

# Research Status and Prospects of Direct Strength Method for Thin-walled Axially Loaded Members

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## Abstract

Direct Strength Method is a structural design approach aimed at assessing the strength of structural members under external loads. Firstly, it requires determining the geometric shape and material properties of the member, such as cross-sectional shape, dimensions, elastic modulus, and yield strength. Then, external loads acting on the structure, such as gravity, wind loads, seismic forces, etc., are determined based on design criteria and specifications. Next, stress analysis is performed to determine the internal stress distribution of the member, considering stress conditions under different load combinations. Finally, utilizing strength theories, the ability of the member to safely withstand these loads is assessed, considering various failure modes such as tension, compression, shear, etc. The Direct Strength Method ensures the safety and stability of structures and finds wide applications in fields such as buildings, bridges, aerospace, etc. This paper will provide a detailed introduction to the Direct Strength Method calculation method for thin-walled members under axial compressive loads.

**Keywords:** Direct strength method; Axially loaded member; Perforated member; Built-up section

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## I. INTRODUCTION

In the structural specifications for cold-formed steel structures, the calculation methods for the load-bearing capacity of axially loaded members mainly include effective width method(EWM) and direct strength method(DSM). The effective width method designs the members based on the effective width of each plate on the cross-section. However, when the cross-sectional shape of the member is complex, the calculation process becomes cumbersome, and the effect of distortional buckling cannot be considered. Chinese scholars have developed GB50018-2002 and JGJ227-2011 based on experimental results. The North American specification AISI S100-16 uses the modified slenderness ratio method to design the load-bearing capacity of cold-formed thin-walled steel built-up sections under axial compression. This method can predict the load-bearing capacity and primary instability modes of the members well through the critical stresses corresponding to overall, local, and distortional buckling.

According to Chapter 1 of the North American specification AISI S100-16, it is recommended to use the direct strength method to calculate the load-bearing capacity of thin-walled axially compressed members, especially for members with complex cross-sections. Since the specific calculation methods for perforated axially loaded members in AISI S100-16 are scattered throughout the specification, this paper will introduce the calculation methods for the load-bearing capacity of unperforated and perforated members within the direct strength method.

## II. DSM FOR THIN-WALLED AXIALLY LOADED MEMBERS

In 1998, Schafer proposed the Direct Strength Method for the design of the load-bearing capacity of cold-formed steel compression members.

$$P_n = \min(P_{ne}, P_{nl}, P_{nd}) \quad (1)$$

In Equation(1),  $P_{ne}$ ,  $P_{nl}$  and  $P_{nd}$  respectively represent the global buckling load, local buckling load, and distortional buckling load.

1) Global buckling load  $P_{ne}$ :

$$\begin{cases} P_{ne} = (0.658\lambda_c^2)P_y & \text{for } \lambda_c \leq 1.5, \\ P_{ne} = \left(\frac{0.877}{\lambda_c^2}\right)P_y & \text{for } \lambda_c > 1.5, \end{cases} \quad (2)$$

In Equation(2),  $\lambda_c = \sqrt{P_y / P_{cre}}$ ,  $P_y = A_g f_y$ ,  $A_g$  represents the gross cross-sectional area of the specimen,  $f_y$  represents the material yield strength. In Equation(3),  $P_{cre}$  represents the global critical load, where  $E$  represents elastic modulus,  $I$  represents the moment of inertia about the weak axis,  $K$  represents the slenderness coefficient, and  $L$  represents the column length.

$$P_{cre} = \frac{\pi^2 EI}{(KL)^2} \tag{3}$$

2) Local buckling load  $P_{nl}$ :

$$\begin{cases} P_{nl} = P_{ne} & \text{for } \lambda_l \leq 0.776, \\ P_{nl} = \left[ 1 - 0.15 \left( \frac{P_{crl}}{P_{ne}} \right)^{0.4} \right] \left( \frac{P_{crl}}{P_{ne}} \right)^{0.4} P_{ne} & \text{for } \lambda_l > 0.776, \end{cases} \tag{4}$$

In Equation(4),  $\lambda_l = \sqrt{P_{ne} / P_{crl}}$ ,  $P_{crl} = A_g f_{crl}$ ,  $P_{crl}$  and  $f_{crl}$  respectively represent the elastic local buckling critical load and the elastic local buckling critical stress of the member. Due to the complexity of the numerical solution provided in the code regarding the elastic local buckling critical stress  $f_{crl}$ , finite strip analysis software CUFSM is utilized to calculate  $f_{crl}$ .

