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Dynamically elastic-deformed displacements of the loaded coiled cylindrical spring

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Abstract
The dynamic elastic-deformational displacements of a loaded helical cylindrical spring are examined. A mathematical model of motions in two directions has been derived: bending and twisting its cross-sections. An initial-boundary problem is formulated based on the consequent conditions. The system of partial differential equations has been fully integrated. Exact solutions of the displacement laws of the spring's median line and the twisting angles of its cross-sections have been obtained in the form of mathematical formulas. This will allow the research engineer to quickly determine the forms of both bending and twisting oscillations, the frequencies of forms and dynamic oscillations, the maximum movement stroke, and visually observe the deformability of the structure taking into account the fixation of the ends in any symbolic algebra package.
Keywords: *bending, torsion, forces, moments, displacements, angles of rotation the cross section, cylindrical spring, homogeneous elastic material, dynamic stress-strain state.*

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NOMENCLATURE

Symbol	**Description**	**Unit**
r	spring wire cross-section radius	m
R	spring radius	m
E	Young modulus	N/m^2
G	shear modulus	N/m^2
J_k	moment of inertia of the cross-sectional area of the wire	m^4
J_1	moment of inertia of the cylinder area	m^4
m_0	mass per unit length of spring	kg/m
w	transverse displacement of the spring wire centerline	m
p	dynamic frequency of the external load mode	rad/s

Greek letter

Symbol	**Description**	**Unit**
ρ	spring material density	kg/m^3
β	angle of rotation the spring wire cross section	rad
η	number of coils of the cylindrical spring	dimensionless value
ψ_1	external dimensionless loads	dimensionless value
ψ_2	external dimensionless loads	dimensionless value

I. INTRODUCTION

Thin rod elements are successfully used in many technical devices under conditions of periodic dynamic modes. The rising useful and harmful oscillations have a significant impact on the strength, reliability, and durability of the structure. The ability to qualitatively describe dynamic stress-deformed states and derive analytical solutions are relevant problems of modern mechanics. The elastic spring is used in many industries: oil and gas, mining, energy, aerospace, etc. Calculations of rod elements, plates, and ring elastic devices are known from courses of material resistance, elasticity theory, and other sections of mechanics. Among the authors, one can note Alfutov N.A., Balabukha L.I., Biderman V.L., Birger I.A., Svetlitsky V.A. and others. These scientists hold the lead in modeling, schematizing, and problem sets for the dynamics of stress-deformed states of rings and cylindrical springs, in the study of preliminary solutions, decomposed into series according to orthogonal function systems, in the use of approximate methods of a small parameter, harmonic balance, etc., in the analysis of frequency equations [1-5].

Stress-deformed states of a helical cylindrical spring made of homogeneous elastic material are considered. It is one of the main structural elements in many technical devices since it participates in the damping of vertical oscillations and softens impacts from extreme external forces. The spring has several functional properties - it provides stable support for power units and must be endowed with optimal rigidity. Such a feature is obliged to create conditions for the best damping of harmful vibrations, stable manageability, and smooth movements during overloads.

To form a system of motion equations, we consider the equilibrium of element ds of a helical cylindrical spring with a helix radius R and a wire radius r, which is in a complex loading mode, Fig.1. In the general case of loading, the spring experiences bending of the wire's median line $w(\varphi,t)$, twisting in the wire's cross-sectionsβ (φ,t) Fig.2, longitudinal and radial extension - compression in the planes of its turns. With sufficiently large magnitudes of applied loads, the spring form may become unstable. In setting the dynamic problem of elasticity theory, the hypothesis of a flat section is accepted during twisting and bending, and the hypothesis of non-compression of layers. Longitudinal and radial extension-compression are independent tasks and are not given here.

