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Modification and Android Application Development of Blaney-Morin-Nigeria Evapotranspiration Estimation Model For Use In Gombe State, Nigeria

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Abstract

*Determination of reference evapotranspiration (ET_o) of an area or region is a vital step in estimating crop water requirement before irrigation. Food and Agricultural Organisation (FAO) has recommended the estimation of ET_o using 56-Penman Monteith (FAO56-PM) method. This method, though acceptable the World over, but require many meteorological inputs which makes it to be cumbersome for the Village Agricultural Extension Agents (VEAs) or Irrigation Project Managers (IPMs). In Nigeria some researchers had made an effort and developed a simplified ET_o estimation for Nigerian environment, a model known as Blaney-Morin-Nigeria (BMN). Some gaps were equally observed, as the BMN may only be suitable as an option for estimating ET_o for an area similar in climate to Kaduna State (an area where its meteorological data was used to develop the model). In this research a modified BMN model for estimating ET_o for Gombe State was therefore developed that is less cumbersome, with minimum error and user friendly to the VEAs and the IPMs. In developing the modified model, Gombe State meteorological data for twelve years were collected from the Nigeria Meteorological Agency (NiMet) which comprised of, temperature, solar radiation, humidity and wind speed. With the data a script using Levenberg-Marquardt Algorithm was written in MATLAB environment to minimise the sum of square deviations in the BMN model and the FAO56-PM model was used as a baseline for calibration. The modified model known as $BMN_{GombeState}$, has input variables of temperature (T), humidity (R) and monthly to annual solar radiation ratio (rf) with empirical constants H and m having values of 458 and 1.20 respectively indicating climatic peculiarity of Gombe State. A correlation coefficient (r) of 0.95 was obtained when the developed $BMN_{GombeState}$ ET_o values were compared with that of FAO 56-PM model. Similarly, Mean Absolute Error (MAE), Root Mean Square Error (RMSE), Chi Square (X^2) with values of 0.28,0.36 and 0.24 respectively were obtained. It could therefore be suggested that the new developed modified model can serve as an option to the VEAs and the IPMs for estimating ET_o for Gombe State, Nigeria. Finally, Java programming language was used to developed an android mobile application (App) using the new model for estimating evapotranspiration rate in Gombe State thereby making a means of reference evapotranspiration estimation, mobile, faster and easier. The App is available on Google Play Store:*https://play.google.com/store/apps/details?id=com.laburtaibrahim63gmail.com.bmngombeevapotranspirationapp

Keywords: *Evapotranspiration, Blaney-Morin-Nigeria model, FAO56-PM model, Levenberg Marquardt Algorithm, Mobile App, Gombe State, Nigeria.*

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I.INTRODUCTION

To determine crop irrigation water requirements, it is necessary to estimate evapotranspiration by using Field Experimental Plots, Evaporation Pans, Water Budget Methods, Soil Depletion Studies, Empirical Methods and so on. The use of an empirical model is an indirect method of estimating evapotranspiration (ET), because it is based on the use of meteorological data such as temperature, solar radiation, humidity and wind speed. As a result of the timeliness and less cumbersomeness in operation of empirical models numerous were developed such as temperature-based (Thornwaite, Blaney-Criddle, Blaney-Morin), radiation- based (Turc, Hargreaves, Priestly-Taylor), and Food and Agriculture Organisation (FAO) Penman Monteith model known as FAO56-PM (Ilesanmi et al.,2012).FAO recommended FAO56-PM model for universal usage as a standard model for ET_o estimation, but FAO56-PM model requires meteorological data that are not often readily available in developing countries. As a result of that, a temperature-based model the Blaney-Morin-Nigeria (BMN) developed by Duru in 1984 for Nigeria is enjoying a wider usage in Nigeria and receiving good recommendations because it is simple and suitable for the Nigerian conditions. (Iwuchukwu et al.,2018).

BMN model was developed using data from a single location (Zaria) and presented for use all over Nigeria, but because environmental factors vary from location to location the reliability of BMN model will be better appreciated if it is calibrated to indicate climatic peculiarities of the location of interest. The aim of this study is to modify a model for the estimation of reference evapotranspiration for Gombe State with the following specific objectives:

- To modify an existing model known as BMN thereby developing a new model specific for Gombe State.
- To compare ET_o using the modified BMN model against BMN andFAO 56-PM model in Gombe State.
- To develop an android mobile phone application (App) for calculating reference evapotranspiration of Gombe State using the modified model.

II.MATERIALS AND METHODS

Study Area

Gombe State lies on latitude $10^o\ 16^{'}\ 59.9988^{''}$N and longitude $11^o\ 10^{'}\ 0.0012^{''}$E and is located in the north-eastern part of Nigeria at an altitude of 461m. It has semi-arid to savannah climate characterised with scattered trees and shrubs. It receives an average precipitation between 900-1000mm in 7 months per year. The mean and maximum monthly temperatures of 37^oC is recorded in April and March, while the minimum monthly temperatures were about 21^oC around December and early February (WeatherSpark.com,2023).

Plate 1: Map of Gombe State.
Source: United Nations, (2016).

Data Used for The Study

The Nigeria Meteorological Agency (NiMet) is responsible for the collation and management of all climatic data in Nigeria. The data for this study was therefore sourced from NiMet. The types of data collected for Gombe State from the Agency from 2010-2021 were mean monthly data of; temperature, humidity, solar radiation and wind speed.

Data Processing

The optimisation script using Levenberg Marquardt Algorithm (LMA) was written in MATLAB environment as thus, the response data for the model (standard for calibration) was the ET_o values of FAO56-PM and the input data for the model (BMN) was the BMN model with the inputs of temperature, humidity and solar radiation monthly to annual ratio, with H and m unknown. Initial guesses for H and m were made and the script was run. The meteorological data (2010-2020) of temperature(T), relative humidity(R), sunshine hours, wind speed and solar radiation(rf) were used in the following units of degree Celsius (^{o} C), percentage, (%), hours, kilometre per day (km/day) and ratio of monthly to annual respectively. After several iterations and minimising the sum of squared deviations it yielded new values of H and m which are the function of the meteorological data. Therefore these new found empirical constants are now the BMN constants that are applicable specifically to only Gombe State. Theprocess of the optimisation is shown by flowcharts in figures 1 and 2. In figure1; H_est, m_est and H_opt, m_opt represents initial guess of H and m before optimisation and after optimisation respectively. In figure 2; f, λ_o, minS, maxIt represents function, lambda, sum of square deviations before iterations and number of iterations respectively.The original BMN model and FAO56-PM models used in the optimisation were presented in equations 1 and 2 below. Thereafter, the new found model

