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Consolidation analysis of composite ground improved by CCSG columns with short core under time-dependent loading

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Abstract: *Ignoring radial flow in the sand-gravel shell and the sand-gravel column beneath it, the consolidation equations and the corresponding solution conditions of the composite ground improved by concrete-core sand-gravel (CCSG) columns with short core are derived under time-dependent loading. These consolidation equations are solved by the method of separation of variables, and the average excess pore water pressures within the surrounding soil, sand-gravel shell, sand-gravel column and the upper and lower improved zones are obtained. The overall average consolidation degree of the composite ground is calculated using the average excess pore water pressures in the two improved zones. Then, the rationality of the proposed solution is verified by comparison of the degenerated solutions with an existing solutions. Finally, the consolidation behavior of the composite ground is investigated by a parametric analysis using the proposed solution. The results indicate that the core-column length ratio has little effect on the consolidation rate of the composite ground. When it is smaller, the changes in the diameter and stiffness of the core pile have insignificant influence on the consolidation rate. When the core-column length ratio is larger, the consolidation rate gradually increases with increasing stiffness of the core pile, and increases at first and then decreases with an increase in the diameter of the core pile. The consolidation rate increases with an increase in the diameter and stiffness of the sand-gravel shell, and decreases with the well resistance in it.*
Key words: *composite ground; CCSG column with short core; consolidation; analytical solution; sand-gravel shell*

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I. INTRODUCTION

Concrete core sand and stone (CCSG) pile is a composite pile composed of two materials: the concrete core pile located in the center and the surrounding sand and stone shell. The length of core pile is usually the same as sand shell, but also shorter or longer than sand shell, which are called equal core CCSG pile, short core CCSG pile and long core CCSG pile respectively. As the core pile has high stiffness and bearing capacity and the sand shell has strong drainage capacity, CCSG pile composite foundation has high bearing capacity, small settlement and fast consolidation, and has been applied in many soft foundation reinforcement projects [1]

Some scholars have studied the bearing capacity and consolidation characteristics of CCSG pile composite foundation. Tang Tongzhi et al[2] studied the settlement, post-construction settlement control effect and pile-soil load transfer characteristics of CCSG pile composite foundation under embankment through field tests. Jiang et al[3] used the three-dimensional finite element method to simulate and analyze the load transfer and consolidation characteristics between core pile, sand shell and soil of composite foundation with soil-soil-core sand pile on rigid foundation. Weng Jiawei et al [4] used the axisymmetric finite element method to simulate and analyze the consolidation characteristics of the composite foundation of bearing and suspended short core CCSG stone piles at the bottom of the embankment.

The development of the consolidation theory of crushed stone pile composite foundation lays a foundation for the establishment of the analytical calculation model of CCSG pile composite foundation consolidation. As early as 2002, Han et al[5] ignored the vertical seepage of soil between piles and proposed an analytical solution for consolidation of the composite foundation of gravel piles under instantaneous load, which considered the effect of well group and smear. At present, the consolidation theory research of gravel pile composite foundation has achieved fruitful results. The proposed consolidation analytic solution can consider the influence of many factors such as radial and vertical combined seepage [6], double-sided semi-permeable boundary [7], nonlinearity [8] and unsaturated [9] of foundation soil, variable load [6][9], pile-soil free strain [10] and foundation stiffness [11].

At present, some progress has been made in the study of analytical consolidation theory of CCSG pile composite foundation. Shi Beixiao et al[12] proposed the analytical solution of consolidation of CCSG pile composite foundation under instantaneous load. Yu Jin et al[13] improved the analytical consolidation solution of Shi Beixiao et al[12], and the proposed analytical consolidation solution could consider the change of the radial permeability coefficient of soil in the smear area. Yang Yanwei et al [14] gave the analytical solution of consolidation of CCSG pile composite foundation under instantaneous load considering non-equal strain between core pile, sand shell and foundation soil. Qi Zhibo et al[15] obtained the analytical solution of consolidation of CCSG pile composite foundation under variable load. CAI Yanyan et al[16] proposed the analytical solution of nonlinear consolidation of CCSG pile composite foundation under instantaneous load. Lu et al [17] established the analytical solution of consolidation of CCSG pile composite foundation with irregular section under variable load. Ye Guanbao et al[18] gave the analytical solution of radial consolidation of CCSG pile composite foundation under vacuum and pile preloading, which is applied instantaneously under both load and vacuum.