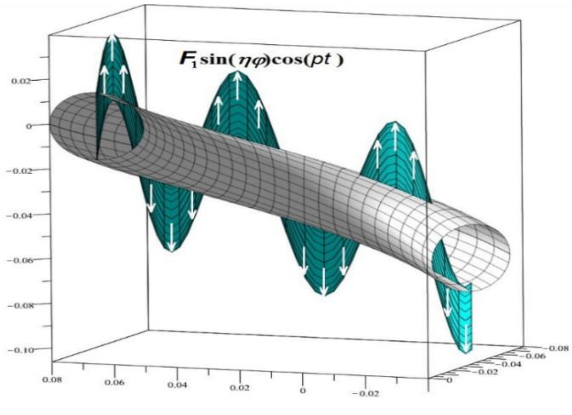
The dynamic equations of motion of the spring element ds are as follows:

$$\frac{1}{R}\frac{\partial^4 w}{\partial\varphi^4}-\frac{GJ_k}{EJ_1 R}\frac{\partial^2 w}{\partial\varphi^2}+\frac{m_0 R^3}{EJ_1}\frac{\partial^2 w}{\partial t^2}+\left(1+\frac{GJ_k}{EJ_1}\right)\frac{\partial^2\beta}{\partial\varphi^2}+\frac{m_0 R^3 r}{EJ_1}\frac{\partial^2\beta}{\partial t^2}=\psi_1\sin(\eta\varphi)\cos(pt), \tag{1}$$

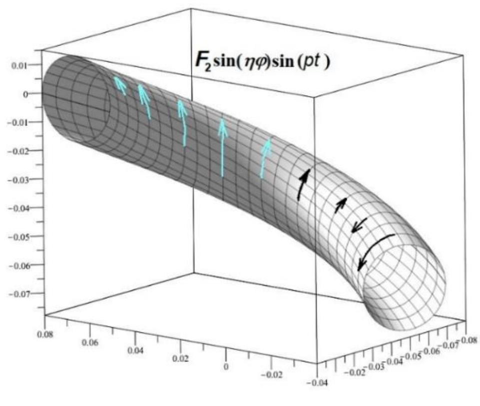
$$\left(1+\frac{GJ_k}{EJ_1}\right)\frac{1}{R}\frac{\partial^2 w}{\partial\varphi^2}-\frac{GJ_k}{EJ_1}\frac{\partial^2\beta}{\partial\varphi^2}+\beta=\psi_2\sin(\eta\varphi)\sin(pt), \tag{2}$$

here η - any non-integer real positive number, p - the frequency of external influences on the spring, ψ_1 and ψ_2 - external dimensionless loads, the spring has 7.125 turns, the angle φ varies from 0 to 14.25 radians Fig. 1, 2.

A mathematical model (1), (2) of bending displacements of points of the contacting plane to the turns and twists, twisting in the spring's cross-sections, has been obtained [1-5].



a)



b)

Figure 1: Loading mode. Bending on *a*, twisting cross-sections on *b*

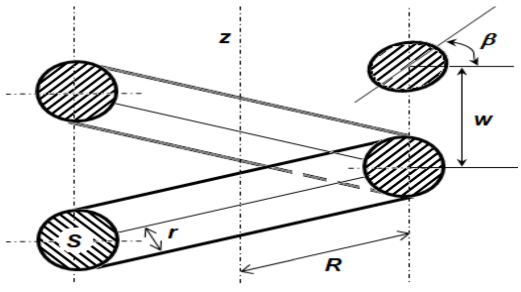


Figure 2: Scheme of parameters used in a mathematical mode

II. DERIVATION OF THE LAWS OF MOTION FOR A CYLINDRICAL SPRING

The canonical representation of the general system of dynamics equations of the stress-deformed state of the spring with a special loading mode [5 - 10]:

$$L(w(\varphi,t)) = Q_1(\varphi,t),\quad L(\beta(\varphi,t)) = Q_2(\varphi,t), \tag{3}$$

here the differential operator

$$L(\square) = \frac{GJ_k}{EJ_1R}\partial^6_{\varphi\varphi\varphi\varphi\varphi\varphi}(\square) + 2\frac{GJ_k}{EJ_1R}\partial^4_{\varphi\varphi\varphi\varphi}(\square) + m_0R^3\frac{\left(GJ_k(R+r)+EJ_1r\right)}{E^2J_1^{\,2}R}\partial^4_{\varphi\varphi tt}(\square) + \frac{GJ_k}{EJ_1R}\partial^2_{\varphi\varphi}(\square) - \frac{m_0R^3}{EJ_1}\partial^2_{tt}(\square), \tag{4}$$

