vertices of a graph such that no two adjacent vertices have the same color. Graph coloring remains a rich area of study in mathematics and computer science with applications in areas such as scheduling, map coloring, register allocation in compilers and more. The concept of graph coloring dates back to the 1850's when Francis Guthrie proposed the "Four Color Problem"[10], which asked whether it is possible to color a map using only four colors in such a way that no two adjacent regions share the same color. This problem sparked interest in graph coloring. Alfred Kempe, an English mathematician, published a flawed proof of the Four Color Theorem in 1879 [10]. Although his proof was ultimately found to be incorrect, his work contributed to the development of graph coloring techniques. Percy Heawood, a British mathematician, exposed a flaw in Alfred Kempe's proof 1890 [9], that had been considered as valid for 11 years. The four color theorem being an open question again, he established the weaker five color theorem. The four-color theorem itself was finally established by a computer-based proof in 1976.

A proper vertex coloring of a graph is no adjacent vertices share the same color. The minimum number of colors are needed for the proper vertex coloring of a graph is called chromatic number [2].

Definition 1.1. [4]. The rainbow neighbourhood number of the graph G, indicated as $r_\chi(G)$ and is denoted as $v \in V(G)$ then closed neighbourhood $N[v]$ in a graph G called a "rainbow neighbourhood" which includes at least one colored vertex of each color class in the chromatic coloring C of G. Let C represents the chromatic coloring of graph G, is the total number of vertices that produce rainbow neighbourhoods.

Definition 1.2. [8]. If vertices of graph G can be divided into r- independent sets $\{v_1, v_2, \dots v_r\}$ and every pair of vertices $\{v_i, v_j\}$ hold the condition $\left||v_i| - |v_j|\right| \leq 1$, then the graph G is said to be equitably r-colorable. The equitable chromatic number of G is the least integer r, for which G is equally r-colorable and it is represented as $\chi_{=}(G)$.

Definition 1.3. [3]. A set $S \subseteq V(G)$ of vertices in graph G is called a dominating set if every $v \in V(G)$ is either an element of S or is adjacent to an element of S.

A subset U of $V(G)$ is said to be C-colorful if every vertex of U receives different colors.

Definition 1.4.[1]. A proper coloring C of a graph G is said to be a gamma coloring of G if there exists a dominating set which is C −colorful. The gamma chromatic number $\chi_{\gamma}(G)$ is the minimum number of colors needed for a gamma coloring.

Definition 1.5. [6]. Let e_i be the set of edges of G, for each edge e_i of G, a new vertex e_i' is taken and the resulting set of vertices is denoted by $E_1(G)$. The line Mycielskian graph $L_\mu(G)$ of a graph G is defined as the graph having vertex set $E(G) \cup E_1(G) \cup \{e\}$ and edge set $E\big(L(G)\big) \cup \{e_i e_i' : e_i e_j \text{ } are \text{ } adjacent \text{ } in \text{ } G\} \cup \{e_i' e : e_i' \in E_1(G)\}$.