$BMN_{GombeState}$ was validated against the FAO suggested standard evapotranspiration model FAO 56-PM using the remaining one-year (2021) meteorological data that was not used in the optimisation process as presented in model validationsection below where analysis of forecast accuracy, evaluation of prediction quality, evaluation of distinction and analysis of association were conducted using statistical tools as presented in equations 1-4.Thereafter the modified model was used to developed a mobile App for evapotranspiration rate estimation in Gombe State using Java programming language as presented in figure 3 and plate 2-5.

$$ET_{o(BMN)} = rf\,\frac{(0.45T+8)(520-R^{1.31})}{100} \quad ...1$$

$ET_{o(BMN)}$ is reference evapotranspiration using BMN model (mm/day)

$$ET_{o(FAO\ 56-PM)} = \frac{0.408 \times \Delta \times (R_n - G) + \gamma \times \frac{900}{T+273} \times u_2 \times (e_s - e_a)}{\Delta + \gamma \times (1 + 0.34 \times u_2)} \quad ...2$$

Where,
T is daily average air temperature ^oC at 2m height.
u_2 is the wind velocity at 2m height (ms^{-1}).
e_s and $e_a$is the saturation vapour pressure and actual vapour pressure kpa.
(e_s - e_a) is deficit in vapour pressure (kpa).
R_n is the net radiation.
G is the soil heat flux.
Δ is the slope of saturation vapour pressure.
γ is the psychometric constant (mb^{o}C).

The optimisation process in MATLAB

Figure 1: Optimisation process for $BMN_{GombeState}$.

Process of Levenberg Marquardt Algorithm on BMN

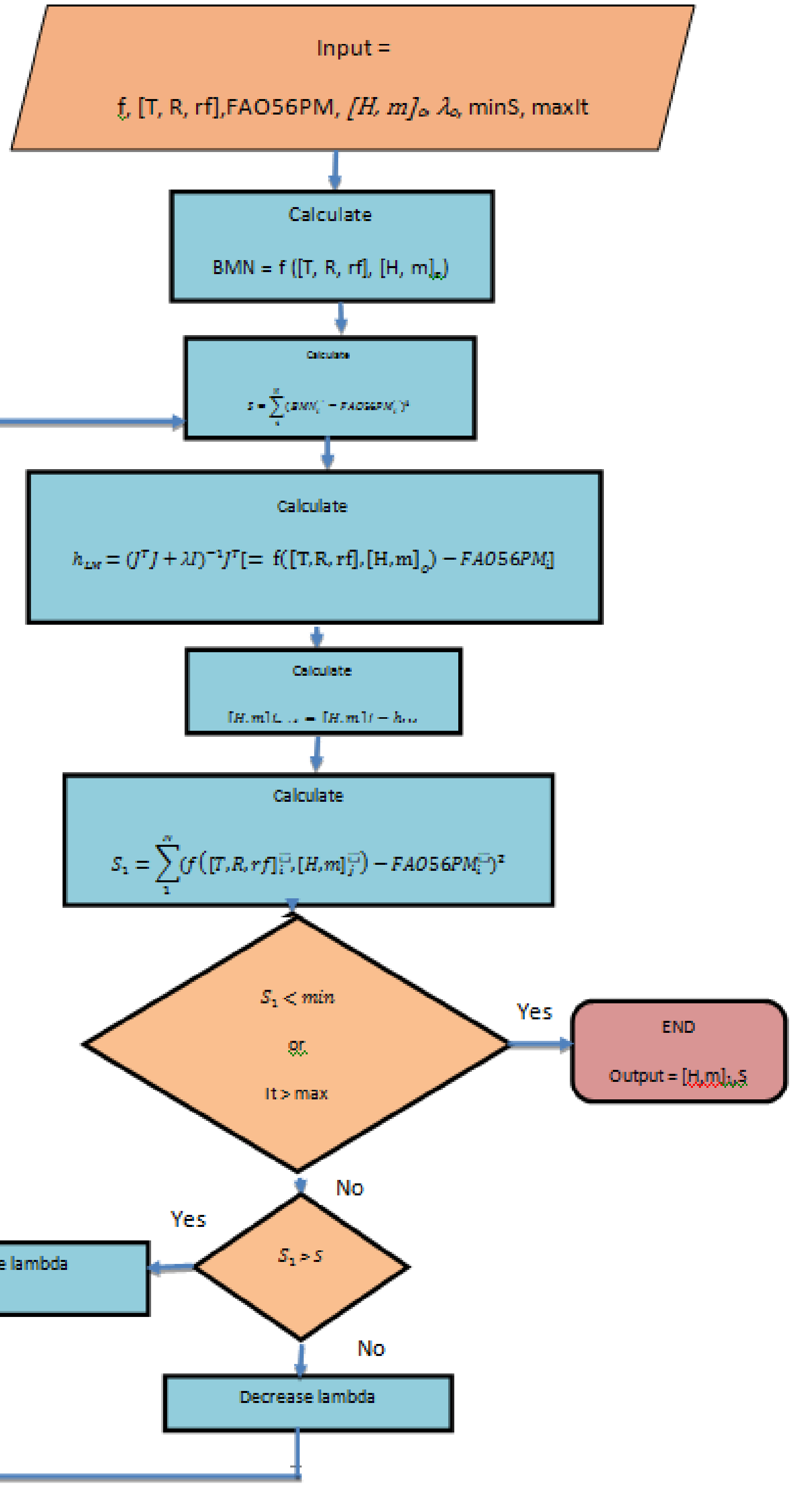

Figure 2: Process of Levenberg Marquardt Algorithm on BMN

The Model Validation

The validation involved; analysis of the forecasted and the actual values, evaluation of the quality of the prediction, analysis of the difference between the observed and the expected result and evaluation of the relationship between the variables as presented below:

Analysis of forecast accuracy

The forecast accuracy analysis was conducted using Mean Absolute Error (MAE) statistical tool as suggested by Ilesanmi et al., (2012), as presented below;

$$MAE = \frac{1}{n}\sum_{i=1}^{n}|F_i - A_i| \ldots 3$$

Where:

F_i are forecasted values ($ET_{o\ (BMNGombeState)}$).

A_i are actual values ($ET_{o\ (FAO56\text{-}PM)}$).

n is the number of observations.

Evaluation of prediction quality

The prediction quality analysis was computed using Root Mean Square Error (RMSE) statistical tool as suggested by Ilesanmi et al., (2012), as presented below;

$$RMSE = \sqrt{\sum_{i=1}^{n}\frac{(MR_{pre,i} - MR_{obs,i})^2}{n}} \ldots 4$$

Where:

MR_{pre} is ith predicted data ($ET_{o\ (BMNGombeState)}$).

MR_{obs} is ith observed data ($ET_{o\ (FAO56\text{-}PM)}$).

n is the number of observations.

Evaluation of distinction

The analysis of the difference between the observed and expected results was carried out using Chi-Square (X^2) statistical tool as suggested by Echiegu et al., (2016), as presented below;

$$X^2 = \sum_{i=1}^{n}\frac{(O_i - E_i)^2}{E_i} \ldots 5$$

Where:

E_i are expected values ($ET_{o\ (BMNGombeState)}$).