and the right sides of formulas (3) are such

$$Q_1(\varphi,t) = -\left(p^2r\frac{m_0R^3}{EJ_1} + \left(1+\frac{GJ_k}{EJ_1}\right)\eta^2\right)\psi_2\sin(\eta\varphi)\sin(pt) - \left(1+\frac{GJ_k}{EJ_1}\eta^2\right)\psi_1\sin(\eta\varphi)\cos(pt), \tag{5}$$

$$Q_2(\varphi,t) = \left(p^2\frac{m_0R^3}{EJ_1} - \frac{1}{R}\eta^4 - \frac{GJ_k}{EJ_1R}\eta^2\right)\psi_2\sin(\eta\varphi)\sin(pt) - \frac{1}{R}\left(1+\frac{GJ_k}{EJ_1}\right)\eta^2\psi_1\sin(\eta\varphi)\cos(pt). \tag{6}$$

By the method of variable separation from the general system (3), a homogeneous system of differential equations with a zero right-hand side is selected

$$L(w^o(\varphi,t)) = 0,\quad L(\beta^o(\varphi,t)) = 0, \tag{7}$$

from which two independent equations are obtained. At the same time, $w^o(\varphi,t) = X_1(\varphi)T_1(t)$, $\beta^o(\varphi,t) = X_2(\varphi)T_1(t)$ are solutions of the homogeneous system (7)

$$X_j^{(6)} + 2X_j^{(4)} + \left(1-\lambda\left((R+r)\frac{GJ_k}{EJ_1R} + \frac{r}{R}\right)\right)X_j'' + \lambda X_j = 0,\quad j = 1,2, \tag{8}$$

$$T_j'' + \lambda\frac{GJ_k}{R^4m_0}T_j = 0, \tag{9}$$

here λ – an arbitrary positive constant, $\lambda > 0$ [5-16], $\delta = \left(\sqrt{\lambda GJ_k/m_0}\right)/R^2$.

We will choose the initial conditions for the homogeneous system (7) as follows:

$$w^o(\varphi,0) = \theta_1(\varphi),\qquad \beta^o(\varphi,0) = \theta_2(\varphi), \tag{10}$$

$$\partial_t(w^o(\varphi,0)) = 0,\qquad \partial_t(\beta^o(\varphi,0)) = 0. \tag{11}$$

Functions $\theta_1(\varphi)$ and $\theta_2(\varphi)$ are based on known experimental data or as a solution to a problem with unknown initial conditions.

As boundary conditions, we use the requirement of the upper fixed end turn:

$$w^o(0,t) = 0,\qquad \partial_\varphi(w^o(0,t)) = 0, \tag{12}$$

$$\partial^2_{\varphi\varphi}\left(w^o(0,t)\right) = 0,\qquad \partial^3_{\varphi\varphi\varphi}\left(w^o(0,t)\right) = 0, \tag{13}$$

$$\beta^o(0,t) = 0,\qquad \partial_\varphi\left(\beta^o(0,t)\right) = 0. \tag{14}$$

These conditions allow determining the general parameter λ uniting the general homogeneous boundary problem (7), (12)-(14), the graphical solution is presented in Fig. 3 and enclosed in a dashed rectangle.

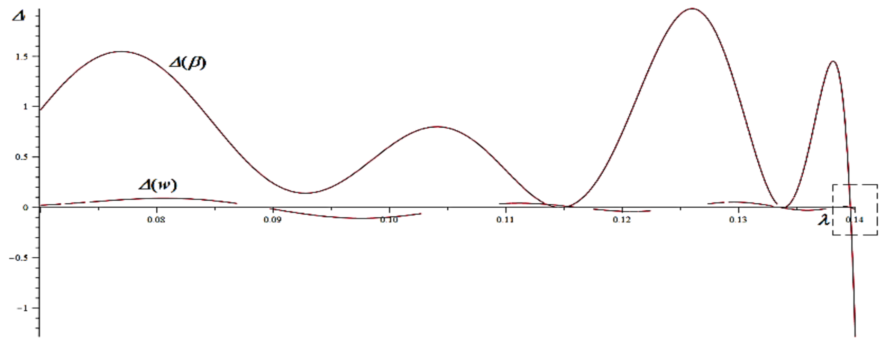


Figure 3: The intersection of the algebraic determinants of the system (12)-(14) at the point λ = 0.139577493899