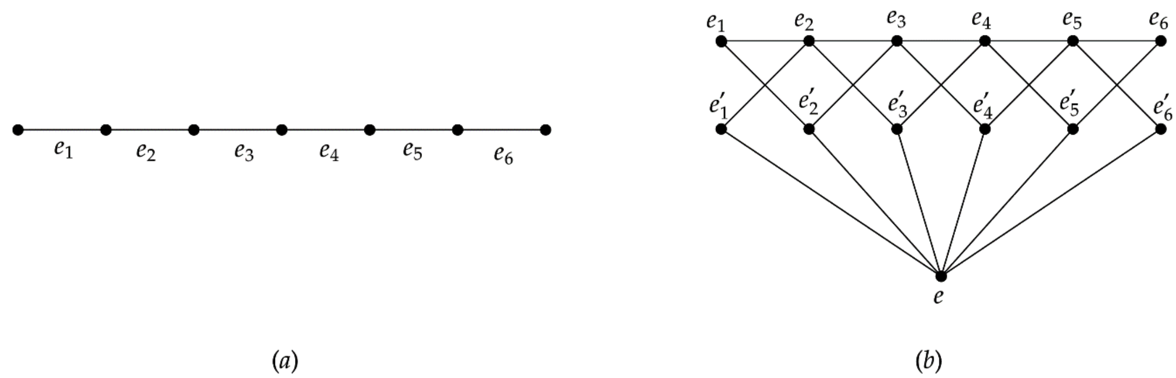

Figure 1. (a) Path P_7 (b) Line Mycielskian graph of $L_\mu(P_7)$

The following are of immediate use

Proposition 1.1.[6]. For $n \geq 2$, $r_\chi(P_n) + r_\chi\big(L(P_n)\big) = 2n - 1$.

Proposition 1.2. [6]. For $n \geq 3$,

$$r_\chi(C_n) + r_\chi\big(L(C_n)\big) = \begin{cases} 2n, & if \ n \ is \ even, \\ 6, & if \ n \ is \ odd. \end{cases}$$

Theorem 1.1[4]. For any graph G, the edge-chromatic number satisfies the inequalities $\Delta \leq \chi' \leq \Delta + 1$.

In the literature, there are various research articles that deal with equitable coloring, rainbow neighbourhood and gamma coloring of graph valued functions see [5, 7, 8, 11, 12, 13].

II. RESULT AND DISCUSSION

2.1 Gamma coloring

Theorem 2.1 *For any graph* G, $\chi_\gamma\big(L_\mu(G)\big) = \chi_\gamma\big(L(G)\big) + 1$ *or* $\chi_\gamma(L_\mu(G)) = \chi_\gamma(L(G))$.

Proof: Let c_1 be the gamma coloring of $L(G)$ with a colorful dominating set S_1. Let b_1 be gamma coloring to $L_\mu(G)$ using $\chi_\gamma(L(G)) + 1$ colors which is as follows.

$$Let, \qquad b_1(e_i) = b_1(e_i') = c_1(e_i) \qquad (1)$$

Assign a new color to the root vertex e. We first prove that b_1 is a proper coloring of $L_\mu(G)$. If $q \in E(L_\mu(G))$, then $q = e_ie_j$ or $q = e_i'e_j$ or $q = ee_i'$.

If $q = e_ie_j$ or $q = e_i'e_j$ then, $e_ie_j \in E(L(G))$ which implies that $c_1(e_i) \neq c_1(e_j)$ and hence from equation 1, $b_1(e_i) \neq b_1(e_j)$.

If $q = e_i'e$ then $b_1(e_i) \neq b_1(e)$, because $b_1(e_i) \neq b_1(e_i')$. Hence, $b_1(e)$ needs new color. This shows b_1 is a proper coloring.

If S_1 is a dominating set of $L(G)$ then, $S_1 \cup \{e\}$ is a dominating set $L_\mu(G)$. S_1 is a $c_1 -$ colorful set in $L(G)$, Since from equation 1, $c_1(e_i) = b_1(e_i)$ for all $e_i \in V(L(G))$. It follows that $S_1 \cup \{e\}$ is a b_1-colorful set in $L_\mu(G)$.

Thus $S_1 \cup \{e\}$ is a colorful dominating set of $L_\mu(G)$ and also line Mycielskian graph has a gamma coloring using $\chi_\gamma(L(G) + 1)$ colors. Hence, $\chi_\gamma(L_\mu(G)) = \chi_\gamma(L(G)) + 1$.

Let us now prove that, $\chi_\gamma(L_\mu(G)) = \chi_\gamma\big(L(G)\big)$. Let b_2 be the minimum gamma coloring of $L_\mu(G)$ with colorful dominating set Z. Let c_2 be the gamma coloring of $L(G)$ using $\chi_\gamma\left(L_\mu(G)\right)$ colors which is as follows.

Let,

$$\begin{cases} b_2(e_i) = c_2(e_i'), & if\ e_i' \in Z, \\ b_2(e_i) = c_2(e_i), & \text{otherwise.} \end{cases} \qquad (2)$$

Now we have to show that c_2 is a proper coloring of $L(G)$. If $e_ie_j \in E(L(G))$ then, $e_ie_j, e_i'e_j, e_ie_j' \in E\left(L_\mu(G)\right)$.