3) Distortional buckling load  $P_{nd}$ :

$$\begin{cases} P_{nd} = P_y & \text{for } \lambda_l \leq 0.561, \\ P_{nd} = \left[ 1 - 0.25 \left( \frac{P_{crd}}{P_y} \right)^{0.6} \right] \left( \frac{P_{crd}}{P_y} \right)^{0.6} P_y & \text{for } \lambda_l > 0.561, \end{cases} \tag{5}$$

In Equation(5),  $\lambda_d = \sqrt{P_y / P_{crd}}$ ,  $P_{crd} = A_g f_{crd}$ ,  $P_{crd}$  and  $f_{crd}$  respectively represent the elastic local buckling critical load and the elastic local buckling critical stress of the member. Due to the complexity of the numerical solution provided in the code regarding the elastic local buckling critical stress  $f_{crd}$ , finite strip analysis software CUFSM is utilized to calculate  $f_{crd}$ .

For back-to-back built-up sections, the North American specification Commentary requires adjustment of the overall slenderness ratio during the calculation of elastic buckling stress to reflect the reduced stiffness of the composite section. To achieve more accurate axial load capacity, the composite section is generally treated as a single entity, reducing the thickness of the web-to-web contact portion. The specific formula for adjusting the overall slenderness ratio is as follows:

$$\left( \frac{KL}{r} \right)_m = \sqrt{\left( \frac{KL}{r} \right)_o^2 + \left( \frac{a}{r_i} \right)^2} \tag{6}$$

In Equation(6),  $(KL/r)_o$  represents the overall slenderness ratio,  $(KL/r)_m$  represents the modified slenderness ratio,  $r$  represents the radius of gyration of the entire cross-section of the composite section,  $r_i$  is the radius of gyration of one limb of the composite section, and  $a$  represents the spacing between fasteners.

### III. DSM FOR THIN-WALLED AXIALLY LOADED MEMBERS WITH PERFORATIONS

Moen and Schafer[1–3], as well as Smith and Moen[4], have expanded upon the current North American specification AISI S100-16 by proposing formulas for the Direct Strength Method for perforated members. These formulations adjust the critical loads for overall, local, and distortional buckling to reflect the detrimental effects of the perforations on the member's load-bearing capacity. Since this calculation method is only applicable to single-web members, the reliability of predicting the load capacity of assembled perforated members has not been fully established.

1) Global buckling load  $P_{ne}$ :

By introducing a weighted section moment of inertia, the reduction of the overall buckling load can be achieved, as illustrated below:

$$I_{avg} = \frac{I_g L_g + I_{net} L_{net}}{L} \tag{7}$$

$$f_{cre} = \frac{\pi^2 E}{(KL/r)^2} \cdot \frac{I_{avg}}{I_g} = \frac{\pi^2 E I_{avg}}{A_g (KL)^2} \tag{8}$$

In Equation(7),  $I_{avg}$  represents the weighted section moment of inertia,  $I_g$  and  $I_{net}$  respectively represent the moment of inertia of the entire cross-section about the weak axis and the net moment of inertia,  $L_g$  and  $L_{net}$  represent the net length of the member and the hole length.

Substituting the weighted section moment of inertia  $I_{avg}$  into Equation(8), the global critical stress  $f_{cre}$  can be obtained. Multiplying the global critical stress  $f_{cre}$  by the gross cross-sectional area  $A_g$  yields the global critical load  $P_{cre}$ . Subsequently, substituting  $P_{cre}$  into Equation(2), the reduced overall buckling load  $P_{ne}$  can be determined.

2) Local buckling load  $P_{nl}$ :

In the CUFSM software, finite strip analysis can only be performed on continuous sections. To induce local buckling in perforated sections, it is necessary to constrain the degrees of freedom at the nodes in CUFSM and adjust the stresses to obtain a reasonable critical stress for elastic local buckling.

The specific procedure is as follows: Firstly, when editing node information in CUFSM, it is necessary to constrain the degrees of freedom at the four corners of the section. As shown in Figure 1(a), for perforated sections with web openings, one direction of freedom needs to be constrained. For perforated sections with flange openings, additional constraints are required for the two directions of independent elements. Secondly, to ensure that the same magnitude of load is applied to perforated and unperforated sections, the stress level applied to the perforated section needs to be adjusted to be higher than that of the unperforated section, as shown in Figure 1(b). Additionally, it is important to note that the nodes at both ends of the hole cannot be directly deleted; instead, they should be connected to adjacent elements, with the thickness set to 0.