The found value of λ allows determining the roots of the characteristic polynomial, composed for the equation (8), and finding dimensionless natural frequencies of oscillations ω_1=1.165786450, ω_2=0.5661533434, ω_3=0.5660497713. The graphical representation of the spring's own vibration forms $X_1(\varphi)$ and $X_2(\varphi)$ is shown in Fig. 4 with the modeled internal loading mode and with the described end fixations.

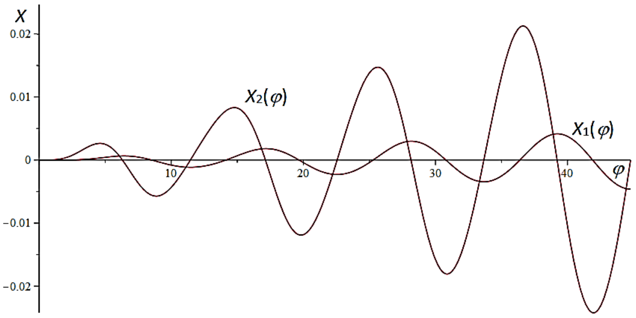


Figure 4: Graph of natural vibration forms $X_1(\varphi)$ and $X_2(\varphi)$

To the boundary conditions (12)-(14), additional conditions should be attached that do not contradict the physical nature of the problem for the final turn of the spring.

$$w^o(14,25\pi,t) = w_0^o\cos(p_1 t)\,. \tag{15}$$

Initial conditions corresponding to the external dynamic load mode $F_1(\varphi,t)$ and $F_2(\varphi,t)$ Fig. 1 and the particular solution of the system (3), are such

$$w^*(\varphi,0) = \theta_3\sin(\eta\varphi), \qquad \partial_t(w^*(\varphi,0)) = \theta_5\sin(\eta\varphi)\,, \tag{16}$$

$$\beta^*(\varphi,0) = \theta_4\sin(\eta\varphi)\,, \qquad \partial_t(\beta^*(\varphi,0)) = \theta_6\sin(\eta\varphi), \tag{17}$$

here θ_i, $i = \overline{1,6}$ are some constants subject to additional definition as a solution to a problem with unknown parameters.

Boundary conditions for the particular solution of the inhomogeneous problem (1), (2), not contradicting its physical nature, for the upper fixed section S and the oscillating opposite section at the other end of the spring with speed v_0 according to the law $z_0\sin(pt+\alpha)$, where $z_0 = v_0/p$, will be the following:

$$w^*(0,t) = 0\,, \qquad w^*(14.25\pi,t) = v_0\sin(pt+\alpha)/p\,, \tag{18}$$

$$\partial^2_{\varphi\varphi}\left(w^*(0,t)\right) = 0\,, \qquad \partial^2_{\varphi\varphi}\left(w^*(14.25\pi,t)\right) = -v_0\eta^2\sin(pt+\alpha)/p\,, \tag{19}$$

$$\beta^*(0,t) = 0, \qquad \partial_\varphi\left(\beta^*(14.25\pi, t)\right) = 0 \cdot \tag{20}$$

