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Equivalence Relation of Einstein and MollerEnergy-Momentum Complexes in Teleparallel Gravity Theory

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Abstract

In this paper, we established the equivalence relation of Einstein and Moller energy-momentum prescriptions via distinct diagonal and non-diagonal space times in teleparallel gravity theory. We perform an extensive investigation of the localization of energy-momentum in various space-times. We determined that both of the prescriptions (Einstein and Moller) provide equivalent results of energy-momentum and super potentials for any space-times (diagonal and non-diagonal). We further explored that the tensor quantity $\Delta_{\mu}^{i\alpha}$ must be equal to zero for non-diagonal space-times in teleparallel gravity theory.

Kywords: Einstein energy-momentum complex, Moller energy-momentum complex, Teleparallel gravity, Space-times.

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I. INTRODUCTION

The energy-momentum localization [1–3] has been a problem for the physics research community gradually after the emergence of the general theory of relativity. Numerous authors,including Einstein [4], Moller [2, 5], Bergmann-Thomson [6], and their collaborators, have attempted to use a variety of space-time measures and energy-momentum formulations [7, 8] to solve the energy-momentum localization issue [9]. They discovered that energy-momentum complexes hold true for well-known and physically significant space times.They evaluated the same major emotions in both the GR and TG theories.

The distribution of energy-momentum has been the subject of several investigations that have been documented in the literature. However, Einstein was the pioneer in this field, and since then, Lnadu-Lifshitz [10], Tolman [11], Papapetrou [12], Moller, Weinberg [13], Bergmen-Thomson, and have all produced various energy and momentum prescriptions. Numerous methods were employed put out since the discovery of general relativity to derive the conservation rules that defined generic systems. But more recently, teleparallel gravitation [TG], a competing theory of gravity, has also raised this issue [14]. The usage of Moller prescriptions therefore appears to be more fascinating, useful, and suitable when discovering the energy-momentum since one of the most significant reasons is that it is not coordinate dependent.

The concept of energy-momentum complexes was heavily questioned for several reasons. First, the nature of symmetric and locally preserved. The object is not tensorial, making its physical meaning unclear [15]. Secondly, various energy-momentum complexes may produce distinct distributions for the same gravitational backdrop [16, 17]. Energy-momentum complexes were shown to be local objects, contrary to popular belief that gravitational fields cannot be localized [18]. Penrose and colleagues created a concept known as quasi-local energy [19, 20]. Although these quasi-local masses are theoretically significant, their definitions have fundamental flaws. Chang et al. [21] shown that energy-momentum complexes are valid formulations of the energy-momentum.

According to Mikhail et al [22, 23], the energy-momentum issue can also be localized inside the teleparallel theory of gravity. This idea was originally proposed to define the energy of the gravitational field. Möller [24] was the first to recognize that the tetrad description of the gravitational field provides a more accurate representation of gravitational energy-momentum than general relativity [GR]. Vargas [25] discovered that the closed FRW spacetimes had zero total energy when using the teleparallel form of Einstein and Landau-Lifshitz complexes. This finding is consistent with prior studies by Cooperstock [26] and Rosen [27]. According to Vargas et al [28], Bianchi types I and II in TPT produce consistent results. Sharif and Jamil [29] discovered that the Lewis-Papapetrou measure yields different outcomes in TPT compared to GR [30].

In our study, we aim to investigate the distribution of energy and momentum within the context of TG, considering various metrics. While researchers have explored this issue in different contexts like GR and TG, our focus is specifically on the energy-momentum distribution in TG and its relationship with diverse metrics. Recently A.T Ali et al [31], showed the equivalence of energy-momentum complexes of Einstein and Tolman in G.R. They proved that energy-momentum tensors defined by Einstein and Tolman in G.R are same for any space-time. They also established that Einstein and Tolman super-potentials are different in general. Keeping in view these points, in this paper we will try to show that Einstein and Moller energy-momentum complexes gives same results for different space-times in teleparallel theory of gravity [TPGT]. In particular we will take different space-time metrics and show that equivalence of Einstein and Moller energy-momentum complexes exists in teleparallel gravity theory. This paper is consists on some sections, in section 3 a general summary of Einstein and Moller energy-momentum complexes in the context of teleparallel gravity theory is given. In section 4 there are some particular space-time examples are given for the verification of the equivalence relation of Einstein and Moller energy-momentum tensors gives same results for the energy-momentum distribution in teleparallel gravity theory. In section 5 conclusion of the whole work is given.

1 Einstein and Moller energy momentum Tensors

1.1 Einstein energy momentum Tensor

The tensor defined by Einstein for energy momentum in teleparallel gravity theory and the super potential in the Moller tetrad theory is given as [25,32,33]

$$\begin{cases} h\Phi^{\mu}_{\nu} & = \frac{2}{\kappa}\Phi^{\mu}_{\nu,\alpha}, \\ h\Phi^{\mu\kappa} & = g^{\nu k}h\Phi^{\mu}_{\nu} = \frac{2}{\kappa}g^{\nu k}\Phi^{\mu\alpha}_{\nu,\alpha}, \end{cases} \tag{1}$$

where κ is the coupling constant and

$$h = Det(h^j_i) = \sqrt{-Det(g_{\mu\nu})} = \sqrt{-g} \ , \tag{2}$$

and the Frued super-potential $\Phi^{[\mu\nu]}_{\alpha}$ is given by

$$\Phi^{[\mu\nu]}_{\alpha} = hS^{[\mu\nu]}_{\alpha} \ , \tag{3}$$

where the tensor $S^{\mu[\nu\lambda]}$ is defined as

$$S^{i[jk]} = \frac{1}{2}(K^{[jk]i} + g^{ij}T^{\alpha k}_{\alpha} - g^{ik}T^{\alpha j}_{\alpha}) \ , \tag{4}$$

where

$$K^{[ij]k} = \frac{1}{2}(T^{j[ik]} + T^{k[ij]} - T^{i[jk]}). \ , \tag{5}$$

Here we note that $h\Phi^0_0$ is the energy momentum density and $h\Phi^0_i$ where $i = 1, 2, 3$ are the momentum density components and the energy current density component is $h\Phi^\alpha_0$.