If $e_i', e_j' \notin Z$ and $e_ie_j \in E\left(L_\mu(G)\right)$ then, $b_2(e_i) \neq b_2(e_j)$ from equation 2, $c_2(e_i) \neq c_2(e_j)$.

If $e_i' \in Z$, $e_j' \notin Z$ and $e_i'e_j \in E\left(L_\mu(G)\right)$ then, $b_2(e_i') \neq b_2(e_j)$ from equation 2, $c_2(e_i) \neq c_2(e_j)$.

If $e_j' \in Z$, $e_i' \notin Z$ and $e_j'e_i \in E\left(L_\mu(G)\right)$ then, $b_2(e_j') \neq b_2(e_i)$ from equation 2, $c_2(e_i) \neq c_2(e_j)$.

If $e_i', e_j' \in Z$ then, $b_2(e_i') \neq b_2(e_j')$, since Z is colorful and from equation 2, $c_2(e_i) \neq c_2(e_j)$.

Thus, whenever we have $e_i, e_j \in E(L(G))$ we have $c_2(e_i) \neq c_2(e_j)$. Hence c_2 is a proper coloring of $L(G)$.

Let $S_2 = \{e_i \in E(L(G)) |\ e_i \in Z\ or\ \ e_j' \in Z\}$. We now to show that S_2 be a colorful dominating set in $L(G)$. We first verify that S_2 is a dominating set of $L(G)$, either there exists a vertex $e_i \in Z$ such that $e_ie_j \in E\left(L_\mu(G)\right)$ or there exists a vertex $e_i' \in Z$ such that $e_i'e_j \in E\left(L_\mu(G)\right)$. In either case, we have $e_i \in S_1$ and such that $e_ie_j \in E(L(G))$ Therefore, S_2 is dominating set in G.

If $e_i, e_j \in S_2$ then $e_i', e_j' \in Z\ or\ e_i' \in Z,\ e_j' \notin Z\ \ or\ e_i' \notin Z,\ e_j' \in Z\ \ or\ \ e_i', e_j' \notin Z$.

If $e_i', e_j' \in Z$ then, we know Z is colorful in $L_\mu(G)$, which implies $b_2(e_i') \neq b_2(e_j')$, from the equation 2, $c_2(e_i) \neq c_2(e_j)$.

If $e_i' \in Z$, $e_j' \notin Z$ then $e_i', e_j \in Z$ and Z is colorful in $L_\mu(G)$, which implies $b_2(e_i') \neq b_2(e_j)$, from the equation 2, $c_2\ (e_i) \neq c_2(e_j)$.

If $e_j' \in Z$, $e_i' \notin Z$ then $e_j', e_i \in Z$ and Z is colorful in $L_\mu(G)$, which implies $b_2(e_i) \neq b_2(e_j')$, from the equation 2, $c_2(e_i) \neq c_2(e_j)$.

If $e_i', e_j' \notin Z$ and $e_i, e_j \in Z$ and Z is colorful in $L_\mu(G)$, which implies $b_2(e_i) \neq b_2(e_j)$, from the equation 2, $c_2(e_i) \neq c_2(e_j)$.

Thus S_2 is a colorful dominating set, hence $\chi_\gamma(L_\mu(G)) = \chi_\gamma(L(G))$.

2.2 Rainbow Neighbourhood

Theorem 2.2. *For any graph G, $r_\chi\left(L_\mu(G)\right) = r_\chi\left(L(G)\right) + 1$.*

Proof: Let $E = \{e_1, e_2, \ldots, e_n\}$ be the vertex set of $L(G)$. Let $E' = \{e_1', e_2', \ldots, e_n'\}$ be the set of newly introduced vertices such that e_i' corresponds to the vertex e_i. If e_k and e_l are adjacent to e_i then, e_i' is adjacent to e_k and e_l.