The above analytical solutions for consolidation of CCSG pile composite foundation are all proposed for equal-

core CCSG pile. The length of the core pile in the actual project should be determined according to the load and the engineering geological conditions of the site, etc., and the length may not be the same as that of the sand and stone shell. For example, when there is liquefied silt under the soft soil layer, the short-core CCSG pile composite foundation reinforcement scheme can be considered. Therefore, it is necessary to study the consolidation calculation method of short core CCSG pile composite foundation. The purpose of this paper is to establish the analytical solution of consolidation of composite foundation with short core CCSG pile, and obtain the consolidation characteristics of composite foundation by using the established analytical solution, so as to further improve the consolidation theory of CCSG pile composite foundation and provide theoretical guidance for engineering design.

II. CONSOLIDATION MODEL AND BASIC ASSUMOTION

2.1 Computational model

Fig. 1 shows the axisymmetric consolidation model of short core CCSG pile composite foundation for formula derivation. The thickness of composite foundation is H, and the radius of influence zone of single pile is r_e. The length and radius of short-core CCSG are H and r_p, respectively, and the corresponding replacement rate is $m=(r_p/r_e)^2$. The radius and length of the core pile were r_c and H_1, and the core length ratio $\beta= H_1/ H$. The length of sand shell around the core pile is H_1, the outer radius is r_p, and the section containing heart rate is $\rho=(r_c/r_p)^2$. The length of sand pile under the core pile is H_2, the radius is r_p, $H=H_1+H_2$. r_s is the outer radius of the coating area, E_c and E_p are the compression modulus of the core pile and the sand shell and the underlying sand pile, respectively, and k_p is the permeability coefficient of the sand shell (pile). The reinforcement is divided into upper and lower parts according to the length of the core pile. k_{hi}, k_{vi} and E_{si} are the diameter, vertical permeability coefficient and compression modulus of the soil between the I-layer (i=1,2) undisturbed piles, respectively. p(t) is the uniform external load on the composite foundation, and r and z are the diameters and vertical coordinates of the system respectively.

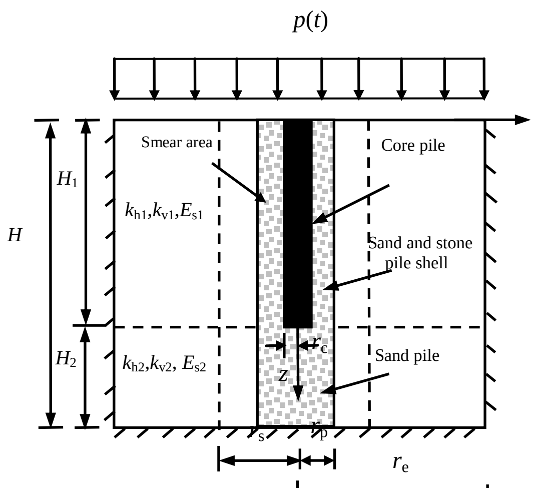


Fig.1 Axisymmetric consolidation model

2.2 Basic assumption

This paper deduces the formula based on the following basic assumptions:

(1) Only vertical deformation occurs in sand shell, core pile, sand pile under core pile and soil between piles. Referring to the study of Lu et al[17], it is assumed that the equal strains in the upper and lower reinforcement zones are established respectively: the same vertical strain εz1 exists in the core pile, sand shell and soil between piles at any depth in the upper reinforcement zone, and the same vertical strain εz2 exists in the sand pile and soil between piles at any depth in the lower reinforcement zone.

(2) There is no radial seepage in the sand shell and the underlying sand pile.

(3) The properties of the soil in the daubed area are the same as those in the undisturbed area except for the radial permeability coefficient.

(4) The soil between piles, the sand shell and the underlying sand and gravel piles are saturated, the seepage follows Darcy's law, and the permeability coefficient and compression modulus remain unchanged in the consolidation process.

(5) The vertical additional stress σ(t) caused by the external load p(t) in the natural foundation is uniformly distributed along the depth.