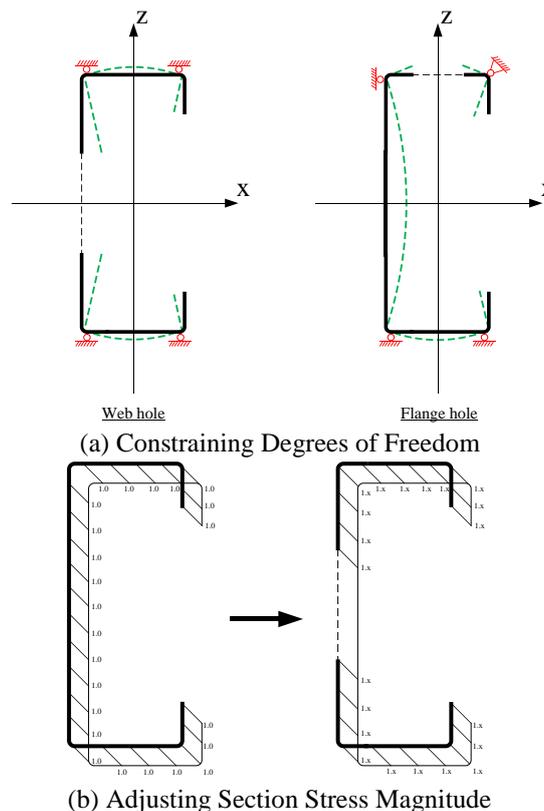


Figure 1: Schematic diagram for calculating the critical stress of elastic local buckling

Then, a finite strip analysis is conducted on the modified section. At this point, the critical stress of elastic local buckling  $f_{crlh}$  can be obtained. It is important to note that the final selection of  $f_{crlh}$  follows the rules provided in Section Appendix 2 of the Commentary of the North American specification. When the hole length is less than half the buckling wavelength corresponding to the critical stress of elastic local buckling, the stress value corresponding to the hole length should be chosen as the correct  $f_{crlh}$  as shown in Figure 2(a). Conversely, when the hole length is greater than half the buckling wavelength, the original stress value should be chosen as the correct  $f_{crlh}$ , as shown in Figure 2(b).

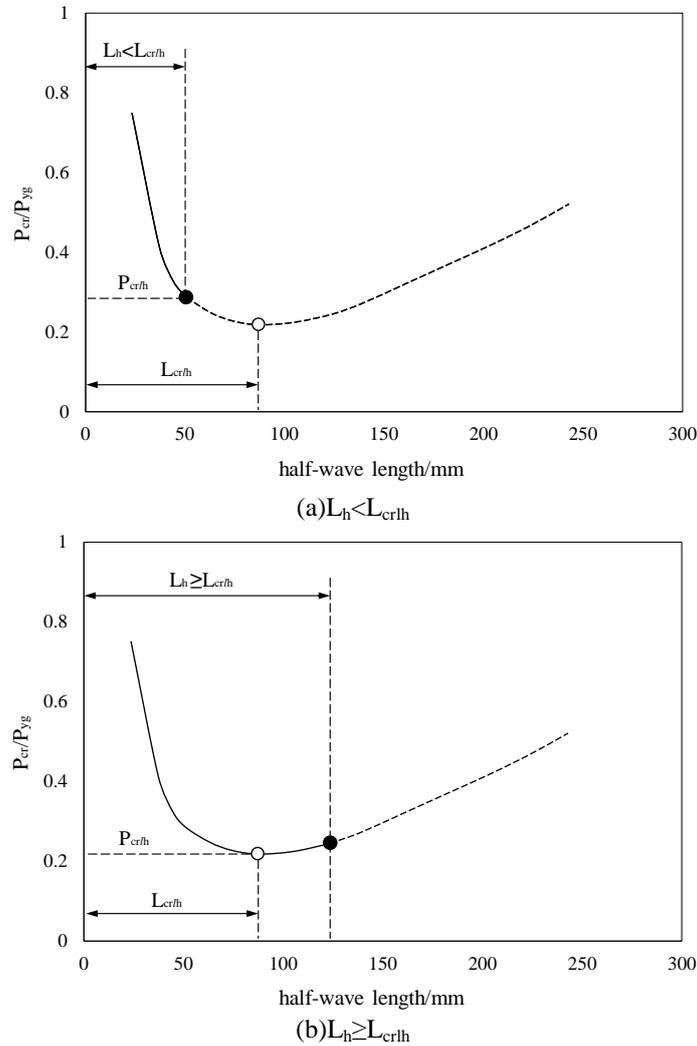


Figure 2: Selection rules for the critical stress of elastic local buckling

Next, the elastic local buckling critical stress  $f_{crlh}$  obtained from the analysis is substituted into the following, and the smaller value is taken as the elastic local buckling critical load  $P_{crl}$ :

$$P_{crl} = \min(P_{crl-nh}, P_{crl-h}) \tag{9}$$

In Equation(9),  $P_{crl-nh} = f_{crl-nh}A_g$ ,  $P_{crl-h} = f_{crl-h}A_{net}$ ,  $f_{crl-nh}$  and  $f_{crl-h}$  respectively represent the elastic local buckling critical stress of the entire section and the net section obtained from finite strip analysis,  $A_g$  and  $A_{net}$  represent the gross and net cross-sectional areas, respectively.