1.2 Moller energy momentum tensor

The Moller energy-momentum tensor in the context of teleparallel theory of gravity Mikhail et al. [22] are given by

$$\Psi_{\mu}^{[i\alpha]} = \frac{h}{4} P_{\beta\gamma\delta}^{ki\alpha} g_{\mu k} [g^{\beta\delta} V^{\gamma} - \lambda K^{\beta\gamma\delta} - 1(1-2\lambda) K^{\delta\lambda\beta}], \tag{6}$$

and it's another important form is

$$\Psi_{\mu}^{[i\alpha]} = \frac{h}{4} P_{\beta\gamma\delta}^{ki\alpha} g_{\mu k} (g^{\beta\delta} V^{\gamma} - K^{\beta\gamma\delta} - \lambda[K^{\beta\gamma\delta} - 2K^{\delta\lambda\beta}]), \tag{7}$$

where

$$P_{jkl}^{i\mu k} = \delta_i^j g_{kl}^{\mu\nu} + \delta_k^i g_{ij}^{\mu\nu} - \delta_i^j g_{jk}^{\mu\nu} \ , \tag{8}$$

and the tensor quantity $g_{kl}^{\mu\nu}$, which is defined as

$$g_{kl}^{\mu\nu} = \delta_k^{\mu} \delta_l^{\nu} - \delta_l^{\mu} \delta_k^{\nu}. \tag{9}$$

Now the Moller energy-momentum density in TPG is defined as

$$\begin{cases} h\Psi_{\nu}^{\mu} & = \frac{2}{\kappa} \Psi_{\nu,\alpha}^{\mu\alpha} \ , \\ h\Psi^{\mu k} & = g^{\nu k} h\Psi_{\nu}^{\mu} = \frac{2}{\kappa} g^{\nu k} \Psi_{\nu,\alpha}^{\mu\alpha} \end{cases} \tag{10}$$

here $h\Psi_0^0$ stands for energy density and $h\Psi_i^0$ where $i = 1, 2, 3$ for momentum density.

2 Some particular space-times Examples

In this section, we will take ten different space-times [34] and to show that the results from the energy-momentum tensors of Einstein and Moller give the same results in all cases

Example 1. Bell-Szekeres Metric [35]. The Cartesian form of the metric is

$$\begin{aligned} ds^2 = \frac{1}{2}dt^2 - Cos^2\{\frac{D(t-z)}{2}\alpha(\frac{t-z}{2}) + \frac{C(t+z)}{2}\alpha(\frac{t+z}{2})\}dx^2 \\ -Cos^2\{\frac{D(z-t)}{2}\alpha(\frac{t-z}{2}) + \frac{C(t+z}{2})\alpha(\frac{t+z}{2})\}dy^2 - \frac{1}{2}dz^2. \end{aligned} \tag{11}$$

By putting the metric coefficients in eq. (3) and eq. (7), we obtained the following Einstein and Moller super potential components

$$\begin{aligned}\Phi_0^{30} =& \Psi_0^{30} = -\frac{1}{8}[Cos\{\frac{D(t-z)}{2}\alpha(\frac{t-z}{2})+\frac{C(t+z)}{2}\alpha(\frac{t+z}{2})\}]Cos\{\frac{D(z-t)}{2}\alpha(\frac{t-z}{2})\\ &+\frac{1}{2}+C(t+z)\alpha(\frac{t+z}{2})\}Tan\{\frac{1}{2}(D(t-z)\alpha(\frac{t-z}{2}))+C(t+z)\alpha(\frac{t-z}{2})\}\\ &(2D\alpha(\frac{t-z}{2}))-2C\alpha(\frac{t+z}{2})+Dt\alpha'(\frac{t-z}{2})-Dz\alpha'(\frac{t-z}{2})-Ct\alpha'(\frac{t+z}{2})\\ &-Cz\alpha'(\frac{t+z}{2})+Tan\{\frac{1}{2}(D(z-t)\alpha(\frac{t-z}{2}))+C(t+z)\alpha(\frac{t+z}{2})\}[(2D\alpha(\frac{t-z}{2})\\ &+2C\alpha(\frac{t+z}{2})+Dt\alpha'(\frac{t-z}{2})-Dz\alpha'(\frac{t-z}{2})+Ct\alpha'(\frac{t+z}{2})+Cz\alpha'(\frac{t+z}{2}])]\\ &\Phi_1^{10}=\Psi_1^{10}=\Phi_1^{31}-(Tan\{\frac{1}{2}(D(z-t)\alpha(\frac{t-z}{2}))+C(t+z)\alpha(\frac{t+z}{2})\})(2D\alpha(\frac{t-z}{2})\\ &\quad+2C\alpha(\frac{t+z}{2})+Dt\alpha'(\frac{t-z}{2})-Dz\alpha'(\frac{t-z}{2})+Ct\alpha'(\frac{t+z}{2})+Cz\alpha'(\frac{t+z}{2})),\\ &\Phi_1^{31}=\Psi_1^{31}\\ &\Phi_2^{20}=\Psi_2^{20}=\Phi_2^{32}+2(-2C\alpha(\frac{t+z}{2})-Ct\alpha'(\frac{t+z}{2})-Cz\alpha'(\frac{t+z}{2})) \qquad ,\\ &\Phi_2^{32}=\Psi_2^{32}.\end{aligned} \tag{12}$$

Now by putting the above values of super-potentials in eq. (1) and eq. (10), we obtained the following Einstein and Moller energy-momentum tensors

$$\begin{aligned}&h\Phi_0^0=h\Psi_0^0=\frac{1}{8\kappa}[Cos\{B(z-t)\alpha(\frac{t-z}{2})\}(2D\alpha(\frac{t-z}{2})+D(t-z)\alpha'(\frac{t-z}{2}))^2+DSin\\ &\{D(z-t)\alpha(\frac{t-z}{2})\}-\{4\alpha'(\frac{t-z}{2})+(z-t)\alpha''(\frac{t-z}{2})\}C\{(CCos\{C(t+z)\alpha(\frac{t+z}{2})\\ &(2\theta(\frac{t+z}{2})+(t+z)\alpha'(\frac{t+z}{2}))^2\}+Sin\{C(t+z)\alpha(\frac{t+z}{2}\}(4\alpha'(\frac{t+z}{2})+(t+z)\\ &\alpha''(\frac{t+z}{2}))\})\}],\\ &h\Phi_3^0=h\Psi_3^0,\\ &h\Phi_i^0=h\Psi_i^0=0, \qquad i=1,2.\end{aligned} \tag{13}$$

Example 2. The general line element of the gravitational waves [36] is given by

$$ds^2=e^{-N}dt^2-e^{-N}dx^2-e^{-P-R}dy^2+e^{-P+R}dz^2, \tag{14}$$

where P,R and N are the functions of t and x only.
By putting the metric coefficients in eq(3) and eq(7), we obtained the following