III. CONSOLIDATION EQUATON

Combined diameter and vertical seepage occurs in soil between piles, and the consolidation equation is as follows[17]:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[\frac{k_{ri}(r)}{\gamma_w} r\frac{\partial u_{si}}{\partial r}\right]+\frac{k_{vi}}{\gamma_w}\frac{\partial^2 \bar{u}_{si}}{\partial z^2}=-\frac{\partial \varepsilon_{zi}}{\partial t}\ (i=1,2) \quad (1)$$

where, γ_w is the water weight, $u_{si}(r, z, t)$and $\bar{u}_{si}(z,t)$ are respectively the excess pore pressure at any point (r, z) and the average excess pore pressure at depth z in the soil between piles at time t, $k_{ri}(r)=f_i(r)k_{hi}$ is the radial permeability coefficient of soil between piles at layers i, and $f_i(r)$ is the function of variation with r.

The radial boundary conditions at the outer boundary of the axisymmetric consolidation model are:

$$r=r_e;\frac{\partial u_{si}}{\partial r}=\frac{\partial \bar{u}_{si}}{\partial r}=0 \quad (i=1, 2) \qquad (2)$$

The continuity condition of pore pressure at the upper sand-gravel shell-soil interface and the lower sand-gravel pile-soil interface is：

$$r=r_e;\ u_{si}=u_{pi}=\bar{u}_{pi} \qquad (i=1, 2) \qquad (3)$$

where, $u_{p1}(r, z, t)$ and $u_{p2}(r, z, t)$ are respectively the excess pore pressure at any point (r, z) in the sand shell and the sand pile below it at time t, $\bar{u}_{p1}(z,t)$ and $\bar{u}_{p2}(z,t)$ are respectively the average excess pore pressure at any depth z in the sand shell and the sand pile below it at time t. Without considering the radial seepage in the sand shell (pile), there is $u_{pi}=\bar{u}_{pi}$ (i=1,2).

According to hypothesis (2), the radial water flow of soil into sand shell (pile) at depth z is equal to the water flow from the latter upward, namely:

$$\left[2\pi r\frac{k_{ri}(r)}{\gamma_w}\frac{\partial u_{si}}{\partial r}\right]_{r=r_p}=-A_{pi}\frac{k_p}{\gamma_w}\frac{\partial^2 \bar{u}_{pi}}{\partial z^2} \quad (i=1, 2) \qquad (4)$$

where, $A_{p1}=\pi(r_p^2 - r_c^2)$ and $A_{p2}=\pi r_p^2$ are respectively the cross-sectional area of the sand shell and the sand pile below it.

Based on the assumption of vertical balance of force and equal vertical strain, we can get:

$$\frac{\partial \varepsilon_{zi}}{\partial t}=\frac{1}{E_{comi}}\left[\frac{d\sigma(t)}{dt}-\frac{\partial \bar{u}_i}{\partial t}\right] \quad (i=1,2) \qquad (5)$$

$$\bar{u}_1=m(1-\rho)\bar{u}_{p1}+(1-m)\bar{u}_{s1} \qquad (6)$$

$$\bar{u}_2=m\bar{u}_{p2}+(1-m)\bar{u}_{s2} \qquad (7)$$

$$E_{com1}=m\left[\rho E_c+(1-\rho)E_p\right]+(1-m)\ E_{s1} \qquad (8)$$

$$E_{com2}=mE_p+(1-m)E_{s2} \qquad (9)$$

where, $\bar{u}_i(z,t)$ is the average excess pore pressure at depth z of the ith reinforcement zone at time t, and E_{comi} is the composite compression modulus of the reinforcement zone.

Referring to the derivation method of Lu et al[17], the consolidation equation can be obtained by integrating equation (1) with respect to r and using equations (2) ~ (5) :

$$A_i\frac{\partial^4\overline{u}_{\mathrm{pi}}}{\partial z^4}+B_i\frac{\partial^3\overline{u}_{\mathrm{pi}}}{\partial t\partial z^2}+C_i\frac{\partial^2\overline{u}_{\mathrm{pi}}}{\partial z^2}+D_i\frac{\partial\overline{u}_{\mathrm{pi}}}{\partial t}=\frac{\mathrm{d}\sigma(t)}{\mathrm{d}t}\quad (i=1,\,2)\qquad(10)$$

$$\overline{u}_i=D_i\overline{u}_{\mathrm{pi}}+B_i\frac{\partial^2\overline{u}_{\mathrm{pi}}}{\partial z^2}\quad (i=1,\,2)\qquad(11)$$