Take another vertex e, which is adjacent to all vertices of E'. Let $C = \{c_1, c_2, \ldots, c_l\}$ be the chromatic coloring of the graph $L(G)$. e_i and e_i' can have the same color when e_i is not adjacent to e_i'. Since e is adjacent to all e_i' no color in C can be assigned to the vertex e. Therefore, we need another color to color e, say c_{l+1}. Hence chromatic coloring of $L_\mu(G)$ is $C = \{c_1, c_2, \ldots, c_l, c_{l+1}\}$. In $L_\mu(G)$ the vertex e is not adjacent to vertices in E. This clearly shows that the vertices in E will not be in the rainbow neighbourhood of $L_\mu(G)$. It can be observed that the corresponding vertex e_i' will be in a rainbow neighbourhood of $L_\mu(G)$. Since e_i' is adjacent to the vertices in all color classes in C and is adjacent to the vertex e having color c_{l+1} Therefore, every vertex in E' belongs to some rainbow neighbourhoods of $L_\mu(G)$. Moreover, the vertex e is adjacent to all vertices of E', which belong to different color classes in C and hence belongs to some rainbow neighbourhoods of $L_\mu(G)$ Therefore, $r_\chi\left(L_\mu(G)\right) = r_\chi\left(L(G)\right) + 1$.

Corollary 2.3. *The rainbow neighbourhood number of path is $r_\chi\left(L_\mu(P_n)\right) = n$.*

Proof: Let P_n be the path with n vertices and $n-1$ edges. From the Proposition 1.1, we have $r_\chi(L(P_n)) = n-1$ and from the Theorem 2.2, the result is obvious.

Corollary 2.4. *The rainbow neighbourhood number of cycle is*

$$r_\chi\left(L_\mu(C_n)\right) = \begin{cases} n+1, & if\ n\ is\ even, \\ 4, & if\ n\ is\ odd. \end{cases}$$

Proof: Let C_n be the cycle with n vertices. Since $L(C_n) = C_n$, From the Proposition 1.2, we have $r_\chi(L(C_n)) = n$, if n is even and $r_\chi\left(L(C_n)\right) = 3$, if n is odd and from the Theorem 2.2, the result is obvious.

Equitable coloring

Theorem 2.5 The equitable chromatic number of Line Mycielskian of cycle is

$$\chi_=\left(L_\mu(C_n)\right) = \begin{cases} 3, & if\ n = 4,6,8, \\ 4, & otherwise. \end{cases}$$

Proof: Let $\{e_1, e_2, \ldots, e_n\}$ be the set of vertices of $L(C_n)$ and $V\left(L_\mu(C_n)\right) = V\left(L(C_n)\right) \cup \{e_1', e_2', \ldots, e_n'\} \cup \{e\}$. Now we partition the vertex set of $L_\mu(C_n)$ as follows:

Case 1. For $n = 4,6,8$.

For $n = 4$, $V_1 = \{e_1, e_3, e\}, V_2 = \{e_2, e_4, e_4'\}, V_3 = \{e_1', e_2', e_3'\}$.

For $n = 6$, $V_1 = \{e_1, e_3, e_5, e\}, V_2 = \{e_2, e_4, e_6, e_4', e_6'\}$ $V_3 = \{e_1', e_2', e_3', e_5'\}$.

For $n = 8$, $V_1 = \{e_1, e_3, e_5, e_7, e\}, V_2 = \{e_2, e_4, e_6, e_8, e_4', e_6'\}$, $V_3 = \{e_1', e_2', e_3', e_5', e_7', e_8'\}$.

The above partition holds the condition $\left||V_i| - |V_j|\right| \leq 1$, where $i = 1,2,3$ and $j = 1,2,3$. Hence, it is equitable coloring.

Case 2. If $n \geq 10$ and n is even.