Einstein and Moller super potential components,

$$\begin{aligned}&\Phi_0^{10}=\Psi_0^{10}=\frac{e^{-P}P_x}{2}\ ,\ \Phi_1^{10}=\Psi_1^{10}=-\frac{e^{-P}P_t}{2}\ ,\ \Phi_2^{20}=\Psi_2^{20}=-\frac{e^{-P}(N_t+P_t-R_t)}{4}\\ &\Phi_2^{12}=\Psi_2^{12}=-\frac{e^{-P}(N_x+P_x-R_x)}{4} \qquad ,\ \Phi_3^{30}=\Psi_3^{30}=-\frac{e^{-P}(N_t+P_t+R_t)}{4}\\ &\Phi_3^{13}=\Psi_3^{13}=-\frac{e^{-P}(N_x+P_x+R_x)}{4},\end{aligned} \tag{15}$$

and Einstein and Moller energy-momentum tensors are obtained by putting the above super potentials in eq. (1) and eq. (10)

$$h\Phi_0^0=h\Psi_0^0=\frac{e^{-P}(P_{xx}-P_x^{\kappa})}{\kappa},\ h\Phi_1^0=h\Psi_1^0=\frac{e^{-P}(P_{tx}-P_xU_t)}{\kappa},\ \ h\Phi_i^0=h\Psi_i^0=0;\ i=2,3. \tag{16}$$

Example 3. Consider the Ruban universe model[17] which is a special form of the Szekeres universe and is given by

$$ds^2=dt^2-Q^2dx^2-R^2(dy^2+H^2dz^2), \tag{17}$$

where

$$H(y) = \frac{sin[\sqrt{K}y]}{\sqrt{k}} \begin{cases} sin[y], & \text{if } K = 1 \\ 0, & \text{if } K = 0 \\ sin[y], & \text{if } K = -1 \end{cases}$$

and K is the curvature parameter.$Q(t,x)$ and $R(t)$ are free functions and will be determined.
By putting the metric coefficients in eq. (3) and eq. (7), we obtained the following Einstein and Moller super potential components

$$\begin{aligned} &\Phi_0^{20} = \Psi_0^{20} = -\frac{QH'}{2} \ , \ \Phi_0^{10} = \Psi_0^{10} = HQRR' \ , \ \Phi_1^{21} = \Psi_1^{21} = -\frac{QH'}{2} \\ &\Phi_2^{20} = \Psi_2^{20} = \frac{HR(R'Q + Q_t R)}{2} = \Phi_2^{20} = \Psi_2^{20}, \end{aligned} \tag{18}$$

and Einstein and Moller energy-momentum tensors are obtained by putting the above super potentials in eq. (1) and eq. (10),

$$\begin{aligned} &h\Phi_0^0 = h\Psi_0^0 = -\frac{QH''}{\kappa} \ , \ h\Phi_1^0 = h\Psi_1^0 = \frac{HRR'Q_x}{\kappa} \ , \ h\Phi_2^0 = h\Psi_2^0 = -\frac{RH'(QR' + RQt)}{\kappa} \\ &h\Phi_3^0 = h\Psi_3^0 = 0. \end{aligned} \tag{19}$$

Example 4. Consider the Szezkers class-I space time which is defined by the line element

$$ds^2 = -dt^2 + e^{2B}(dx^2 + dy^2) + e^{2A}dz^2, \tag{20}$$

where $A = A(x,y,z,t)$ and $B = B(x,y,z,t)$.
By putting the metric coefficients in eq. (3) and eq. (7), we obtained the following Einstein and Moller super potential components,

$$\begin{aligned} &\Phi_0^{10} = \Psi_0^{10} = \frac{e^A(A_x + B_x)}{2} \ , \ \Phi_0^{20} = \Psi_0^{20} = \frac{e^A(A_y + B_y)}{2} \ , \ \Phi_0^{30} = \Psi_0^{30} = e^{-A+2B}B \\ &\Phi_1^{10} = \Psi_1^{10} = \frac{e^{A+2B}(-A_t - B_t)}{2} = \Phi_2^{20} = \Psi_2^{20} \ , \ \Phi_1^{21} = \Psi_1^{21} = \frac{e^A A_y}{2}, \\ &\Phi_1^{31} = \Psi_1^{31} = \frac{e^{-A+2B}B_z}{2} = \Phi_2^{32} = \Psi_2^{32}, \\ &\Phi_2^{12} = \Psi_2^{12} = \frac{e^A A_x}{2} \ , \ \Phi_3^{30} = \Psi_3^{30} = -e^{A+2B}B_t \ , \ \Phi_3^{13} = \Psi_3^{13} = \frac{e^A B_x}{2} \ , \ \Phi_3^{23} = \Psi_3^{23} = \frac{e^A B_y}{2} \end{aligned} \tag{21}$$

and Einstein and Moller energy-momentum tensors are obtained by putting the above super potentials in eq. (1) and eq. (10)

$$\begin{aligned} &h\Phi_0^0 = h\Psi_0^0 = h\frac{e^{-A}[2e^{2B}(2B_z^2 - B_z A_z + B_{zz}) + e^{2A}(A_y^2 + A_y B_y + A_{yy} + B_{yy} + A_x^2 + A_x B_x + A_{xx} + B_{xx})}{\kappa} \\ &h\Phi_1^0 = h\Psi_1^0 = \frac{e^{A+2B}[(A_x + 2B_x)(A_t + B_t) + A_{ty} + B_{ty}]}{\kappa} \\ &h\Phi_2^0 = h\Psi_2^0 = \frac{e^{A+2B}[(A_y + 2B_y)(A_t + B_t) + A_{ty} + B_{ty}]}{\kappa} \\ &h\Phi_3^0 = h\Psi_3^0 = \frac{2(e^{A+2B}(A_z B_t + 2B_z B_t + B_{tz}))}{\kappa}. \end{aligned} \tag{22}$$

Example 5. Consider the line element for spatially homogeneous and anisotropic Bianchi type I space time [37] is define by,

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + \Gamma^2dz^2, \tag{23}$$

where A,B and Γ are depends upon t only. By putting the metric coefficients in eq. (3) and eq. (7), we obtained the following Einstein and Moller super potential components

$$\Phi_0^{10}=\Psi_0^{10}=\frac{B_x\Gamma+\Gamma_x B}{2A}\ ,\ \Phi_0^{20}=\Psi_0^{20}=\frac{A_y\Gamma+\Gamma_y A}{2B}\ ,\ \Phi_0^{30}=\Psi_0^{30}=\frac{A_zB+B_zA}{2\Gamma}$$
$$\Phi_1^{10}=\Psi_1^{10}=-\frac{\Gamma(B_t\Gamma+\Gamma_tB)}{2}\ ,\ \Phi_1^{21}=\Psi_1^{21}=\frac{A\Gamma_y}{2B}\ ,\ \Phi_1^{31}=\Psi_1^{31}=\frac{AB_z}{2\Gamma}$$
$$\Phi_2^{20}=\Psi_2^{20}=-\frac{B(A_t\Gamma+\Gamma_tA)}{2}\ ,\ \Phi_2^{12}=\Psi_2^{12}=\frac{B\Gamma_x}{2A}\ ,\ \Phi_2^{32}=\Psi_2^{32}=\frac{BA_z}{2\Gamma}=-\Phi_3^{12}=-\Psi_3^{12}$$
$$\Phi_3^{30}=\Psi_3^{30}=-\frac{\Gamma(A_tB+B_tA)}{2}\ ,\ \Phi_3^{13}=\Psi_3^{13}=\frac{\Gamma B_x}{2A}\ ,\ \Phi_3^{23}=\Psi_3^{23}=\frac{\Gamma A_y}{2B},\tag{24}$$