Combining particular solutions and solutions of the homogeneous system of differential equations (1), (2), we obtain the final general solutions of the problem in the following form:

$$w(\varphi,t) = \cos(p_1 t)\Big(\sigma_1\cos(\omega_1\varphi) + \sigma_2\sin(\omega_1\varphi) + \sigma_3\cos(\omega_2\varphi) + \sigma_4\sin(\omega_2\varphi) + \\ + \sigma_5\cos(\omega_3\varphi) + \sigma_6\sin(\omega_3\varphi)\Big) + \left(\frac{q_{11}}{\delta^2 - p^2}\sin(pt) + \frac{q_{12}}{\delta^2 - p^2}\cos(pt)\right)\sin(\eta\varphi), \tag{21}$$

$$\beta(\varphi,t) = \cos(p_1 t)\Big(h_{21}\left(\sigma_1\cos(\omega_1\varphi)+\sigma_2\sin(\omega_1\varphi)\right) + h_{22}\left(\sigma_3\cos(\omega_2\varphi)+\sigma_4\sin(\omega_2\varphi)\right) + \\ + h_{23}\left(\sigma_5\cos(\omega_3\varphi)+\sigma_6\sin(\omega_3\varphi)\right)\Big) + \left(\frac{q_{21}}{\delta^2 - p^2}\sin(pt) + \frac{q_{22}}{\delta^2 - p^2}\cos(pt)\right)\sin(\eta\varphi). \tag{22}$$

The presented analytical calculation (21)-(22) of the helical cylindrical spring has methodological significance for the educational process, as the derived laws of motion of displacement and angle of twisting of the cross section are very useful for facilitating understanding of its operation in complex operating circumstances. The formulas and data obtained in general form can be used for the design of vibration isolators and car suspensions.

The calculation methodology was developed and verified on the experimental base of the tractor plant, as evidenced by the utility model patent [15] and comparison of experimental data with the analytical formulae [16].

III. PRACTICAL CALCULATIONS

For the numerical realization of solutions of the stress-strain state of the spring, which is in the conditions of the external load regime, let's choose the following coefficients: $E = 2.1\cdot10^{+11}$ N/m^2, $G = 8.0\cdot10^{+10}$ N/m^2, $\rho = 7.86\cdot10^3$ kg/m^3, $J_1 = 1.0\ 10^{-4}$ m^4, $J_k = 7.95\ 10^{-8}$ m^4, $m_0 = 5.56$ kg/m, $R = 0.08$ m, $r = 0.015$ m, $w_0^o = 0.004$ m, $\psi_1 = 2\cdot10^{-6}$, $\psi_2 = 2\cdot10^{-4}$, $\eta = 0.04167$, $p = 1.8$ rad/s, $p_1 = 1975.321114$ rad/s, $\delta = 219.8838$ rad/s.

The laws of motion w(φ,t) and β(φ,t) based on formulas (21), (22) are presented below:

$$w(\varphi,t) = \sin(0.04166666667\cdot\varphi)\cdot[0.05296773801\cdot\sin(1.8\cdot t) + 0.3050018428\cdot\cos(1.8\cdot t)] + \\ + [-0.00004909524584\cdot\sin(1.16578645\cdot\varphi) + 0.8951342009\cdot\sin(0.5661533434\cdot\varphi) - \\ - 0.8951968745\cdot\sin(0.5660497713\cdot\varphi)]\cdot\cos(1975.321114\cdot t), \tag{23}$$

$$\beta(\varphi,t) = \sin(0.04166666667\cdot\varphi)\cdot[0.00134982133\cdot\sin(1.8\cdot t) + 0.006620967223\cdot\cos(1.8\cdot t)] + \\ + [-0.000833950339\cdot\sin(1.16578645\cdot\varphi) + 3.58720156\cdot\sin(0.5661533434\cdot\varphi) - \\ - 3.586140392\cdot\sin(0.5660497713\cdot\varphi)]\cdot\cos(1975.321114\cdot t). \tag{24}$$

Their visual image is shown in Fig. 5.

The correctness of the obtained solutions can be verified by substituting formulas (23) and (24) into the equations of the original problem (1), (2), and all its boundary conditions.

The spatial stress-strain state of the cylindrical spring (23), (24), due to forced dynamic loading, is shown in Fig. 5 and 6 using the Maple symbolic algebra package. For all calculations, the loads are greatly exaggerated by excessive forces without considering their critical values for the best visual observation in animation files for the displacement of spring points and twisting of wire cross-sections. The forces and moments exceed the magnitudes of 1312.5 N and 10500.0 N·m for a steel spring with a length of 0.85 m, a diameter of 16 cm, and a wire cross-section diameter of 3 cm.