We know $\chi'(G) = L(G)$, From the Theorem 1.1, the chromatic number of $L(C_n)$ is 2, when n is even, $(Say\ R_1\ and\ R_2)$. So, in $L_\mu(C_n)$, the vertices of line graph of C_n are colored with $R_1\ and\ R_2$. The newly introduced vertex e will be colored with $R_1\ or\ R_2$. If $\chi_=\left(L_\mu(C_n)\right) = 3$. The newly introduced vertices $\{e_1', e_2', \ldots, e_n'\}$ are colored with R_3. We conclude that the partition of $V\left(L_\mu(C_n)\right)$ into three sets satisfying $\left||V_i| - |V_j|\right| \leq 1, i \neq j$. is not possible because the partition does not satisfy the condition $\left||V_i| - |V_j|\right| \leq 1$, $i \neq j$ a contradiction. Hence, $\chi_=\left(L_\mu(C_n)\right) \geq 4$.

For equality,

$V_1 = \{e_1, e_3, e_5, \ldots, e_{n-1}\} \cup \{e\}, V_2 = \{e_2, e_4, e_6, \ldots, e_n\}, V_3 = \{e_1', e_3', e_5' \ldots, e_{n-1}'\}$ and $V_4 = \{e_2', e_4', e_6' \ldots, e_n'\}$.

The above partition holds the condition $\left||V_i| - |V_j|\right| \leq 1$, where $i = 1,2,3,4$ and $j = 1,2,3,4$. Hence, it is equitable coloring.

When n is odd,

From the Theorem 1.1, the chromatic number of $L(C_n)$ is 3, when n is odd, (Say $R_1, R_2\ and\ R_3$) So, in $L_\mu(C_n)$, the vertices of line graph of C_n are colored with $(R_1, R_2\ and\ R_3)$. The newly introduced vertex e can be colored with $(R_1\ or\ R_2\ or\ R_3)$ and set of newly introduced vertices $\{e_1', e_2', \ldots, e_n'\}$ is colored with R_4. Hence, $\chi_=\left(L_\mu(C_n)\right) \geq 4$.

The following partition gives the equitable partition of Line myscielskian of cycle.

$V_1 = \{e_1, e_3, e_5, \ldots, e_{n-2}\} \cup \{e\}$, $V_2 = \{e_2, e_4, e_6, \ldots, e_{n-1}\}$,

$V_3 = \{e_3', e_5' \ldots, e_n'\} \cup \{e_n\}$ and $V_4 = \{e_2', e_4', e_6' \ldots, e_{n-3}'\} \cup \{e_1'\}$.

The above partition holds the condition $\left||V_i| - |V_j|\right| \leq 1$, where $i = 1,2,3,4$ and $j = 1,2,3,4$. Hence, it is equitable coloring.

This implies $\chi_=\left(L_\mu(C_n)\right) = 4$.

Theorem 2.6. *The equitable chromatic number of Line Mycielskian* mycielskian *of path is*

$$\chi_=\left(L_\mu(P_n)\right) = \begin{cases} 2, & if\ n = 2, \\ 3, & if\ n \leq 12, n \neq 11, \\ 4, & if\ n \geq 11, n \neq 12. \end{cases}$$

Proof. Let $V(L(P_n)) = \{e_1, e_2, \dots, e_n\}$ and $V\left(L_\mu(P_n)\right) = V(L(P_n)) \cup \{e_1', e_2', \dots, e_n'\} \cup \{e\}$.

Now we partition the vertex set of $L_\mu(P_n)$ as follows.

Case 1. For $n = 2$, $V_1 = \{e_1, e\}$ $V_2 = \{e_1'\}$. It is obvious.

Case 2. Consider the following partitions for $\chi_=\left(L_\mu(P_n)\right) = 3$.

For $n = 3$, $V_1 = \{e_1, e\}$, $V_2 = \{e_2, e_2'\}$, $V_3 = \{e_1'\}$.

For $n = 4$, $V_1 = \{e_1, e_3, e\}$, $V_2 = \{e_2, e_2'\}$, $V_3 = \{e_1', e_3'\}$.

For $n = 5$, $V_1 = \{e_1, e_3, e\}$, $V_2 = \{e_2, e_4, e_2'\}$, $V_3 = \{e_1', e_3', e_4'\}$.