and Einstein and Moller energy-momentum tensors are obtained by putting the above super potentials in eq. (1) and eq. (10)

$$h\Phi_0^0=h\Psi_0^0=\frac{1}{2A^2B^2\Gamma^2}[A^2\{2B_z^2B_Z+B^2(A_z\Gamma_z+\Gamma)A_{zz})-\Gamma^2Ay_{By}$$
$$+B\Gamma^2(2A_y\Gamma_y+\Gamma A_{yy})\}+A_2\{B_2(\Gamma_zB_{zz}-B_z\Gamma_z+)-\Gamma^2B_y\Gamma_y+$$
$$B\Gamma^2\Gamma_{yy}\}-B^2\Gamma^2A_x(\Gamma^2B_x+B\Gamma_x)+A^2B^2\Gamma^2(2B_x\Gamma_x+_{xx}+B\Gamma_{xx})]$$
$$h\Phi_1^0=h\Psi_1^0=\frac{1}{\kappa}[BA_x\Gamma_t+\Gamma(A_xB_t+AB_{tx})+A(\Gamma_xB_t+B_x\Gamma_t+B\Gamma_{tx})]\tag{25}$$
$$h\Phi_2^0=h\Psi_2^0=\frac{1}{\kappa}[AB_x\Gamma_t+\Gamma(B_yA_t+B_{ty})+B(\Gamma_yA_t+A_y\Gamma_t+A\Gamma_{yt})]$$
$$h\Phi_3^0=h\Psi_3^0=\frac{1}{\kappa}[\Gamma_z(BA_t+AB_t)+\Gamma(B_zA_t+A_zB_t+BA_{tz}+AB_{tz})].$$

Example 6. Consider the metric function,

$$ds^2=-A^2dt^2+B^2dx^2+F^2dy^2+G^2dz^2,\tag{26}$$

where A,B,F and G are the functions of t,x,y and z. By putting the metric coefficients in eq. (3) and eq. (7), we obtained the following Einstein and Moller super potential components

$$\begin{aligned}
\Phi_0^{10}=\Psi_0^{10}&=\frac{A(GF_x+FG_x)}{2B},\Phi_0^{20}=\Psi_0^{20}=\frac{A(GB_y+BG_y)}{2F}\\
\Phi_0^{30}=\Psi_0^{30}&=\frac{A(FB_z+BF_z)}{2G},\Phi_1^{10}=\Psi_1^{10}=\frac{-B(GF_t+FG_t)}{2A}\\
\Phi_2^{21}=\Psi_1^{21}&=\frac{B(GA_y+AG_y)}{2F},\Phi_1^{31}=\Psi_1^{31}=\frac{B(FA_z+AF_z)}{2G}\\
\Phi_2^{20}=\Psi_2^{20}&=\frac{-F(GB_t+BG_t)}{2A},\Phi_2^{12}=\Psi_2^{12}=\frac{F(GA_x+AG_x)}{2B}\\
\Phi_2^{32}=\Psi_2^{32}&=\frac{F(BA_z+AB_z)}{2G},\Phi_3^{30}=\Psi_3^{30}=\frac{-G(FB_t+BF_t)}{2A}\\
\Phi_3^{13}=\Psi_3^{13}&=\frac{G(FA_x+AF_x)}{2B},\Phi_3^{23}=\Psi_3^{23}=\frac{G(BA_y+AB_y)}{2F},
\end{aligned}\tag{27}$$

and Einstein and Moller energy-momentum tensors are obtained by putting the above super potentials in eq. (1) and eq.(10)

$$h\Phi^0_0 = h\Psi^0_0 = \frac{A}{G^2F^2B^2}\{4GF^2B^2(FB_y + BF_y) - 4F^2B^2(FB_z + BF_z)G_z + 4GF^2B^2$$
$$(2B_zF_z + FB_{zz} + BF_{zz}) + 4G^2FB^2(GB_y + BG_y) - 4G^2B^2(GB_y + BG_y)F_y + 4G^2$$
$$FB^2(2B_yGy + GB_{yy} + BG_{yy}) + 4G^2F^2B(GF_x + FG_x) - 4G^2F^2(GF_x + FG_xB_x)+$$
$$4G^2F^2B(2F_xG_x + GF_{xx} + FG_{yy})\}$$
$$h\Phi^0_1 = h\Psi^0_1 = \frac{1}{2A^2}\{AB(GF_t + FG_t) + B(-FA_xG_t + G(-A_xF_t + AF_{tx}) + A(G_xF_t + F_x$$
$$G_t + FG_{tx}))\}$$
$$h\Phi^0_2 = h\Psi^0_2 = \frac{1}{2A^2}\{AF_y(GB_t + BG_t) + F(-BA_yG_t + G(-A_yB_t + AB_{ty}) + A(G_yB_t + B_y$$
$$G_t + BG_{ty}))\}$$
$$h\Phi^0_3 = h\Psi^0_3 = \frac{1}{2A^2}\{-GA_z(FB_t + BF_t) + AG_z(FB_t + BF_t) + AG(F_zB_t + B_zF_t + FB_{tz}+$$
$$BF_{tz})\}. \qquad (28)$$

Example 7. Now let us consider the special type of Ferrari-Ibanez [38] degenerated spacetime is given by,

$$ds^2 = (1 + \delta sint)^2(dt^2 - dz^2) - \frac{1 - \delta sint}{1 + \delta sint}dx^2 - cos^2z(1 + \delta sint)^2dy^2, \qquad (29)$$

where δ is an arbitrary constant. By putting the metric coefficients in eq. (3) and eq. (7), we obtained the following Einstein and Moller super potential components

$$\begin{aligned}
\Phi^{30}_0 = \Psi^{30}_0 &= \frac{-\sqrt{\frac{1-\delta sint}{1+\delta sint}}(1 + \delta sintcost)}{2}\\
\Phi^{10}_1 = \Psi^{10}_1 &= \delta sintcosz\sqrt{\frac{1 - \delta sint}{1 + \delta sint}}\\
\Phi^{31}_1 = \Psi^{31}_1 &= \frac{-\sqrt{1 - \delta^2 sin^2 t}}{2}\\
\Phi^{20}_2 = \Psi^{20}_2 = \Phi^{30}_3 = \Psi^{30}_3 &= \frac{\delta sintcosz\{\sqrt{1 - \delta^2 sin^2 t} - 2(\sqrt{\frac{1-\delta sint}{1+\delta sint}})\}}{2(1 + \delta sint)},
\end{aligned} \qquad (30)$$

and Einstein and Moller energy-momentum tensors are obtained by putting the above super potentials in eq. (1) and eq. (10)

$$\begin{aligned}
h\Phi^0_0 = h\Psi^0_0 &= -\frac{cosz\sqrt{1 - \delta^2 sin^2 t}}{\kappa}\\
h\Phi^0_3 = h\Psi^0_3 &= -\frac{\delta sintcosz\{\sqrt{1 - \delta^2 sin^2 t} - 2(\sqrt{\frac{1-\delta sint}{1+\delta sint}})\}}{\kappa(1 + \delta sint)}\\
h\Phi^0_1 = h\Psi^0_1 &= h\Phi^0_2 = h\Psi^0_2 = 0.
\end{aligned} \qquad (31)$$