Finally, substituting the elastic local buckling critical load  $P_{crl}$  into Equation(4) while ensuring  $P_{nl} \leq P_{ynet}$ , the reduced local buckling load  $P_{nl}$  can be obtained.

3) Distortional buckling load  $P_{nd}$ :

To convert a perforated section into a continuous section, it is necessary to reduce the thickness of the plate at the hole locations. The specific method for reducing the thickness is illustrated in Figure 3 and described by Equation(10).

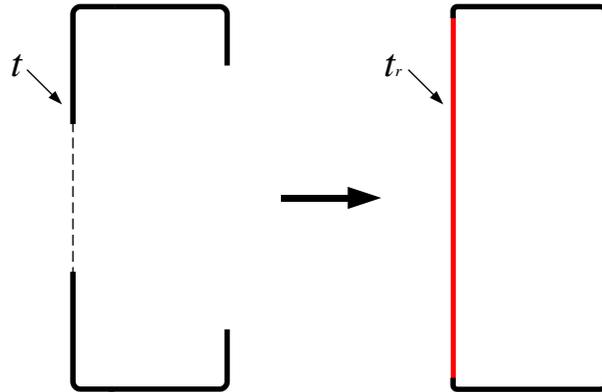


Figure 3: Thickness reduction in elastic distortional buckling critical stress calculation

$$t_r = \left(1 - \frac{L_h}{L_{crd}}\right)^{\frac{1}{3}} t \tag{10}$$

In Equation(10),  $t_r$  and  $t$  respectively represent the reduced thickness of the flange and the original thickness of the flange,  $L_h$  and  $L_{crd}$  respectively represent the hole length and the distortional buckling half-wavelength of the unperforated member.

After thickness reduction, using CUFSM can yield the elastic distortional buckling critical stress  $f_{crd}$  of the perforated section. However, it's important to note that this critical stress value is not correct. The perforated section and the unperforated section have a unique distortional buckling half-wavelength. The correct  $f_{crd}$  value should be obtained by selecting the stress value corresponding to the distortional buckling half-wavelength of the unperforated section on the curve of the perforated section. Taking Figure 4 as an example, the final  $f_{crd}$  should be selected as the stress value corresponding to 700mm on the red dashed line.