$$\overline{u}_{\mathrm{si}}=\overline{u}_{\mathrm{pi}}-L_i\frac{\partial^2\overline{u}_{\mathrm{pi}}}{\partial z^2}\quad (i=1,\,2)\qquad(12)$$

$$\begin{cases}A_i=\dfrac{L_ik_{\mathrm{v}i}E_{\mathrm{com}i}}{\gamma_w},\ \ B_i=-(1-m)L_i\quad (i=1,2)\\ C_i=-\dfrac{k_{\mathrm{v}i}E_{\mathrm{com}i}}{\gamma_w}(1+\dfrac{2L_i}{r_e^2F_{\mathrm{c}i}}\dfrac{k_{\mathrm{h}i}}{k_{\mathrm{v}i}})\quad (i=1,2)\\ D_1=1-m\rho,\quad D_2=1.0\\ L_1=\dfrac{r_e^2F_{\mathrm{c}1}k_{\mathrm{p}}m(1-\rho)}{2k_{\mathrm{h}1}(1-m)},L_2=\dfrac{r_e^2F_{\mathrm{c}2}k_{\mathrm{p}}m}{2k_{\mathrm{h}2}(1-m)}\end{cases}\qquad(13)$$

where, F_{ci} (i=1,2) is the parameter reflecting smudging effect caused by short core CCSG pile construction in the i reinforcement zone. Xie et al[19] gave the formula for calculating F_{c} when the radial permeability coefficient of smear area was distributed in three modes: constant, linear and parabolic.

IV. THE CONSOLIDATION EQUATON IS SOLVED

4.1 The conditions for solving the consolidation equation

If the top surface of composite foundation is drained and the bottom surface is not drained, the definite solution conditions of the above consolidation equation are as follows:

(1) Vertical boundary conditions

$$z=0:\overline{u}_{\mathrm{s}1}=0,\overline{u}_{\mathrm{p}1}=0,\overline{u}_1=0\qquad(14)$$

$$z=H:\frac{\partial\overline{u}_{\mathrm{s}2}}{\partial z}=0,\frac{\partial\overline{u}_{\mathrm{p}2}}{\partial z}=0,\frac{\partial\overline{u}_2}{\partial z}=0\qquad(15)$$

(2) The horizontal pressure and seepage continuity conditions of core pile end are as follows:

$$z=H_1:\begin{cases}\overline{u}_{\mathrm{p}1}=\overline{u}_{\mathrm{p}2}\\ \overline{u}_{\mathrm{s}1}=\overline{u}_{\mathrm{s}2}\\ (1-\rho)k_{\mathrm{p}}\dfrac{\partial\overline{u}_{\mathrm{p}1}}{\partial z}=k_{\mathrm{p}}\dfrac{\partial\overline{u}_{\mathrm{p}2}}{\partial z}\\ k_{\mathrm{v}1}\dfrac{\partial\overline{u}_{\mathrm{s}1}}{\partial z}=k_{\mathrm{v}2}\dfrac{\partial\overline{u}_{\mathrm{s}2}}{\partial z}\end{cases}\qquad(16)$$

(3) Initial conditions

$$t=0:\overline{u}_i=0;\ \ \overline{u}_{\mathrm{pi}}=0;\ \ \overline{u}_{\mathrm{si}}=0\ (i=1,\,2)\qquad(17)$$

4.2 The solution to the consolidation equation

The consolidation equations (10) ~ (12) were solved by using the separation of variables method under the definite solution condition equations (14) ~ (17). According to the solution method of non-homogeneous partial differential equation, the general form of pressure solution of sand holes in the upper and lower reinforcement areas satisfying vertical boundary conditions (14) and (15) can be written as follows:

$$\overline{u}_{\mathrm{pi}}=\sum_{n=1}^{\infty}T_n(t)V_{ni}(z)\quad (i=1,\,2)\qquad(18)$$

$$V_{n1}(z)=a_{n1}sh(\eta_{n1}z)+d_{n1}\sin(\xi_{n1}z)\qquad(19)$$

$$V_{n2}(z)=b_{n2}ch[\eta_{n2}(H-z)]+c_{n2}\cos[\xi_{n2}(H-z)]\qquad(20)$$

$$\begin{cases} \eta_{ni} = \sqrt{\dfrac{-(C_i - \beta_n B_i) + \sqrt{(C_i - \beta_n B_i)^2 + 4A_i\beta_n D_i}}{2A_i}} \\ \xi_{ni} = \sqrt{\dfrac{(C_i - \beta_n B_i) + \sqrt{(C_i - \beta_n B_i)^2 + 4A_i\beta_n D_i}}{2A_i}} \end{cases} \quad (i=1,\ 2) \qquad (21)$$

where, β_n、 a_{n1}、d_{n1}、b_{n2} and c_{n2} Are undetermined constants.