Example 8. Consider the general Bianchi type diagonal space-time,

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{2\alpha x}dy^2 + H^2e^{2\beta x}dz^2, \qquad (32)$$

where A,B and F are only depends on t. and α,β are constants. Now by putting the metric coefficients in eq. (3) and eq. (7), we obtained the following Einstein and Moller super potential components

$$\begin{aligned}
\Phi^{10}_1 = \Psi^{10}_1 &= -\frac{1}{2}e^{\alpha x+\beta x}A(HB_t + H_tB)\\
\Phi^{20}_2 = \Psi^{20}_2 &= -\frac{1}{2}e^{\alpha x+\beta x}B(HA_t + H_tA)\\
\Phi^{30}_3 = \Psi^{30}_3 &= -\frac{1}{2}e^{\alpha x+\beta x}H(BA_t + B_tA),
\end{aligned} \qquad (33)$$

and Einstein and Moller energy-momentum tensors are obtained by putting the above super potentials in eq. (1) and eq. (10)

$$h\Phi^0_i = h\Psi^0_i = 0 \quad ; \quad i = 0, 1, 2, 3. \qquad (34)$$

Example 9. Now by putting $\alpha = \beta = 0$ in eq. (32) which describe the well known diagonal space-time of Bianchi type I_0,

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + H^2dz^2, \tag{35}$$

where A,B and F are only depends on t. Now by putting the metric coefficients in eq. (3) and eq. (7), we obtained the following Einstein and Moller super potential components

$$\begin{aligned} \Phi_1^{10} = \Psi_1^{10} &= \frac{2A(HB_t + BH_t)}{\kappa} \\ \Phi_2^{20} = \Psi_2^{20} &= -\frac{1}{\kappa}B(HA_t + AH_t) \\ \Phi_3^{30} = \Psi_3^{30} &= -\frac{1}{\kappa}H(BA_t + AB_t) \end{aligned} \tag{36}$$

and Einstein and Moller energy-momentum tensors are obtained by putting the above super potentials in eq. (1) and eq. (10)

$$h\Phi_i^0 = h\Psi_i^0 = 0 \quad ; \quad i = 0, 1, 2, 3. \tag{37}$$

Example 10. By putting $\alpha = -1$,$\beta = 0$ in eq. (32),we obtained the diagonal space-time which describe the well known Bianchi type III, which is given as,

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{-2x}dy^2 + H^2dz^2, \tag{38}$$

where the functions A,B and H are only depends on t. Now by putting the metric coefficients in eq. (3) and eq. (7), we obtained the following Einstein and Moller super potential components,

$$\begin{aligned} &\Phi_0^{10} = \Psi_0^{10} = \frac{e^{-x}BH}{2A} = \Phi_3^{13} = \Psi_3^{13} \ ; \ \Phi_1^{10} = \Psi_1^{10} = -\frac{1}{2}e^{-x}A(HB' + BH') \\ &\Phi_2^{20} = \Psi_2^{20} = -\frac{1}{2}e^{-x}B(HA' + AH') \ ; \ \Phi_3^{30} = \Psi_3^{30} = -\frac{1}{2}e^{-x}H(BA' + AB'), \end{aligned} \tag{39}$$

and Einstein and Moller energy-momentum tensors are obtained by putting the above super-potentials in eq. (1) and eq. (10),

$$h\Phi_0^0 = h\Psi_0^0 = \frac{BHe^x}{\kappa A} \ ; \ \Phi_1^0 = \Psi_1^0 = \frac{e^x A(HB_t + A(BH_t))}{\kappa} \ , \ h\Phi_2^0 = h\Psi_2^0 = h\Phi_3^0 = h\Psi_3^0 = 0. \tag{40}$$

Example 11.Now if the space-time is non-diagonal in the context of tele-parallel gravity the Einstein $\Phi_\mu^{[i\alpha]}$ and Moller $\Psi_\mu^{[i\alpha]}$ super potentials and energy momentum densities are same through the relation $\Psi_\mu^{i\alpha} = \Phi_\mu^{i\alpha} + \lambda\Delta_\mu^{i\alpha}$, where $\Delta_\mu^{i\alpha} = (\frac{h}{2})g_{\mu k}(K^{[ki]\alpha} + K^{[\alpha k]i} + K^{[i\alpha]k})$ and the Einstein and Moller energy momentum densities is$h\Psi_\mu^i = h\Phi_\mu^i + \lambda\Delta_\mu^i$ if the super potential difference $\Delta_\mu^{[i\alpha]} = 0$.Now for this we consider the non diagonal space time is given as,

$$ds^2 = Adt^2 - e^{2B}dx^2 - e^{2B}dy^2 - Hdz^2 + 2Gdzdt, \tag{41}$$

where A, B, H and G are the functions of t, x. After some lengthy calculations, we obtained the non vanishing components of the super potentials $\Delta_\mu^{i\alpha}$, $\Phi_\mu^{i\alpha}$ and