O_i are observed values ($ET_{o\ (FAO56\text{-}PM)}$).

n is the number of observations.

Analysis of association

The relationship of the variables was calculated using Correlation Coefficient (r) statistical tool as suggested by Ilesanmi et al., (2012) and Echiegu et al., (2016) as presented below;

$$r = \frac{\sum PO - \frac{\sum P \sum O}{n}}{\sqrt{(\sum P^2) - \frac{(\sum P)^2}{n} \times (\sum O^2) - \frac{(\sum O)^2}{n}}} \ldots 6$$

Where:

P are predicted values ($ET_{o\ (BMNGombeState)}$).

O are observed values ($ET_{o\ (FAO56\text{-}PM)}$).

n is number of observations.

Process of BMNGombeState App Development Using Java

The new model was then used to developed an android App for calculating reference evapotranspiration of Gombe State using Java Programming Language as shown by the flowchart below. The App only takes the values of temperature, relative humidity and solar radiation ratio from the user and provides a result at a click of a button.

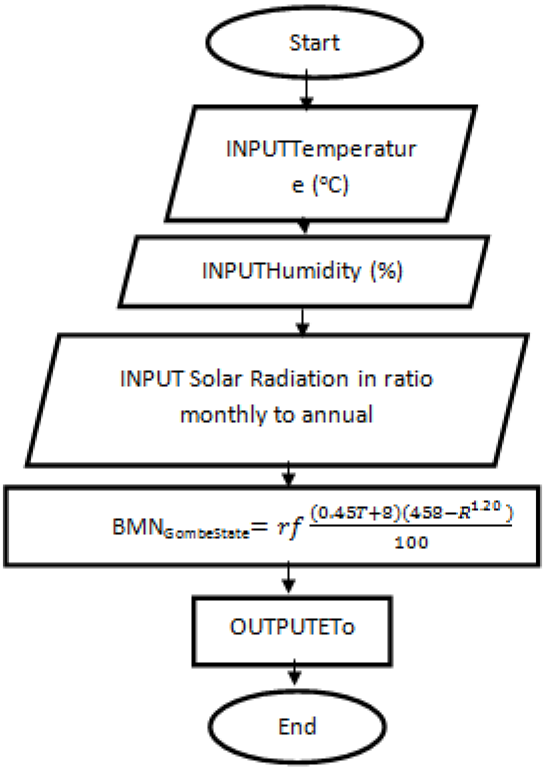

Figure 3: Flowchart of $BMN_{GombeState}$ App Development

III.RESULTS AND DISCUSSIONS

Weather Data of Gombe State
The meteorological data covering the study period for 2021 is tabulated in Table 1 showing mean monthly temperature (T), relative humidity (R), wind speed and monthly to annual ratio of solar radiation (rf) with monthly ET_o values of FAO56-PM, BMN and$BMN_{GombeState}$

Table 1: The Mean Monthly Metrological Data of Gombe State (2021).

Month	Temperature (°C)	Humidity (%)	Solar Radiation Ratio (rf)	Wind Speed (km/day)	FAO56-PM (mm/day)	BMN (mm/day)	BMNGombe (mm/day)
January	26.25	24	0.08	209	6.39	7.42	6.54
February	25.80	15	0.09	233	7.16	8.17	7.63
March	30.35	30	0.09	205	7.06	8.22	7.77
April	31.80	39	0.08	203	6.89	7.12	6.73
May	30.50	60	0.09	206	6.26	5.96	6.29
June	28.05	69	0.08	193	5.21	4.53	4.90
July	25.90	68	0.08	192	4.7	4.12	4.72
August	25.65	64	0.08	172	4.79	4.62	4.86
September	26.30	67	0.08	177	4.71	4.36	4.80
October	28.30	60	0.09	174	5.39	5.56	6.01
November	28.95	52	0.08	207	6.04	6.01	5.78
December	24.65	24	0.08	187	5.8	6.97	6.30

Source: NiMet

From Table 1 it can be seen that the temperature varies from 24.65 °C in December to 31.80 °C in April, the relative humidity from 24 % in January to 69 % in June and finally solar radiation is averagely 0.08.

Model Optimisation

Table 2 shows the output of the optimisation based on the objective i.e., to minimise the sum of squared deviations. The algorithm used was LMA, H (H_est) and m (m_est) are initial guesses, Sum of Squared Deviations before optimisation (SSD_{Bopt}) and Sum of Squared Deviations after optimisation (SSD_{Aopt}), the new value of H (H_opt) and m (m_opt) are tabulated as presented below,

Table 2: Outputs of the Optimisation of BMN for Gombe State.

Algorithm (LMA)	H_est	m_est	SSD_{Bopt}	SSD_{Aopt}	H_opt	m_opt
	0	0	5390.36	189.846	365.5927	-76.054
	1	1	6541.35	189.846	365.5927	-5.1638
	300	1.31	1831.57	76.5216	458.1102	1.1974
	520	1.31	134.13	76.5216	458.1102	1.1974
	700	1.31	1531.51	76.5216	458.1102	1.1974

The optimisation script written in MATLAB was simulated using eleven years meteorological data of temperature, humidity, solar radiation and wind speed using the reference evapotranspiration results obtained for these eleven years using FAO 56-PM model as standard. Averages of the meteorological data was not used but rather the entire data meaning a total of 924 values comprising temperature, humidity, solar radiation and wind speed.

From Table 2 it can be seen that based on the initial guesses the highest SSD is 6541.35 and the least SSD is 76.5216 and this represent the minimisation of errors by 98.83% which gave an optimised value of H as 458.1102 and m as 1.1974 which agrees with the values obtained by Ilesanmi et al (2012)(418,1.24) and Echiegu et al (2016)(392.2,1.19) which are all lower than the original value of the model by Duru (1984),where H is 520 and m is 1.31.The lowest SSD_{Bopt} of 134.13 was obtained when H_est is 520 and m_est is 1.31 this shows that these values are the best initial guess to make and no need for other guesses.