By substituting equation (18) into Equation (11) and Equation (12), we get:

$$\bar{u}_i = \sum_{n=1}^{\infty} T_n(t) W_{ni}(z) \quad (i=1,\ 2) \qquad (22)$$

$$\bar{u}_{si} = \sum_{n=1}^{\infty} T_n(t) N_{ni}(z) \quad (i=1,\ 2) \qquad (23)$$

$$W_{n1}(z) = a_{n1}(D_1 + B_1\eta_{n1}^2)\mathrm{sh}(\eta_{n1}z) + d_{n1}(D_1 - B_1\xi_{n1}^2)\sin(\xi_{n1}z) \qquad (24)$$

$$W_{n2}(z) = b_{n2}(D_2 + B_2\eta_{n2}^2)\mathrm{ch}[\eta_{n2}(H-z)] + c_{n2}(D_2 - B_2\xi_{n2}^2)\cos[\xi_{n2}(H-z)] \quad (25)$$

$$N_{n1}(z) = a_{n1}(1 - L_1\eta_{n1}^2)\mathrm{sh}(\eta_{n1}z) + d_{n1}(1 + L_1\xi_{n1}^2)\sin(\xi_{n1}z) \qquad (26)$$

$$N_{n2}(z) = b_{n2}(1 - L_2\eta_{n2}^2)\mathrm{ch}[\eta_{n2}(H-z)] + c_{n2}(1 + L_2\xi_{n2}^2)\cos[\xi_{n2}(H-z)] \qquad (27)$$

By substituting equations (18) and (22) into equations (11), we can get:

$$W_{ni} = B_i V_{ni}^{(2)} + D_i V_{ni} \quad (i=1,\ 2) \qquad (28)$$

The homogeneous equation corresponding to equation (10) is solved by the separation of variables method, thus:

$$A_i V_{ni}^{(4)} + C_i V_{ni}^{(2)} = \beta_n [B_i V_{ni}^{(2)} + D_i V_{ni}] \quad (i=1,\ 2) \qquad (29)$$

β_n、 a_{n1}、d_{n1}、b_{n2} and c_{n2} were determined by continuity conditions. By substituting equations (18) and (23) into equations (16), the following linear equations for solving a_{n1}、d_{n1}、b_{n2} and c_{n2} can be obtained:

$$\boldsymbol{S} \times \boldsymbol{X}^{\mathrm{T}} = 0$$

（30）

$$\boldsymbol{X} = (a_{m1}, d_{m1}, b_{m2}, c_{m2})$$

（31）

where, S is a 4×4 coefficient matrix containing β_n.. In order to obtain the non-zero solutions of a_{n1}、d_{n1}、b_{n2} and c_{n2}, the determinant of matrix S should be *0*, that is:

$$|\boldsymbol{S}| = 0 \qquad (32)$$

From equation (32), β_n（n=1, 2, 3,…,∞）. By substituting β_n into equation (30) and setting a_{n1}=1.0, the other three undetermined constants $_{n1}$、b_{n2} and c_{n2} can be solved.

$T_n(t)$ is determined by the initial conditions. Equation (18) is substituted into equation (10), and by means of equation (28) and (29), we get:

$$\sum_{n=1}^{\infty} [T_n(t)\beta_n + T_n'(t)] W_{ni} = \frac{\mathrm{d}\sigma(t)}{\mathrm{d}t} \quad (i=1,2) \qquad (33)$$

W_{n1} and $W_{n1\backslash 2}$ have the following weighted orthogonal relations：

$$\int_0^{H_1} \frac{1}{E_{\mathrm{com1}}} W_{n1} W_{j1} \mathrm{dz} + \int_{H_1}^{H} \frac{1}{E_{\mathrm{com2}}} W_{n2} W_{j2} \mathrm{dz} = 0;\ (n \neq j) \qquad (34)$$