$\Psi_{\mu}^{i\alpha}$ are,

$$\Delta_0^{10} = \frac{1}{8\sqrt{A}\sqrt{H}\left(G^2+AH\right)}\left(\sqrt{G}\left(A\left(\sqrt{A}+\sqrt{H}\right)HG_x+G^{3/2}\left(-HA_x+AH_x\right)-G\right.\right.$$
$$\left.\left.\left(H^{3/2}A_x+At^{3/2}H_x\right)\right)\right),$$
$$\Delta_0^{13} = \frac{1}{8\sqrt{A}\sqrt{G}\left(G^2+AH\right)}\left(\sqrt{H}\left(-A\left(\sqrt{A}+\sqrt{H}\right)HG_x+G^{3/2}\left(HA_x-AH_x\right)+G\right.\right.$$
$$\left.\left.\left(H^{3/2}A_x+A^{3/2}H_x\right)\right)\right)=-\Delta_1^{03},$$
$$\Delta_0^{31} = -\frac{1}{8\sqrt{A}\sqrt{G}\sqrt{H}\left(G^2+AH\right)}\left(e^{2B}\left(-A\left(\sqrt{A}+\sqrt{H}\right)HG_x+\right.\right.$$
$$\left.\left.G^{3/2}\left(HA_x-AH_x\right)+G\left(H^{3/2}A_x+A^{3/2}H_x\right)\right)\right)=-\Delta_3^{01},$$
$$\Delta_1^{30} = -\frac{1}{8\sqrt{G}\sqrt{H}\left(G^2+AH\right)}\left(\sqrt{A}\left(-A\left(\sqrt{A}+\sqrt{H}\right)HG_x+G^{3/2}\left(HA_x-AH_x\right)+G\right.\right.$$
$$\left.\left.\left(H^{3/2}A_x+A^{3/2}H_x\right)\right)\right),$$
$$\Delta_3^{13} = \frac{1}{8\sqrt{A}\sqrt{H}\left(G^2+AH\right)}\left(\sqrt{G}\left(-A\left(\sqrt{A}+\sqrt{H}\right)HG_x+G^{3/2}\left(HA_x-AH_x\right)+G\right.\right.$$
$$\left.\left.\left(H^{3/2}A_x+A^{3/2}H_x\right)\right)\right),$$

$$\Phi_0^{10} = \frac{1}{\left(8\sqrt{A}\left(-G+\sqrt{A}\sqrt{H}\right)\sqrt{H}\left(G^2+AH\right)\right)}\left(e^{-2B}\left(-e^{2B}G+e^{2B}\sqrt{A}\sqrt{H}\right)\left(4\sqrt{A}\right.\right.$$
$$G^3\sqrt{H}log(e)B_x+4A^{3/2}GH^{3/2}log(e)B_x+A\sqrt{G}\left(\sqrt{A}+\sqrt{H}\right)HG_x+2A^{3/2}H^{3/2}G_x-$$
$$2A^2H\left(2Hlog(e)\ B_x+H_x\right)-G^2\left(H\left(A_x+4Alog(e)B_x\right)-2\sqrt{A}\sqrt{H}G_x+AH_x\right)-G^{3/2}$$
$$\left.\left(H^{3/2}A_x+A^{3/2}\ H_x)\right)\right),$$
$$\Phi_0^{13} = \frac{1}{\left(8\sqrt{A}\sqrt{G}\left(-G+\sqrt{A}\sqrt{H}\right)\sqrt{H}\left(G^2+AH\right)\right)}\left(e^{-2B}\left(-e^{2B}G+e^{2B}\sqrt{A}\sqrt{H}\right)\left(2\sqrt{A}\right.\right.$$
$$G^2HG_x+A\left(\sqrt{A}-\sqrt{H}\right)H^2G_x-2\sqrt{A}G^3H_x+G^{3/2}H\left(HA_x-AH_x\right)+G\left(H^{5/2}A_x-A^{3/2}\right.$$
$$HH_x))),$$
$$\Phi_0^{30} = \Psi_0^{30} = \frac{2G\left(-e^{2B}G+e^{2B}\sqrt{A}\sqrt{H}\right)log(e)B_t}{2\left(G^2+AH\right)},$$
$$\Phi_0^{31} = -\frac{1}{\left(8\sqrt{A}\sqrt{G}\left(-G+\sqrt{A}\sqrt{H}\right)\sqrt{H}\left(G^2+AH\right)\right)}\left(\left(-e^{2B}G+e^{2B}\sqrt{A}\sqrt{H}\right)\right.$$
$$\left.\left(A\left(\sqrt{A}+\sqrt{H}\right)HG_x+G^{3/2}\left(-HA_x+AH_x\right)-G\left(H^{3/2}A_x+A^{3/2}H_x\right)\right)\right),$$
$$\Phi_1^{10} = \Psi_1^{10} = -\frac{1}{\left(4\left(-G+\sqrt{A}\sqrt{H}\right)\sqrt{H}\left(G^2+AH\right)\right)}\left(\left(-e^{2B}G+e^{2B}\sqrt{A}\sqrt{H}\right)\left(2GH^{3/2}log(e)B_t+\right.\right.$$
$$\left.\left.\sqrt{G}HG_t-G^{3/2}H_t+H\left(\sqrt{H}G_t-\sqrt{A}\left(2Hlog(e)B_t+H_t\right)\right)\right)\right),$$
$$\Phi_1^{03} = \Psi_1^{03} = 1\left(8\sqrt{A}\sqrt{G}\left(-G+\sqrt{A}\sqrt{H}\right)\sqrt{H}\left(G^2+AH\right)\right)\left(e^{-2B}\left(-e^{2B}G+e^{2B}\sqrt{A}\sqrt{H}\right)\left(-2\sqrt{A}\right.\right.$$
$$G^2HG_x+A\left(-\sqrt{A}+\sqrt{H}\right)H^2G_x+2\sqrt{A}G^3H_x+G^{3/2}H\left(-HA_x+AH_x\right)+G\left(-H^{5/2}A_x+\right.$$
$$\left.\left.\left.A^{3/2}HH_x\right)\right)\right),$$
$$\Phi_1^{30} = \frac{1}{\left(8\sqrt{A}\sqrt{G}\left(-G+\sqrt{A}\sqrt{H}\right)\sqrt{H}\left(G^2+AH\right)\right)}\left(e^{-2B}\left(-e^{2B}G+e^{2B}\sqrt{A}\sqrt{H}\right)\left(2G^3\sqrt{H}\right.\right.$$
$$\left.\left.A_x-2AG^2\sqrt{H}G_x+A^2\left(\sqrt{A}-\sqrt{H}\right)HG_x+AG^{3/2}\left(-HA_x+AH_x\right)+G\left(AH^{3/2}A_x-A^{5/2}H_x\right)\right)\right),$$
$$\Phi_1^{31} = \frac{1}{\left(4\sqrt{G}\left(G-\sqrt{A}\sqrt{H}\right)\sqrt{H}\left(G^2+AH\right)\right)}-\left(\left(-e^{2B}G+e^{2B}\sqrt{A}\sqrt{H}\right)\left(-2G^{5/2}\sqrt{H}log(e)B_t+\right.\right. = \Psi_1^{31},$$

$$AHG_t - AGH_t + G^{3/2}\left(-\sqrt{H}G_t + \sqrt{A}\left(2Hlog(e)B_t + H_t\right)\right)\Big)\Big),$$

$$\Phi_2^{20} = -\frac{1}{\left(4\left(-G+\sqrt{A}\sqrt{H}\right)\sqrt{H}\left(G^2+AH\right)\right)}\left(\left(-e^{2B}G + e^{2B}\sqrt{A}\sqrt{H}\right)\left(2GH^{3/2}log(e)B_t + \sqrt{G}H\right.\right.$$