Plot of the optimisation before and after is as presented in Figures 4 and 5 below,

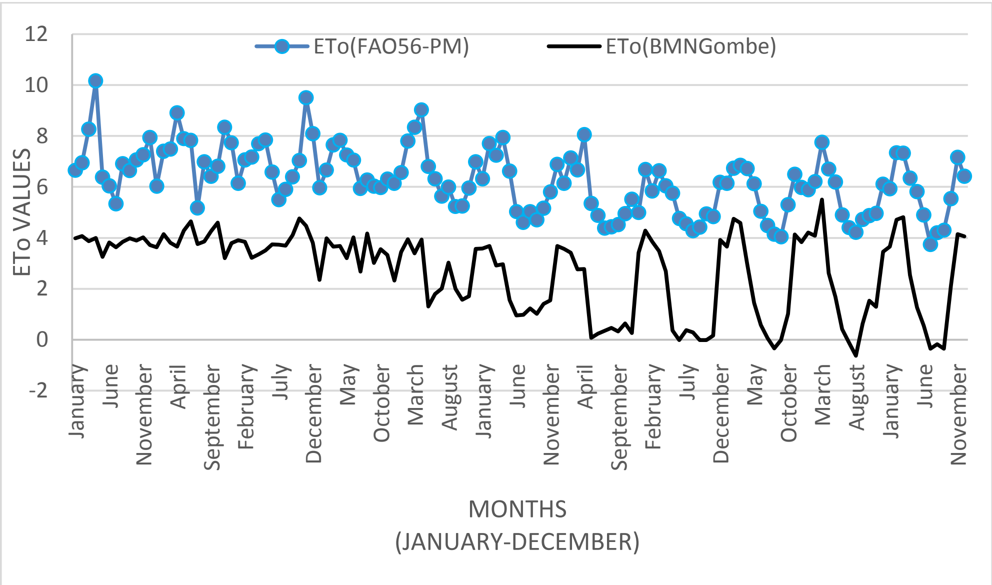

Figure 4: Plot of Optimisation Process before Optimisation of BMN in Gombe State.

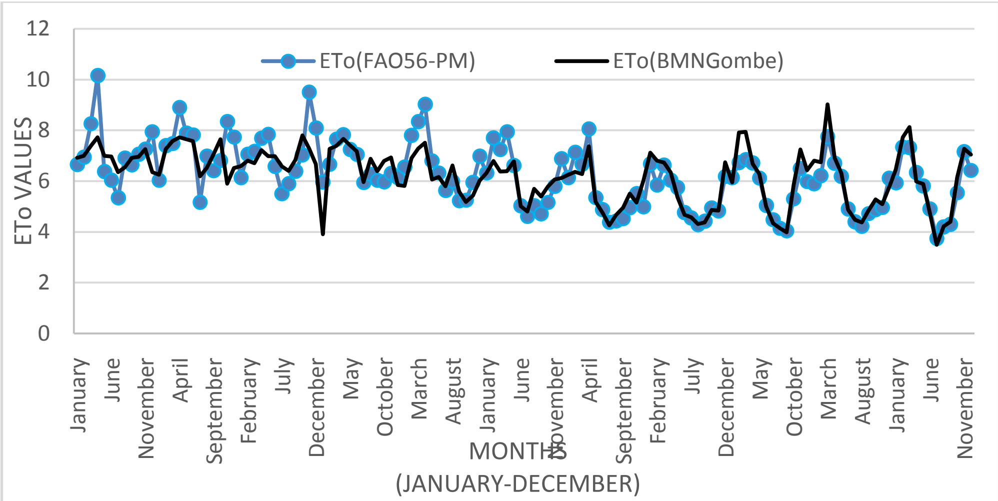

Figure 5: Plot of the Optimised $\text{BMN}_{\text{GombeState}}$.

It can be seen from Figure 4 that the model being modified did not fit with the standard, but as the errors were minimised it fits as shown in Figure 5. This agrees with Transtrum and Sethna, (2012), that for a successful optimisation the resultant graph will be a best fit.
The new model is now presented as,

$$ET_{o(BMNGombeState)} = rf\,\frac{(0.45T+8)(458-R^{1.20})}{100} \quad ...7$$

Where,
$ET_{o(BMNGombeState)}$ is reference evapotranspiration using $\text{BMN}_{\text{GombeState}}$ model (mm/day),
rf is the ratio of monthly radiation to annual radiation.
T is the mean monthly temperature (^{o}C).
R is the mean monthly relative humidity (%)
The outcome of the optimisation to get the new model constants specific for Gombe State was validated successful as shown in appendice 1, because the exit flag (3) being positive and the check on nearby points producing results greater than the objective function value at solution (fval) strongly agree with MATLAB Inc. (2022) that a positive exit flag and check on nearby points producing values greater than fval corresponds to a successful outcome.

As is common with all temperature-based models $\text{BMN}_{\text{GombeState}}$ will be best applied at ten days, monthly and longer and not hourly or daily (Allen et al,1998). The model is applicable to the whole state, because of similar climatic conditions (Bello et al.,2020 and Yusuf et al.,2018).

Model Validation Values

The remaining meteorological data of Gombe State (2021) that was not used in the optimisation of the model was used to validate the model and the results in comparison with the results obtained by BMN against FAO56-PM and $\text{BMN}_{\text{GombeState}}$ against FAO56-PM is presented in Table 3,

Table 3: Model Validation Values of $\text{BMN}_{\text{GombeState}}$ in Comparison to BMN

Models	H	m	MAE	RMSE	X^2	r
BMN	520	1.31	0.66	0.79	1.00	0.94
$\text{BMN}_{\text{GombeState}}$	458	1.20	0.28	0.36	0.24	0.95

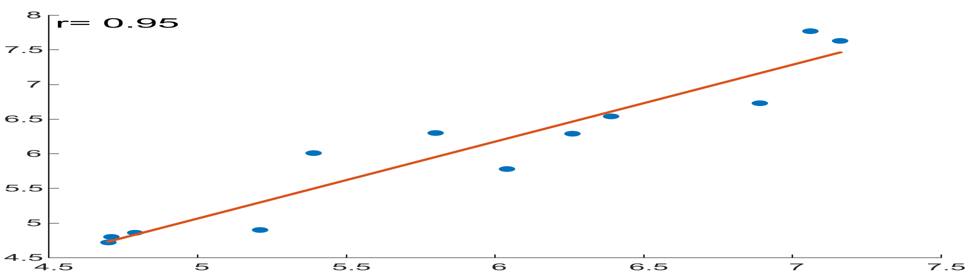

Figure 6: Plot of Correlation between $\text{BMN}_{\text{GombeState}}$ and FAO56-PM

Although the value of r (0.94) for BMN and r (0.95) for $BMN_{GombeState}$ indicates a strong linear relationship between the models and FAO56-PM the values of r alone cannot be used to validate the model. The goal is to have less errors in prediction, therefore $BMN_{GombeState}$ with MAE (0.28), RMSE (0.36), and Chi-Square (0.24) as against BMN with MAE (0.66), RMSE (0.79), and Chi-square (1.00) is a better model for Gombe State (Ilesanmi et al,2012, Echiegu et al.,2016, Chu 2020 and Glen 2020).

The lowest MAE, RMSE reported by Duru, (1984) are 0.40,0.40 respectively (Echiegu et al.,2016), as can be seen in Table 3 the MAE, RMSE of $BMN_{GombeState}$ of 0.28,0.36 respectively are lower. It is also lower than the combine BMN model of MAE of 0.55 and RMSE of 1.06 respectively presented by Ilesanmi et al, (2012). The MAE is slightly higher than that of Echiegu et al, (2016) of 0.13 and the same with RMSE of 0.36 respectively. The X^2 which was not measured by Duru (1984), was compared with the one measured by Echiegu et al., (2016) which is 0.14 and with the value of 0.24 as shown in Table 3 it is slightly higher, the closer the values of MAE, RMSE and X^2 are to zero the better the model and also the closer the value of r is to 1 the better the model (as cited in Echiegu et al.,2016).