By using orthogonal relation (34) to organize equation (33), we can get:

$$T_n'(t) + \beta_n T_n(t) = M_n \frac{\mathrm{d}\sigma(t)}{\mathrm{d}t} \qquad (35)$$

$$M_n = \frac{\sum_{i=1}^{2} \dfrac{K_{ni}}{E_{\mathrm{com}i}}}{\sum_{i=1}^{2} \dfrac{Q_{ni}}{E_{\mathrm{com}i}}} \qquad (36)$$

$$K_{n1} = \int_0^{H_1} W_{n1} dz = \frac{1}{1-\rho} \left\{ \frac{a_{n1}}{\eta_{n1}} (D_1 + B_1\eta_{n1}^2) [ch(\eta_{n1}H_1) - 1] + \frac{d_{n1}}{\xi_{n1}} (D_1 - B_1\xi_{n1}^2) [1 - \cos(\xi_{n1}H_1)] \right\} \qquad (37)$$

$$K_{n2}=\int_{H_1}^{H}W_{n2}dz=(1+B_2\eta_{n2}^2)\frac{b_{n2}}{\alpha_{n2}}sh(\eta_{n2}H_2)+(1-B_2\xi_{n2}^2)\frac{c_{n2}}{\xi_{n2}}\sin(\xi_{n2}H_2) \quad (38)$$

$$Q_{n1}=\int_0^{H_1}W_{n1}^2dz=a_{n1}^2(D_1+B_1\eta_{n1}^2)^2\left[\frac{sh(2\eta_{n1}H_1)}{4\eta_{n1}}-\frac{H_1}{2}\right]+d_{n1}^2(D_1-B_1\xi_{n1}^2)^2\left[\frac{H_1}{2}-\frac{\sin(2\xi_{n1}H_1)}{4\xi_{n1}}\right]$$
$$+\frac{2a_{n1}d_{n1}(D_1+B_1\eta_{n1}^2)(D_1-B_1\xi_{n1}^2)}{\eta_{n1}\left[1+(\frac{\xi_{n1}}{\eta_{n1}})^2\right]}\times\left[ch(\eta_{n1}H_1)\sin(\xi_{n1}H_1)-\frac{\xi_{n1}}{\eta_{n1}}sh(\eta_{n1}H_1)\cos(\xi_{n1}H_1)\right]$$

(39)

$$Q_{n2}=\int_{H_1}^{H}W_{n2}^2dz=b_{n2}^2(D_2+B_2\eta_{n2}^2)^2\left[\frac{H_2}{2}+\frac{sh(2\eta_{n2}H_2)}{4\eta_{n2}}\right]+c_{n2}^2(D_2-B_2\xi_{n2}^2)^2\left[\frac{H_2}{2}+\frac{\sin(2\xi_{n2}H_2)}{4\xi_{n2}}\right]$$
$$+\frac{2b_{n2}c_{n2}}{\eta_{n2}{}^2+\xi_{n2}{}^2}(D_2+B_2\eta_{n2}^2)(D_2-B_2\xi_{n2}^2)\times\left[\xi_{n2}ch(\eta_{n2}H_2)\sin(\xi_{n2}H_2)+\eta_{n2}sh(\eta_{n2}H_2)\cos(\xi_{n2}H_2)\right]$$

(40)

By substituting equation (22) into initial condition equation (17), we get：:

$$t=0:T_n(t)=0 \quad (41)$$

Under the initial condition formula (41), the nonhomogeneous ordinary differential solution formula (35) is obtained：

$$T_n(t)=M_ne^{-\beta_nt}\int_0^t\frac{\mathrm{d}\sigma(t)}{\mathrm{d}\tau}e^{\beta_nt}d\tau \quad (42)$$

By substituting equation (42) into Equation (18), (22) and (23), each excess pore pressure can be obtained as follows：

$$u_{\mathrm{p}i}=\sum_{n=1}^{\infty}M_ne^{-\beta_nt}V_{ni}(z)\int_0^t\frac{\mathrm{d}\sigma}{\mathrm{d}t}e^{\beta_nt}\mathrm{d}\tau \quad (i=1,2) \quad (43)$$

$$u_{\mathrm{s}i}=\sum_{n=1}^{\infty}M_ne^{-\beta_nt}N_{ni}(z)\int_0^t\frac{\mathrm{d}\sigma}{\mathrm{d}t}e^{\beta_nt}\mathrm{d}\tau \quad (i=1,\ 2) \quad (44)$$

$$u_i=\sum_{n=1}^{\infty}M_ne^{-\beta_nt}W_{ni}(z)\int_0^t\frac{\mathrm{d}\sigma}{\mathrm{d}t}e^{\beta_nt}\mathrm{d}\tau \quad (i=1,\ 2) \quad (45)$$