$$\left.\left.G_t - G^{3/2}H_t + H\left(\sqrt{H}G_t - \sqrt{A}\left(2Hlog(e)B_t + H_t\right)\right)\right)\right) = \Psi_2^{20},$$

$$\Phi_2^{12} = \Psi_2^{12} = -\frac{2\left(-e^{2B}G + e^{2B}\sqrt{A}\sqrt{H}\right)\left(HA_x - 2\sqrt{A}\sqrt{H}G_x + AH_x\right)}{2\left(-4e^{2B}\sqrt{A}G\sqrt{H} + 4e^{2B}AH\right)},$$

$$\Phi_2^{32} = -\frac{1}{\left(4\sqrt{G}\left(G-\sqrt{A}\sqrt{H}\right)\sqrt{H}\left(G^2+AH\right)\right)}\left(\left(-e^{2B}G + e^{2B}\sqrt{A}\sqrt{H}\right)\left(-2G^{5/2}\sqrt{H}log(e)B_t + \right.\right.$$

$$\left.\left.AHG_t - AGH_t + G^{3/2}\left(-\sqrt{H}G_t + \sqrt{A}\left(2Hlog(e)B_x + H_x\right)\right)\right)\right) = \Psi_2^{32},$$

$$\Phi_3^{01} = -\frac{1}{\left(8\sqrt{A}\sqrt{G}\left(-G+\sqrt{A}\sqrt{H}\right)\sqrt{H}\left(G^2+AH\right)\right)}\left(\left(-e^{2B}G + e^{2B}\sqrt{A}\sqrt{H}\right)\right.$$

$$\left.\left(-A\left(\sqrt{A}+\sqrt{H}\right)HG_x + G^{3/2}\left(HA_x - AH_x\right) + G\left(H^{3/2}A_x + A^{3/2}H_x\right)\right)\right),$$

$$\Phi_3^{30} = \frac{2\left(-e^{2B}G + e^{2B}\sqrt{A}\sqrt{H}\right)Hlog(e)B_t}{2\left(G^2+AH\right)},$$

$$\Phi_3^{10} = \frac{1}{\left(8\sqrt{A}\sqrt{G}\left(-G+\sqrt{A}\sqrt{H}\right)\sqrt{H}\left(G^2+AH\right)\right)}\left(e^{-2B}\left(-e^{2B}G + e^{2B}\sqrt{A}\sqrt{H}\right)\left(-2G^3\sqrt{H}A_x + \right.\right.$$

$$\left.\left.2AG^2\sqrt{H}G_x + A^2\left(-\sqrt{A}+\sqrt{H}\right)HG_x + AG^{3/2}\left(HA_x - AH_x\right) + G\left(-AH^{3/2}Ax + A^{5/2}H_x\right)\right)\right) = \Psi_3^{10},$$

$$\Phi_3^{13} = \frac{1}{\left(8\sqrt{A}\left(-G+\sqrt{A}\sqrt{H}\right)\sqrt{H}\left(G^2+AH\right)\right)}\left(e^{-2B}\left(-e^{2B}G + e^{2B}\sqrt{A}\sqrt{H}\right)\left(-2AH^2A_x + \right.\right.$$

$$4\sqrt{A}G^3\sqrt{H}log(e)B_x + 4A^{3/2}GH^{3/2}log(e)B_x - 4A^2H^2log(e)B_x - A\sqrt{G}\left(\sqrt{A}+\sqrt{H}\right)HG_x +$$

$$\left.\left.2A^{3/2}H^{3/2}G_x - G^2\left(H\left(A_x + 4Alog(e)B_x\right) - 2\sqrt{A}\sqrt{H}G_x + AH_x\right) + G^{3/2}\left(H^{3/2}A_x + A^{3/2}\ H_x\right)\right)\right),$$

and the energy-momentum components $\Delta_\mu^i, h\Phi_\mu^i$ and $h\Psi_\mu^i$ are,

$$h\Phi_0^0 = \frac{1}{\left(8\kappa A^{3/2}\sqrt{G}H^{3/2}\left(G^2+AH\right)^2\right)}\left(A^3\left(\sqrt{A}+\sqrt{H}\right)H^3G_x^2 + 8A^{3/2}G^{11/2}H^{3/2}log(e)B_{xx} + 16\right.$$

$$A^{5/2}G^{7/2}H^{5/2}log(e)B_{xx} + 4A^2G^{3/2}H^2\left(G_x\left(-HA_x + AH_x\right) + 2A^{3/2}H^{3/2}log(e)B_{xx}\right) + 2AG^3\left(\sqrt{A}\right.$$

$$\left.+\sqrt{H}\right)H\left(G_x\left(HA_x + AH_x\right) + AHG_{xx}\right) + 2A^2GH^2\left(-2G_x\left(H^{3/2}A_x + A^{3/2}H_x\right) + A\left(\sqrt{A}+\sqrt{H}\right)\right.$$

$$\left.HG_{xx}\right) + AG^{5/2}H\left(A^2H_x^2 + H^2\left(3A_x^2 - 8Alog(e)A_xB_x - 2A\left(A_{xx} + 8Alog(e)B_{xx}\right)\right) + 8A^{3/2}H^{3/2}\right.$$

$$\left(2log(e)B_xG_x + G_{xx}\right) + 2AH\left(-2\left(A_x + 2Alog(e)B_x\right)H_x - 3AH_{xx}\right)\Big) + G^4\left(-2AH^{3/2}A_xH_x + A^{5/2}H_x^2\right.$$

$$\left.+H^{5/2}\left(A_x^2 - 2AA_{xx}\right) - 2A^{3/2}H\left(A_xH_x + AH_{xx}\right)\right) + G^{9/2}\left(A^2H_x^2 + H^2\left(A_x^2 - 4Alog(e)A_xB_x - 2A\right.\right.$$

$$\left(A_{xx} + 4Alog(e)B_{xx}\right)\Big) + 4A^{3/2}H^{3/2}\left(2log(e)B_xG_x + G_{xx}\right) - 2AH\left(\left(A_x + 2Alog(e)B_x\right)H_x\right.$$

$$\left.+A\ H_{xx}\right)\Big) - AG^2H\left(3AH^{3/2}G_x^2 - 3A^{5/2}H_x^2 + H^{5/2}\left(-3A_x^2 + 2AA_{xx}\right) + A^{3/2}H\left(3G_x^2 + 2AH_{xx}\right)\right)$$

$$-2A^3\sqrt{G}H^2\left(-AH_x^2 + 2H^2log(e)\left(A_xB_x + 2AB_{xx}\right) - 2\sqrt{A}\ H^{3/2}\left(2log(e)B_xG_x + G_{xx}\right) + H\right.$$