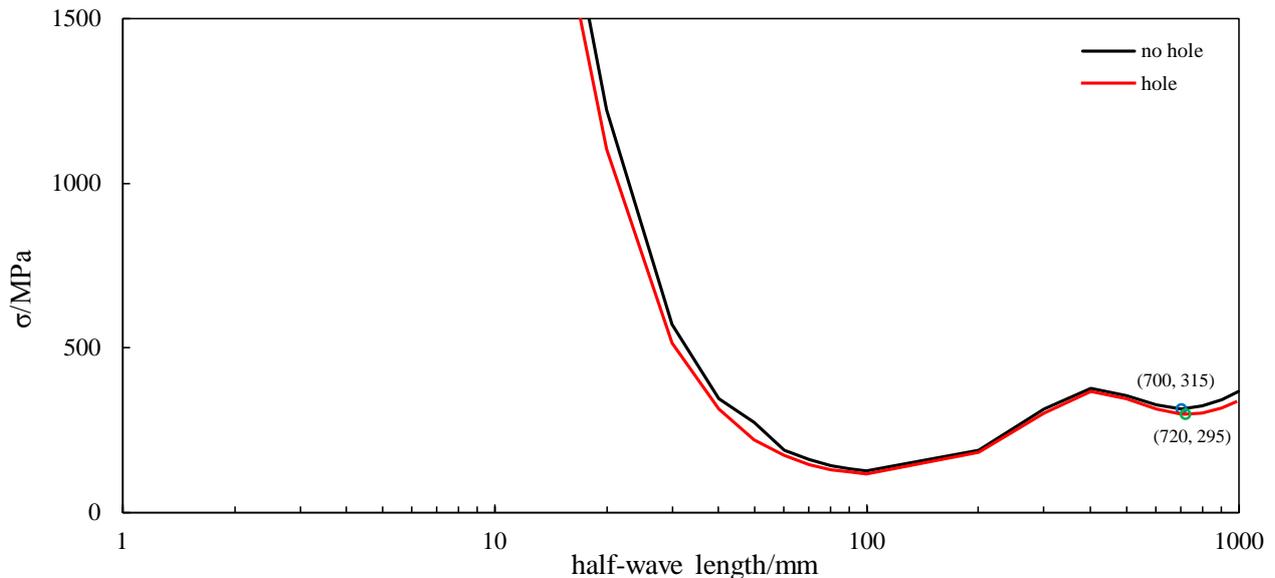


Figure 4: Selection Rules for Elastic Distortional Buckling Critical Stress

Finally, substituting the obtained elastic distortional buckling critical stress  $f_{crd}$  into the following equation yields the reduced distortional buckling load  $P_{nd}$ .

$$\left\{ \begin{array}{ll} P_{nd} = P_{ynet} & \text{for } \lambda_d \leq \lambda_{d1}, \\ P_{nd} = P_{ynet} - \left( \frac{P_{ynet} - P_{d2}}{\lambda_{d2} - \lambda_{d1}} \right) (\lambda_d - \lambda_{d1}) & \text{for } \lambda_{d1} < \lambda_d \leq \lambda_{d2}, \\ P_{nd} = \left[ 1 - 0.25 \left( \frac{P_{crd}}{P_y} \right)^{0.6} \right] \left( \frac{P_{crd}}{P_y} \right)^{0.6} P_y & \text{for } \lambda_{d2} \leq \lambda_d, \end{array} \right. \quad (11)$$

In Equation(11),  $\lambda_{d1} = 0.561(P_{ynet}/P_y)$ ,  $\lambda_{d2} = 0.561 \left[ 14(P_y/P_{ynet})^{0.4} - 13 \right]$ ,  $P_{ynet} = f_y A_{net}$ ,  $P_{crd} = f_{crd} A_{Modified}$ ,  $P_{d2} = 0.561 [1 - 0.25(1/\lambda_{d2})^{1.2}] [(1/\lambda_{d2})^{1.2}] P_y$ .

#### IV. SIMPLIFIED CALCULATION OF BUILT-UP AXIALLY LOADED MEMBERS

When using the direct strength method for calculation, accurately extracting the critical buckling stress is a prerequisite for obtaining reliable calculation results. For composite section members assembled using connectors, the connectors can be regarded as non-continuous constraints in the axial direction. Built-up sections with such non-continuous constraint relationships cannot directly obtain the critical buckling stress of the section through finite strip methods. Therefore, it is necessary to reasonably simplify the built-up section[5–8]. The steps of simplification are as follows: consider the assembled section as a whole. If the thickness of a single component is "t", then the thickness at the joint positions of the composite section is "2t". Based on the constraint strength of the joint positions, the thickness at the joint positions can be moderately reduced. According to the recommendations of literature[6,7], the thickness of the flange of the I-shaped composite section can be set to 1.0t or 1.2t, As shown in Figure 5.

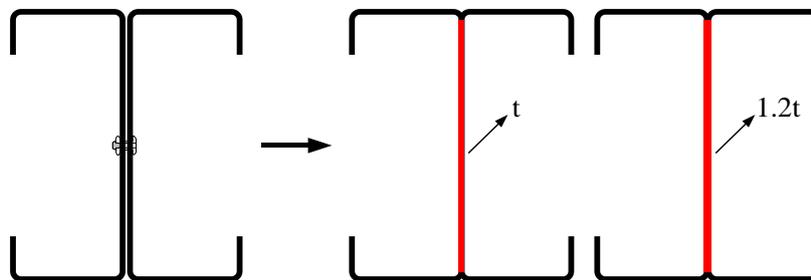


Figure 5: Simplification of I-shaped built-up section

#### V. CONCLUSIONS

- (1) The Direct Strength Method for axially loaded members without perforations provides relatively accurate predictions of load capacity for flat-webbed components.
- (2) The Direct Strength Method for axially loaded members with perforations is overly complex, making it impractical for real-world engineering applications. Further development of software to assist with calculations or simplification of calculation formulas is needed.
- (3) The applicability and accuracy of the Direct Strength Method for axially loaded members with perforations need further improvement. This may be because the perforations in I-shaped built-up section are located away from the centroid of the section, resulting in no reduction in the moment of inertia, which cannot be accounted for in the calculations.

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