The highest r reported by BMN is 0.80 for Lagos and the lowest is 0.62 for Gombe State (Echiegu et al.,2016). With r of 0.95 as can be seen in Table 3 and Figure 6 obtained by $BMN_{GombeState}$ it means it is higher than that of BMN, $BMN_{Combine}$ of 0.88 and BMN_{Enugu} of 0.82 as reported by Ilesanmi et al., (2012) and Echiegu et al., (2016) respectively. The excellent performance of $BMN_{GombeState}$ can be attributed to the reliability of meteorological data as a result of modern meteorological equipment and the usage of meteorological data of Gombe State as against a data from a single location for the development of the model and applying it all over the country (Idike,2005).Also the use of MATLAB platform, a programme developed for science and engineering eliminated the errors that are likely to occur from manual iterations as identified by Ilesanmi et al.,(2012) in respect to the development of BMN.

Comparison between FAO 56-PM and $BMN_{GombeState}$

From Table 1 it can be seen that the ET_o estimate of FAO 56-PM varies from 4.70 mm/day in July to 7.16 mm/day in February with an annual average of 5.87 mm/day while that of $BMN_{GombeState}$ varies from 4.72 mm/day in July to 7.77 mm/day in March and an annual average of 6.03 mm/day. They also have approximately the same estimate in May. Furthermore, the ET_o estimates of FAO 56-PM are lower than that of $BMN_{GombeState}$ in January, February, March, May, July, August, September, October and December but higher in April, June and November. For both models, during the dry season January to May higher ET_o estimates were recorded, and lower during rainy season May to September this agrees with Allen et al., (1998), that solar radiation and temperature are the major factors of evapotranspiration. The mean ET_o of $BMN_{GombeState}$ being higher than that of FAO 56-PM satisfied the statement made by Duru that the model that overpredicts to a lesser degree should be preferred to the model that underpredicts from design and safety viewpoint (Ilesanmi et al.,2012 and Echiegu et al,2016).

A graphical plot of the result of the new model $BMN_{GombeState}$ with FAO56-PM model shows an impressive agreement between the models.

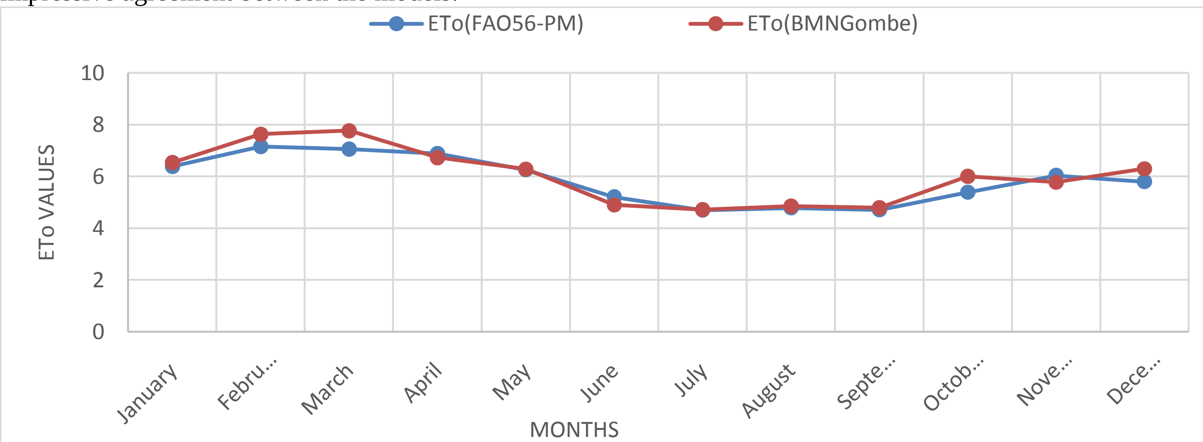

Figure 7: Plot of $ET_{o\ (FAO56\text{-}PM)}$ and ET_o ($_{BMNGombeState}$) of Gombe State 2021

$BMN_{GombeState}$ Reference Evapotranspiration App

The new BMN model was used to developed an android mobile App for calculating ET_o using Java programming language. The new developed App is as presented below,

Plate 3: Icon $\text{BMN}_{\text{GombeState}}$ ETo App

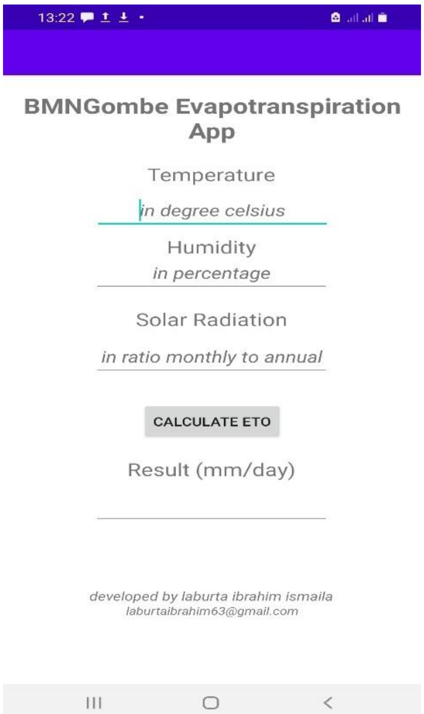

Plate 4: Interface $\text{BMN}_{\text{GombeState}}$ ET_o App

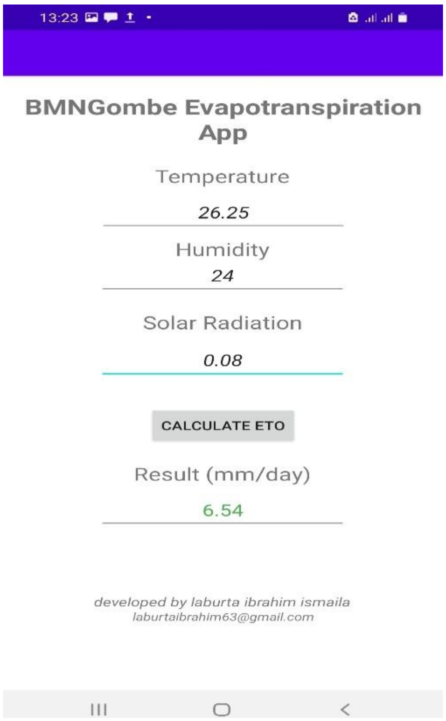

Plate 5: Result $\text{BMN}_{\text{GombeState}}$ ET_o App

The $\text{BMN}_{\text{GombeState}}$ ET_o App is simpler, effective and user friendly as oppose to the mobile App by Rodrigues et al. (2018), that requires up to fifteen user inputs and Lykhovyd (2022), that requires only a temperature input which might be inadequate depending on location though it was designed specifically for Ukraine.