In all figures, the uppermost fixation of the spring at $\varphi = 0$ rad does not change its position at the origin of the chosen coordinate system $x = 0.08$ m, $y = 0.0$ m, $z = 0.0$ m, as according to the problem conditions and its boundary conditions, it is fixed and immobile. The first figure (Fig. 5) shows the loaded state at the initial moment $t = 0$ s, while the others demonstrate the evolving general oscillations at subsequent moments.

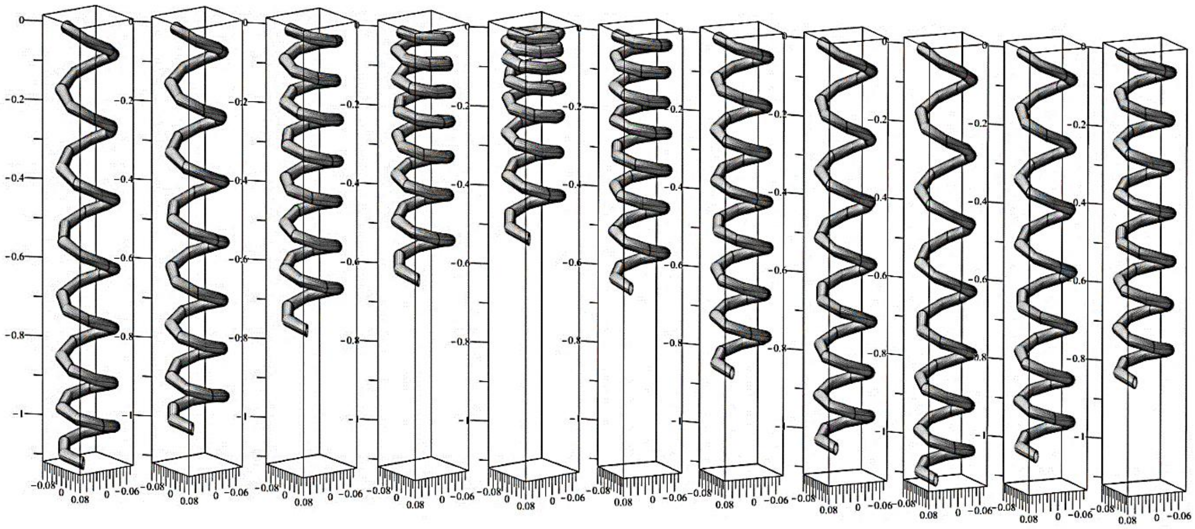


Figure 5: Spatial general oscillations of a cylindrical spring with displacements and twists of its coils at different times

Let's find out how the cross-section of the spring wire is twisted at an angle $\beta\,(\varphi,\,t)$ in the selected high-load mode for the convenience of visual observation. For this, let's consider a strongly twisted cross-section in the zone $\varphi = 3.25\,\pi$ and compare it with a similar untwisted section in Fig. 6. The longitudinal lines characterize the degree of rotation of the cross-section and its deformation, which can be determined using the Cauchy-Green deformation tensor for the cylindrical coordinate system $(\rho;\varphi;z)$.

$$\varepsilon_{\rho\rho} = \frac{\partial u_\rho}{\partial \rho}, \quad \varepsilon_{\varphi\varphi} = \frac{1}{\rho}\frac{\partial u_\varphi}{\partial \varphi} + \frac{u_\rho}{\rho}, \quad \varepsilon_{zz} = \frac{\partial u_z}{\partial z} = 0,$$

$$2\varepsilon_{\varphi z} = \frac{1}{\rho}\frac{\partial u_z}{\partial \varphi} + \frac{\partial u_\varphi}{\partial z}, \; 2\varepsilon_{\rho z} = \frac{\partial u_\rho}{\partial z} + \frac{\partial u_z}{\partial \rho} = 0, \; 2\varepsilon_{\rho\varphi} = \frac{\partial u_\varphi}{\partial \rho} - \frac{u_\varphi}{\rho} + \frac{1}{\rho}\frac{\partial u_\rho}{\partial \varphi}. \tag{25}$$