For $n = 6$, $V_1 = \{e_1, e_3, e_5, e\}$, $V_2 = \{e_2, e_4, e_2', e_4'\}$, $V_3 = \{e_1', e_3', e_5'\}$.

For $n = 7$, $V_1 = \{e_1, e_3, e_5, e\}$, $V_2 = \{e_2, e_4, e_6, e_2', e_4'\}$, $V_3 = \{e_1', e_3', e_5', e_6'\}$.

For $n = 8$, $V_1 = \{e_1, e_3, e_5, e_7, e\}$, $V_2 = \{e_2, e_4, e_6, e_2', e_4'\}$, $V_3 = \{e_1', e_3', e_5', e_6', e_7'\}$.

For $n = 9$, $V_1 = \{e_1, e_3, e_5, e_7, e\}$, $V_2 = \{e_2, e_4, e_6, e_8, e_2', e_4'\}$, $V_3 = \{e_1', e_3', e_5', e_6', e_7', e_8'\}$.

For $n = 10$, $V_1 = \{e_1, e_3, e_5, e_7, e_9, e\}$,$V_2 = \{e_2, e_4, e_6, e_8, e_2', e_4'\}$, $V_3 = \{e_1', e_3', e_5', e_6', e_7', e_8', e_9'\}$.

For$n = 12$, $V_1 = \{e_1, e_3, e_5, e_7, e_9, e_{11}, e\}$,$V_2 = \{e_2, e_4, e_6, e_8, e_{10}, e_2', e_4', e_6'\}$,

$V_3 = \{e_1', e_3', e_5', e_7', e_8', e_9', e_{10}', e_{11}'\}$.

From the above partition the condition $\left||V_i| - |V_j|\right| \leq 1$ hold, where $i = 1,2,3$ and $j = 1,2,3$. Hence, it is equitable coloring.

Case 3. Consider the partitions for $\chi_=\left(L_\mu(P_n)\right) = 4$.

From the Theorem 1.1, The chromatic number of $L(P_n)$ is 2, $(Say\ R_1\ and\ R_2)$. In $L_\mu(P_n)$, the vertices of line graph of P_n are colored with $R_1\ and\ R_2$, then newly introduced vertex e will be colored with $R_1\ or\ R_2$. If $\chi_=\left(L_\mu(P_n)\right) = 3$, then another set of newly introduced vertices $\{e_1', e_2', \dots, e_n'\}$ colored with R_3. Hence we conclude that the partition of $V\left(L_\mu(P_n)\right)$ into three sets satisfying

$\left||V_i| - |V_j|\right| \leq 1, i \neq j$ is not possible because the partition does not satisfy the condition $\left||V_i| - |V_j|\right| \leq 1, i \neq j$ a contradiction. Hence $\chi_=\left(L_\mu(P_n)\right) \geq 4$.

For equality, the following partition gives the equitable partition of line Mycielskian of path, $P_n \geq 11$ but $n \neq 12$.

$V_1 = \{e_1, e_3, e_5, \dots, e_n\} \cup \{e\}$, $V_2 = \{e_2, e_4, e_6, \dots, e_{n-1}\}$,

$V_3 = \{e_3', e_5' \dots, e_{n-1}'\} \cup \{e_n'\}$ and $V_4 = \{e_2', e_4', e_6' \dots, e_n'\} \cup \{e_1'\}$.

The above partition holds the condition $\left||V_i| - |V_j|\right| \leq 1$, where $i = 1,2,3,4$ and $j = 1,2,3,4$. Hence, it is equitable coloring. This implies $\chi_=\left(L_\mu(P_n)\right) = 4$.

III. CONCLUSION

Graph coloring has practical applications in numerous fields such as scheduling, register allocation in compiler design, map coloring and solving various optimization problems. It is also a subject of theoretical interest in mathematics and computer science with many open problems and research areas related to coloring and its variants. In this paper gamma coloring and rainbow neighbourhood coloring of line Mycielskian graph is computed and also, we have obtained results on equitable coloring of line Mycielskian graph of cycle and path.

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