IV.SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Summary

The study successfully achieved its objectives as shown below,

i. The Blaney Morin Nigeria model was modified by minimising the sum of squared deviations using Levenberg Marquardt Algorithm and a new model specific for Gombe State and areas with similar climatic conditions was obtained.

ii. The new model ($\text{BMN}_{\text{GombeState}}$) performed excellently in comparison with FAO 56-PM model in Gombe State.

iii. An android mobile phone application for calculating reference evapotranspiration was developed for Gombe State using the modified model.

Conclusions

The following conclusions were made from the study,

i. Blaney Morin Nigeria is a model that is better applied peculiarly to a location.

ii. Blaney Morin Nigeria model is an excellent alternative to FAO 56-PM model and can even replace it where necessary.

iii. The developed android mobile phone application will make the estimation of reference evapotranspiration easier and accessible to many at an instant.

Recommendations

Based on the outcome of the study the following recommendations were made,

i. An updated version of the app where the meteorological data are obtained automatically via internet will make the estimation further easier and simple.

ii. A software for calibrating BMN model using the script written for this work should be developed as that will make available a means of calibrating the model faster and easier where the need arises for any location anywhere in Nigeria and beyond.

iii. A further study to modify BMN model for North Eastern Nigeria and beyond will make an optimised single model available for many locations.

REFERENCES

[1]. Akpootu, D. O., &Iliyasu, M. I. (2017). A Comparison of Various Evapotranspiration Models for Estimating Reference Evapotranspiration in Sokoto, North Western, Nigeria. *Physical Science International Journal,* 1-14.

[2]. Allen, R. G., Pereira, L. S., Raes, D., & Smith, M. (1998). Crop Evapotranspiration (Guidelines for Computing Crop Water Requirements). *Food and Agriculture Organisation of the United Nations Rome, 300(9), D05109.*

[3]. Ayotamuno, M. J. (2003). Ranking Potential Evapotranspiration Models in the Prediction of Maize Water Requirements in the Port Harcourt Area of Nigeria. *Outlook on agriculture,* 32(1), 51-56.

[4]. Beck, A. (2014). *Introduction to Nonlinear Optimization: Theory, Algorithms, And Applications with MATLAB*. Society for Industrial and Applied Mathematics.

[5]. Bello, Y., Adebayo, A. A., & Abubakar, B. (2020). Analysis of Rainfall and Temperature Changes in Gombe State State, Nigeria. *FUDMA Journal of Sciences, 4(1), 632-646*.

[6]. Bhatt, R., & Hossain, A. (2019). Concept and Consequence of Evapotranspiration for Sustainable Crop production in the Era of Climate Change. Adv. Evapotranspiration Methods *Appl, 1, 1-13.*

[7]. Chugh, A. (2020). MAE, MSE, RMSE, Coefficient of Determination, Adjusted R Squared—Which Metric is Better?.Retrieved January 2023,from https://medium.com/analytics-vidhya/mae-mse-rmse-coefficient-of-determination-adjusted-r-squared-which-metric-is-better-cd0326a5697e

[8]. Córdova, M., Carrillo-Rojas, G., Crespo, P., Wilcox, B., &Célleri, R. (2015). Evaluation of The Penman-Monteith (FAO 56 PM) Method for Calculating Reference Evapotranspiration Using Limited Data. *Mountain Research and Development*, *35*(3), 230-239.

[9]. Cunha, F. F. D., Magalhães, F. F., Castro, M. A. D., & Souza, E. J. D. (2017). Performance of Estimative Models for Daily Reference Evapotranspiration in the City of Cassilândia, Brazil. *EngenhariaAgrícola*, 37, 173-184.

[10]. Curtis, F.E. (2018). *Algorithms for NonsmoothOptimisation* [PowerPoint slides] Lehigh University presented at Center for Optimisation and Statistical Learning, Northwestern University.https://coral.ise.lehigh.edu/frankecurtis/files/talks/nu_tutorial_18.pdf

[11]. Dase, E., Wariri, O., Onuwabuchi, E., Alhassan, J. A., Jalo, I., Muhajarine, N., ... &ElNafaty, A. U. (2020). Applying the WHO ICD-PM classification system to stillbirths in a major referral Centre in Northeast Nigeria: a retrospective analysis from 2010-2018. BMC pregnancy and childbirth, 20(1), 1-10.

[12]. Dike, F. I. (2005). Blaney-Morin-Nigeria (BMN) Evapotranspiration Model. *Nigerian Journal of Technology,* 24, 2.

[13]. Divesh Choudhary (2018). Methods of Evapotranspiration DOI: 10.13140/RG.2.2.14533.76007.Retrieved (July,2020) from https://www.researchgate.net/publication/323945435_Methods_of_Evapotranspiration

[14]. Duru, J. O. (1984). Blaney-Morin-Nigeria Evapotranspiration Model. *Journal of Hydrology,* 70(1-4), 71-83.

[15]. Echiegu, E. A., Ede, N. C., and Ezenne, G. I. (2016). Optimisation of Blaney-Morin-Nigeria (BMN) Model for Estimating Evapotranspiration in Enugu, Nigeria. *African Journal of Agricultural Research,* 11(20), 1842-1848.

[16]. Fapohunda, H. O. (2001). Comparison of Potential Evapotranspiration Calculation Methods in a Tropical Region. *Journal of Agricultural Engineering Research and Technology*, 9, 20-31

[17]. Gavin, H. P. (2019). The Levenberg-Marquardt Algorithm for Nonlinear Least Squares Curve-Fitting Problems. Department of Civil and Environmental Engineering, Duke University, 1-19.

[18]. Ghiat, I., Mackey, H. R., & Al-Ansari, T. (2021). A Review of Evapotranspiration Measurement Models, Techniques and Methods for Open and Closed Agricultural Field Applications. *Water*, *13*(18), 2523.

[19]. Glen, S. (2020). Absolute Error & Mean Absolute Error (MAE). StatisticsHowTo. Co. Retrieved January 2023 from https://www.statisticshowto.com/absolute-

[20]. Glen, S. (2020). Chi-square statistic: How to calculate it/distribution. Retrieved January 2023 from https://www.statisticshowto.com/probability-and-statistics/chi-square/

[21]. Glen, S. (2020). RMSE: Root Mean Square Error. StatisticsHowTo. Com. Retrieved January 2023 from,https://www.statisticshowto.com/probability-and-statistics/regression-analysis/rmse-root-mean-square-error/

[22]. Gombe Climate, Weather by Month, Average Temperature (Nigeria) – Weather Spark [Weather Report]. Retrieved January 2023 fromhttps://weatherspark.com/y/68815/Average-Weather-in-Gombe-Nigeria-Year-Round

[23]. Gulliver, J. S., Erickson, A. J., & Weiss, P. (2010). Stormwater Treatment: Assessment and Maintenance. University of Minnesota, St. Anthony Falls Laboratory. Minneapolis, MN.