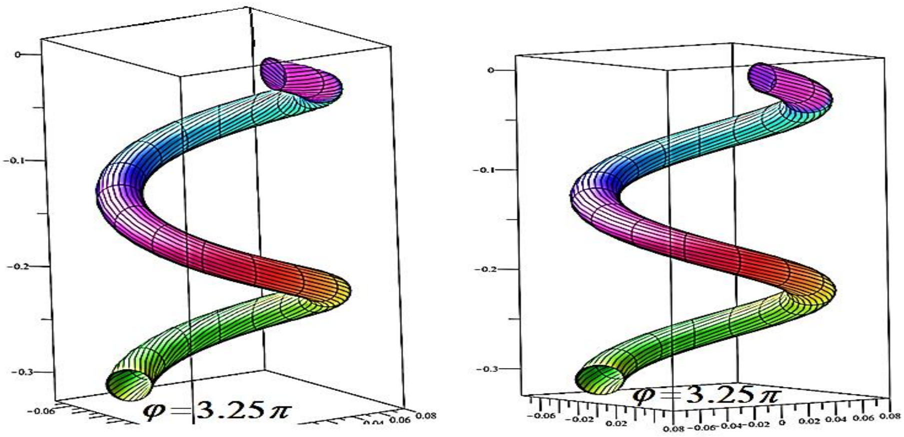


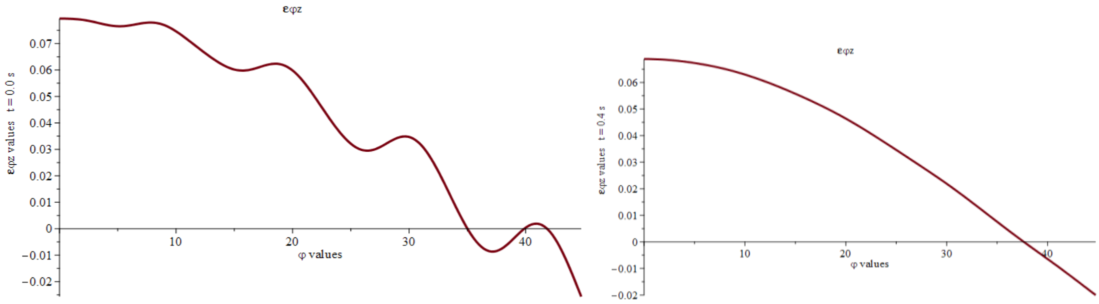
Figure 6: The strongly twisted lower cross-section at φ =3.25 π is compared with a similar untwisted section

The components of displacements, taking into account the hypothesis of plane sections, are as follows:

$$u_\varphi = 0, \;\; u_\rho = 0, \;\; u_z = w(\varphi,t) + r(1-\cos(\beta(\varphi,t)), \tag{26}$$

here, r is the radius of the cross-section of the wire of the cylindrical spring, which is equal to 0.015 m, ρ is the distance from the stationary center of the cylindrical coordinate system to the surface of the wire, therefore $\rho \in [0.065; 0.095]$ m.

The relative deformation of the cross-section in the zone $\varphi = 3.25\ \pi$ can be represented only by the tangential component $\varepsilon_{\varphi z}$ on Fig. 7 in dimensionless quantities. For the other components of the tensor $\boldsymbol{\varepsilon}$, partial derivatives with respect to the corresponding variables must be determined, which are not considered here.



a) *b*)

Figure 7: The deformations $\varepsilon_{\varphi z}$ of the cylindrical spring at (*a*) the initial moment in time, and at (*b*) 0.4 seconds into the time period

IV. CONCLUSION

Final analytical formulas for the laws of displacements and rotations of transverse sections of spring coils made of homogeneous elastic material have been obtained and derived based on the presented mathematical model of the three-dimensional stress-strain state of a cylindrical spring under complex dynamic loading conditions.

The values of the deformation tensor component are presented, along with a graphical clear visible rotation of points of the spring's transverse section and the displacement of the spring itself with the change in the external shape of its homogeneous elastic material.

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