$$\left.\left(\left(A_x + 2Alog(e)B_x\right)H_x + 2AH_{xx}\right)\right)\Big),$$

$$\Psi_0^0 = \Phi_0^0 + \lambda\Delta,$$

where Δ is use for all coefficients of λ contains in $h\Psi_0^0$,

$$
\begin{aligned}
h\Phi_1^0 = &\frac{1}{\left(4\kappa\sqrt{A}\sqrt{G}H^{3/2}\left(G^2+AH\right)^2\right)}\left(e^{2B}\left(A^{3/2}H^3G_xG_t+4\sqrt{A}G^{7/2}H^{3/2}log(e)\left(\left(2Hlog(e)B_x\right.\right.\right.\right.\\
&\left.+H_x\right)B_t+HB_{tx}\right)+4\sqrt{A}G^{3/2}H^2\left(2\sqrt{A}Hlog(e)G_xB_t-\sqrt{H}G_xG_t+\sqrt{A}G_xH_t+H^{3/2}log(e)\left(\right.\right.\\
&-\left(A_x-2Alog(e)B_x\right)B_t+AB_{tx}))+\sqrt{A}G^3H\left(\left(4Hlog(e)B_x+H_x\right)G_t+G_xH_t+2HG_{tx}\right)\\
&+\sqrt{A}GH^2\left(-A\left(H_xG_t+3G_xH_t\right)+2H\left(-\left(A_x-2Alog(e)B_x\right)G_t+AG_{tx}\right)\right)+\sqrt{A}G^4\left(\left(-4Hlog(e)B_x\right.\right.\\
&\left.+H_x\right)H_t-2HH_{tx})+\sqrt{A}G^2H\left(3AH_xH_t-H\left(3G_xG_t-2\left(A_x-2Alog(e)B_x\right)H_t+2AH_{tx}\right)\right)\\
&+\sqrt{G}\left(-2\sqrt{A}H^{7/2}A_xG_t+AH^3A_x\left(2Hlog(e)B_t+H_t\right)+2A^{3/2}H^{7/2}\left(2log(e)\left(G_xB_t+B_xG_t\right)+G_{tx}\right)\right.\\
&\left.-A^2H^2\left(-H_xH_t4H^2log(e)\left(2log(e)B_tB_t+B_{tx}\right)+2H\left(log(e)H_xB_t+2log(e)B_xH_t+H_{tx}\right)\right)\right)\\
&-G^{5/2}H\left(-2\sqrt{A}\sqrt{H}H_xG_x+AH_xH_t+2H^2log(e)\left(\left(A_x+4Alog(e)B_x\right)B_x+2AB_{tx}\right)\right.\\
&+2\sqrt{A}H^{3/2}\left(2log(e)G_xB_x-2log(e)B_tG_x-G_{tx}\right)+H\left(A_xH_x+2A\left(3log(e)H_xB_x\right.\right.\\
&\left.\left.\left.+2log(e)\;\;B_xH_t+H_{tx}\right)\right)\right)),
\end{aligned}
$$

where Δ is use for all coefficients of λ contains in $h\Psi_1^0$,

$$h\Phi_2^0 = h\Psi_2^0 = 0,$$

$$
\begin{aligned}
h\Phi_3^0 = &\frac{1}{\left(8\kappa A^{3/2}G^{3/2}H^{3/2}\left(G^2+AH\right)^2\right)}\left(A^3\left(-\sqrt{A}+\sqrt{H}\right)H^4G_x^2\right.\\
&+2A^2G^{3/2}H^3G_x\left(HA_x-AH_x\right)+2AG^{7/2}H^2G_x\left(-HA_x+AH_x\right)+4A^{3/2}G^5H^2G_{xx}\\
&+2A^2GH^{7/2}\left(G_x\left(HA_x-AH_x\right)+A\left(\sqrt{A}-\sqrt{H}\right)\sqrt{H}G_{xx}\right)+2AG^3H^2\left(G_x\left(-\left(\sqrt{A}+2\sqrt{H}\right)HA_x\right.\right.\\
&\left.\left.-A\left(3\sqrt{A}+2\sqrt{H}\right)H_x\right)+A\left(3\sqrt{A}-\sqrt{H}\right)HG_{xx}\right)+2A^{3/2}G^6\left(H_x^2-2HH_{xx}\right)\\
&+AG^2H^2\left(A^{5/2}H_x^2+AH^{3/2}\left(5G_x^2+2A_xH_x\right)+H^{5/2}\left(-3A_x^2+2AA_{xx}\right)+A^{3/2}H\left(G_x^2-2AH_{xx}\right)\right)\\
&+AG^{5/2}H^2\left(A^2H_x^2+H^2\left(-3A_x^2+2AA_{xx}\right)+2AH\left(A_xH_x-AH_{xx}\right)\right)\\
&-G^{9/2}H\left(A^2H_x^2+H^2\left(A_x^2-2AA_{xx}\right)+2AH\left(-A_xH_x+AH_{xx}\right)\right)\\
&+G^4H\left(4AH^{3/2}A_xH_x+5A^{5/2}H_x^2-H^{5/2}\left(A_x^2-2AA_{xx}\right)\right.\\
&\left.\left.-2A^{3/2}\;H\left(G_x^2-A_xH_x+3AH_{xx}\right)\right)\right),
\end{aligned}
$$

$$h\Psi_3^0 = h\Psi_3^0 + \lambda\Delta,$$

where Δ is use for all coefficients of λ contains in Ψ_3^0.

III. Discussion

The solution of the problem of energy-momentum [39] localization is a great challenge for many researchers but still unsolved. A number of scientist in GR and TG interested in the solution of localization of energy-momentum problem and try to provide a unique way out which gives same result for all space time,but they could not provide a distinctive solution. Now in this paper inquire into the work of *ali et al* [31],they showed energy-momentum equivalent prescriptions of Einstein as well as Tolman in GR. They established that energy and momen-tum complexes formulated by Einstein and Tolman in GR are similar provided any space-time within the framework of GR that extend all space-time config- urations, show equality. Additionally, they proved that the general properties of Einstein and Tolman super-potentials vary. Therefore keeping in views these points we will show that Einstein and Moller energy-momentum complexes and super-potentials gives similar results for different space-times in TG. For this specifically we shall take different metrics and verify that these results will be also gives same results of the energy-momentum distributions for both Einstein and Moller energy-momentum distributions in the context of TG, which foundin the previous work of *ali et al* in GR. In example 1 to 10 we consider differ- ent types metrics which gives same prescriptions for both Einstein and Moller energy-momentum distributions. In example 11 we considered a non-diagonal space-time, which also gives the same energy-momentum distributions for both Einstein and Moller complexes by taking the tensor $\Delta^{i\alpha} = 0$. From the above work we have concluded that Einstein and Moller energy-momentum complexesare same for any space-time in teleparallel gravity theory.

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