[24]. Harwood Engineering (2018). Primary Versus Secondary Standards. Retrieved April, 2022 from https://www.harwoodeng.com/articles/primary-vs-secondary-standards/#:~:text=To%20put%20it%20simply%2C%20a,previously%20by%20the%20primary%20standard.

[25]. Ilesanmi, O. A., Oguntunde, P. G., and Olufayo, A. A. (2012). Re-examination of the BMN Model for Estimating Evapotranspiration. *International Journal of Agriculture and Forestry* 2012, 2(6): 268-272 DOI: 10.5923/j.ijaf.20120206.01

[26]. Isikwue, C. B., Audu, O. M., and Isikwue, O. M. (2014). Evaluation of Evapotranspiration using FAO Penman-Monteith Method in Kano Nigeria. *International Journal of Science and Technology,* 3(11), 698-703.

[27]. Iwuchukwu, E. I., Asoegwu, S. N., & Okereke, N. A. A. (2018). Modelling Monthly Relative Humidity of Imo and Enugu States for Evapotranspiration estimation. *Int J Current Trends Eng Res, 4(10), 1-8.*

[28]. J.Sauppe, editor (2006-18). Mathematical Programming Glossary. INFORMS Computing Society, http://glossary.computing.society.informs.org/, Originally authored by Harvey J. Greenberg,1999-2006.Retrieved 30th March, 2020 from https://glossary.informs.org/ver2/mpgwiki/index.php/Generalized_reduced_gradient_method

[29]. Jasechko, S., Sharp, Z. D., Gibson, J. J., Birks, S. J., Yi, Y., & Fawcett, P. J. (2013). Terrestrial Water Fluxes Dominated by Transpiration. *Nature*, *496*(7445), 347-350.

[30]. Kochenderfer, M. J., and Wheeler, T. A. (2019). Algorithms for Optimisation. MIT. Press.

[31]. Leif Zinn-Bjorkman. (2010). Numerical Optimisation Using Levenberg-Marquardt Algorithm.EES-16.LA-UR-11-12010.Powerpoint Presentation. Retrieved from https://www.google.com/url?sa=t&source=web&rct=j&url=https://mads.lanl.gov/presentations/Leif_LM_presentation_m.pdf&ved=2ahUKEwjwlPnV_dDzAhWGEWMBHYBSBBAQFnoECAQQAQ&usg=AOvVaw0UXBz4Ju5h76gtHu2uRPgj

[32]. Lykhovyd, P. (2022). Comparing Reference Evapotranspiration Calculated in ETo Calculator (Ukraine) Mobile App with the Estimated by Standard FAO-Based Approach. *Agri Engineering*, 4(3), 747-757.

[33]. MathWorks Inc. (2022). MATLAB (Version R2022b). Least Square (Model Fitting Algorithms).https://in.mathworks.com/help/optim/ug/least-squares-model-fitting-algorithms.html,https://www.mathworks.com/help/optim/ug/when-the-solver-might-have-succeeded.html,https://www.mathworks.com/help/optim/ug/exit-flags-and-exit-messages.html

[34]. Messac, A. (2015). Optimisation in Practice with MATLAB®: for Engineering Students and Professionals. Cambridge University Press.

[35]. Mikhailov, E. E. (2018). Programming with MATLAB for Scientists: A Beginner's Introduction. CRC Press.

[36]. Moroke, T. S., Schwartz, R. C., Brown, K. W., &Juo, A. S. R. (2005). Soil Water Depletion and Root Distribution of Three Dryland Crops. *Soil Science Society of America Journal*, *69*(1), 197-205

[37]. Nayak, M., Patel, H., Kumar, A., & Prakash, V. (2016). Study of Soil Moisture Depletion Under Different Irrigation Levels by CROPWAT Simulation Model.

[38]. Nocedal, J., & Wright, S. (2006). Numerical Optimisation. Springer Science & Business Media

[39]. .Novick, K. A., Oren, R., Stoy, P. C., Siqueira, M. B. S., &Katul, G. G. (2009). Nocturnal Evapotranspiration in Eddy-Covariance Records from Three Co-Located Ecosystems in The South-eastern US: Implications for Annual Fluxes. Agricultural and Forest Meteorology, 149(9), 1491-1504.

[40]. Obioma, C. P., Okereke, C. D., and Nwaigwe, K. N. (2015). Comparative Analysis of Direct and Four Indirect methods for Determination of Evapotranspiration of Waterleaf (Talinum triangulare). *International Journal of Research in Engineering and Technology*, 4(2).

[41]. Pereira, A., & Pires, L. (2011). Evapotranspiration and Water Management for Crop Production. in Evapotranspiration-From Measurements to Agricultural and Environmental Applications. *Intech Open.*

[42]. Janet Peter (2018) Literature Review and Proposal on Core Java. Retrieved March 2022 fromhttps://articles.abilogic.com/amp/317671/literature-review-proposal-core-java.html

[43]. Poncea, V. M., & Shetty, A. V. (1995). A Conceptual Model of Catchment Water Balance: 1. Formulation and Calibration. *Journal of Hydrology*, *173*(1-4), 27-40.

[44]. Powell, S. G., & Batt, R. J. (2011). Modelling for insight: a master class for business analysts. John Wiley & Sons. https://onlinelibrary.wiley.com/doi/pdf/10.1002/9780470387207.app1

[45]. Poyen, E. F. B., Ghosh, A. K., and Palash Kundu, P. (2016). Review on Different Evapotranspiration Empirical Equations. *International Journal of Advanced Engineering, Management and Science*, 2(3), 239382.

[46]. Ranganathan, A. (2004). The Levenberg-Marquardt Algorithm. Tutorial On LM Algorithm, 11(1), 101-110.

[47]. Rao, S. (2009). Engineering Optimisation: Theory and Practice. 4th edn. John Wiley & Sons.

[48]. Rockafellar, T. (1994). Nonsmoothoptimisation. *Mathematical Programming: State of the Art, 248-258.*

[49]. Rodrigues¹, D. F. B., de Albuquerque Moura, G. B., Gusmão, L., & dos Santos Araújo, D. C. (2018). EvapoCalc: An Android Application to Estimate Evapotranspiration by Different Methods. *EPiC Series in Engineering*, 3, 1772-1777.