4.3 Average consolidation degree of composite foundation

The composite foundation has two overall average consolidation degrees, U_{p} and U_{s}, defined by pore pressure and settlement, and their calculation formulas are as follows:

$$U_{\mathrm{p}}(t)=\frac{\int_0^{H_1}[\sigma(t)-\bar{u}_1]\mathrm{d}z+\int_{H_1}^{H}[\sigma(t)-\bar{u}_2]\mathrm{d}z}{\int_0^H\sigma_{\mathrm{u}}\mathrm{d}z} \quad (46)$$

$$U_{\mathrm{s}}(t)=\frac{\int_0^{H_1}\frac{\sigma(t)-\bar{u}_1}{E_{\mathrm{com1}}}\mathrm{d}z+\int_{H_1}^{H}[\frac{\sigma(t)-\bar{u}_2}{E_{\mathrm{com2}}}]\mathrm{d}z}{\int_0^{H_1}\frac{\sigma_{\mathrm{u}}}{E_{\mathrm{com1}}}\mathrm{d}z+\int_{H_1}^{H}\frac{\sigma_{\mathrm{u}}}{E_{\mathrm{com2}}}\mathrm{d}z} \quad (47)$$

where, σ_{u} is the final value of vertical additional stress in the foundation.

4.4 Solution under single stage loading

The curve of vertical additional stress σ(t) in foundation caused by single-order load over time is shown in Figure 2, which can be expressed by the following formula:

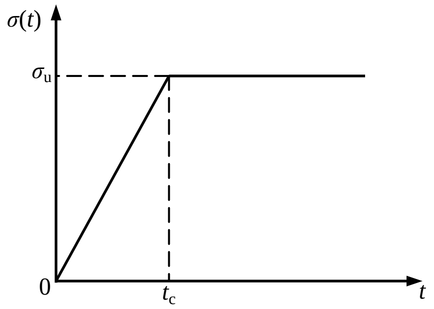


Fig.2 $\sigma(t)$-t curve under ramp loading

$$\sigma(t)=\begin{cases}\dfrac{t}{t_c}\sigma_{\mathrm{u}} & (t\leqslant t_{\mathrm{c}})\\ \sigma_{\mathrm{u}} & (t\geqslant t_{\mathrm{c}})\end{cases} \tag{48}$$

where, t_c is the loading time. By substituting equation (48) into equation (43) ~ (45), each excess pore pressure can be obtained as follows:

$$\bar{u}_{pi}=\begin{cases}\dfrac{\sigma_u}{t_c}\sum_{n=1}^{\infty}\dfrac{M_n}{\beta_n}V_{ni}(1-e^{-\beta_n t}) & (0<t\le t_c)\\ \dfrac{\sigma_u}{t_c}\sum_{n=1}^{\infty}\dfrac{M_n}{\beta_n}V_{ni}e^{-\beta_n t}(e^{\beta_n t_c}-1) & (t\ge t_c)\end{cases}\quad (i=1,2) \tag{49}$$

$$\bar{u}_{si}=\begin{cases}\dfrac{\sigma_u}{t_c}\sum_{n=1}^{\infty}\dfrac{M_n}{\beta_n}N_{ni}(1-e^{-\beta_n t}) & (0<t\le t_c)\\ \dfrac{\sigma_u}{t_c}\sum_{n=1}^{\infty}\dfrac{M_n}{\beta_n}N_{ni}e^{-\beta_n t}(e^{\beta_n t_c}-1) & (t\ge t_c)\end{cases}\quad (i=1,2) \tag{50}$$

$$\bar{u}_{i}=\begin{cases}\dfrac{\sigma_u}{t_c}\sum_{n=1}^{\infty}\dfrac{M_n}{\beta_n}W_{ni}(1-e^{-\beta_n t}) & (0<t\le t_c)\\ \dfrac{\sigma_u}{t_c}\sum_{n=1}^{\infty}\dfrac{M_n}{\beta_n}W_{ni}e^{-\beta_n t}(e^{\beta_n t_c}-1) & (t\ge t_c)\end{cases}\quad (i=1,2) \tag{51}$$