[50]. Russenschuck, S. (1999). Mathematical Optimisation Techniques (No. CERN-OPEN-2000-152). CERN

[51]. .S.'Odea (2022). Smartphone users in Nigeria 2014-2025 Retrieved January, 2023 fromhttps://www.statista.com/statistics/467187/forecast-of-smartphone-users-in-nigeria/

[52]. Sakhaei, Z., Azin, R., &Osfouri, S. (2016). Assessment of empirical/theoretical relative permeability correlations for gas-oil/condensate systems. In Proceedings of the 1st Biennial Persian Gulf Oil, Gas and Petrochemical Energy and Environment Conference, Bushehr, Iran (Vol. 20).

[53]. Samani, Z. (2000). Estimating Solar Radiation and Evapotranspiration Using Minimum Climatological Data. *Journal of Irrigation and Drainage Engineering*, *126*(4), 265-267

[54]. .Schlesinger, W. H., &Jasechko, S. (2014). Transpiration in the Global Water Cycle. *Agricultural and Forest Meteorology,* 189, 115-117.

[55]. Schrader, F., Durner, W., Fank, J., Gebler, S., Pütz, T., Hannes, M., &Wollschläger, U. (2013). Estimating Precipitation and Actual Evapotranspiration from Precision Lysimeter Measurements. *Procedia Environmental Sciences*, *19*, 543-552.

[56]. Shiri, J. (2017). Evaluation of FAO56-PM, empirical, semi-empirical and gene expression programming approaches for estimating daily reference evapotranspiration in hyper-arid regions of Iran. *Agricultural Water Management*, 188, 101-114.

[57]. Sun, Z., Wang, Q., Batkhishig, O., & Ouyang, Z. (2016). Relationship Between Evapotranspiration and Land Surface Temperature Under Energy-And Water-Limited Conditions in Dry and Cold Climates. *Advances in Meteorology, 2016*.

[58]. Tabari, H., &Talaee, P. H. (2011). Local calibration of the Hargreaves and Priestley-Taylor equations for estimating reference evapotranspiration in arid and cold climates of Iran based on the Penman-Monteith model. *Journal of Hydrologic Engineering, 16(10), 837.*

[59]. Tafesse, N., Gebrehiwet, M., Habtu, S., &Alemaw, B. F. (2018). Water Balance Model: Implications for Groundwater Recharge Estimation in Data Scarce Arid Catchment, Northern Ethiopia. *Asian Review of Environmental and Earth Sciences,* 5, 34-46.

[60]. Team, F. (2006). Optimisation, Nonlinear Fitting and Equations Solving. *Tutorial on Numerical Analysis with Optimiz. xla*, 1.

[61]. Transtrum, M. K., & Sethna, J. P. (2012). Improvements to the Levenberg-Marquardt Algorithm for Nonlinear Least-squares Minimization. arXiv preprint arXiv:1201.5885.
[62]. Transtrum, M. K., Machta, B. B., & Sethna, J. P. (2011). Geometry of Nonlinear Least Squares with Applications to Sloppy Models and Optimisation. *Physical Review E,* 83(3), 036701.
[63]. Umego, O. M., Ewemoje, T. A., & Ilesanmi, O. A. (2018). Comparative Study of Evapotranspiration Variation and its Relationship with other Climatic Parameters in Asaba and Uyo. DOI: http://dx.doi.org/10.46792/fuoyejet.v3i2.221
[64]. United Nations Office for the Coordination of Humanitarian Affairs (2016). Retrieved January,2023 from https://www.humanitarianresponse.info/en/node/119748?_gl=1*rsmo6u*_ga*MjA3MTQyNDAyOS4xNjczMDE2MTI3*_ga_E60ZNX2F68*MTY3MzAxNjEyNy4xLjAuMTY3MzAxNjEyNy42MC4wLjA.
[65]. Venter, G. (2010)Review of Optimization Techniques. Retrieved October 2021 from https://scholar.google.com/scholar?hl=en&as_sdt=0%2C5&q=venter+g+review+of+optimization+techniques+2010&btnG=#d=gs_qabs&t=1674704644910&u=%23p%3DTVSJkVcQS04J
[66]. Yan, H., & Shugart, H. H. (2010). An Air Relative- Humidity- Based Evapotranspiration Model from Eddy Covariance Data. *Journal of Geophysical Research: Atmospheres*, *115*(D16).
[67]. Yusuf, A. S., Edet, C. O., Oche, C. O., &Agbo, E. P. (2018). Trend analysis of temperature in Gombe state using Mann Kendall trend test. *Journal of Scientific Research & Reports* 20(3): 1-9, 2018; Article no. JSRR.42029 ISSN: 2320-0227
[68]. Zakwan, M. (2017). Assessment of Dimensionless form of Kostiakov Model. *Aquademia,* 1(1), 01.
[69]. Zhou, S., Liu, W., & Lin, W. (2017). The Ratio of Transpiration to Evapotranspiration in A Rainfed Maize Field on the Loess Plateau of China. *Water Science and Technology: Water Supply*, *17*(1), 221-228.
[70]. Zotarelli, L., Dukes, M. D., Romero, C. C., Migliaccio, K. W., & Morgan, K. T. (2010). Step by Step Calculation of the Penman-Monteith Evapotranspiration (FAO-56 Method). Institute of Food and Agricultural Sciences. University of Florida.
[71]. Zou, M., Niu, J., Kang, S. *Et Al.* The Contribution of Human Agricultural Activities to Increasing Evapotranspiration Is Significantly Greater Than Climate Change Effect Over Heihe Agricultural Region. *Sci Rep* **7,** 8805 (2017). Https://Doi.Org/10.1038/S41598-017-089525

APPENDICE 1: Summary of Optimisation Outputs of BMN for Gombe State in MATLAB

Iteration	Func-count	Residual	First-Order optimality	Lambda	Norm of step
0	3	134.13	76.5216	0.01	-
1	6	86.6889	98.1	0.001	34.9271
2	9	76.767	38.5	0.0001	24.7138
3	12	76.5218	1.3	1e-05	2.21106
4	15	76.5216	0.0032	1e-06	0.0374687
5	18	76.5216	1.16e-05	1e-07	0.000370169
6	21	76.5216	5.29e-07	1e-08	1.71726e-06

```
H_est = 520 m_est = 1.31
<stopping criteria details>
H_opt = 458.1103
m_opt = 1.1974
resnorm = 76.5216
residual =
0.4542
0.2208
0.2597
-1.8428…
exitflag = 3
output =
struct with fields:
iterations: 6
funcCount: 21
stepsize: 1.7173e-06
cgiterations: []
firstorderopt: 5.2938e-07
 algorithm: 'levenberg-marquardt'
message: … '↵Optimisation stopped because the relative sum of squares (r) is changing by less than options. Function Tolerance = 1.000000e-15. ↵↵'
lambda =
struct with fields:
upper: [2×1 double]
lower: [2×1 double]
fval =
0.0148   -2.2521
0.0146   -2.0892
0.0173   -1.5911
0.0129   -0.8927…
ans =
7.1042   7.8950   6.3137
6.2508   7.0335   5.4684
6.3997   7.2011   5.5987
4.1172   4.9415   3.2931…
```