By substituting equations (48) and (51) into equations (46) and (47), we get:

$$U_p(t)=\begin{cases}\dfrac{t}{t_c}-\dfrac{1}{Ht_c}\sum_{n=1}^{\infty}\dfrac{M_n(K_{n1}+K_{n2})}{\beta_n}(1-e^{-\beta_n t}) & (0<t\le t_c)\\ 1-\dfrac{1}{Ht_c}\sum_{n=1}^{\infty}\dfrac{M_n(K_{n1}+K_{n2})}{\beta_n}e^{-\beta_n t}(e^{\beta_n t_c}-1) & (t\ge t_c)\end{cases} \tag{52}$$

$$U_s(t)=\begin{cases}\dfrac{t}{t_c}-\dfrac{1}{t_c}\sum_{n=1}^{\infty}\dfrac{M_nR_n}{\beta_n}(1-e^{-\beta_n t}) & (0<t\le t_c)\\ 1-\dfrac{1}{t_c}\sum_{n=1}^{\infty}\dfrac{M_nR_n}{\beta_n}e^{-\beta_n t}(e^{\beta_n t_c}-1) & (t\ge t_c)\end{cases} \tag{53}$$

$$R_n=\frac{\sum_{i=1}^{2}\dfrac{K_{ni}}{E_{\mathrm{com}i}}}{\sum_{i=1}^{2}\dfrac{H_i}{E_{\mathrm{com}i}}} \tag{54}$$

4.5 Discussion of solutions

The above consolidation analytical solution is discussed below, and the degenerate consolidation degree solution is compared with the calculated results of the existing consolidation analytical solution to verify its rationality.

(1) If the core length ratio β=1.0, the short-core CCSG pile becomes equal-core CCSG pile, and the consolidation analytic solution degrades to the consolidation analytic solution of equal-core CCSG pile composite foundation under varying load.

(2) If the section contains heart-rate ρ=0, the short core CCSG pile becomes double-layer granular material pile, and the consolidation analytical solution degenerates to the consolidation analytical solution of double-layer granular material pile composite foundation under variable load.

(3) If k_p=0 and E_p takes soil-cement compression modulus, short core CCSG pile becomes short core stiffly stirred (TDM) pile, and the consolidation analytical solution degenerates to the consolidation analytical solution of short core TDM pile composite foundation under variable load. Furthermore, β=1.0 is taken as the analytical solution for the consolidation of the composite foundation of equal-core TDM piles.

(4) If β=ρ=1.0, the short core CCSG pile becomes undrained pile, and the consolidation analytic solution degenerates to the consolidation analytic solution of undrained pile composite foundation under variable load.

It can be seen that the consolidation analysis of equal-core CCSG pile composite foundation, double-layer granular material pile composite foundation, stiffly stirred pile composite foundation and undrained pile composite foundation under varying loads are all special cases of the consolidation analytical solution in this paper.

Below, the consolidation degree of the composite foundation of equal-core CCSG is calculated by using the consolidation analytic solution presented in this paper, and the consolidation degree of the composite foundation of short-core TDM pile is calculated by using the regression solution presented in this paper. The calculated results are compared with the existing consolidation analytic solution, and the abscissa is lg$t$. The thickness of composite foundation is H=20m, the radius of equal-core CCSG pile and short-core TDM pile is r_p=0.25m, and the radius of influence zone of single pile is r_e=1m. The radius and compression modulus of the two kinds of pile are r_c=0.115m and E_c =20GPa, respectively. The compressive modulus E_{s1}=E_{s2}=3MPa, and the vertical and radial permeability coefficients are k_{v1}=k_{v2}=10^{-7} cm/s and k_{h1}=k_{h2}=2×10^{-7} cm/s ,respectively. When calculating the consolidation degree of the composite foundation of equal-core CCSG piles, the radial permeability coefficient of the smear area is a constant, r_s=0.5 m，k_s=0.5×10^{-7} cm/s. The compression modulus and permeability coefficient of the crushed rock shell are E_p=60MPa，k_p=10^{-2}cm/s, respectively. When calculating the consolidation degree of short-core TDM pile composite foundation, the length of core pile H_1=12m, soil-cement shell compression modulus is 150MPa. In the two examples, the load is applied in a single stage, and the loading duration is t_c=120d.

Fig. 3 shows the comparison between the two kinds of composite foundation consolidation degree calculated by analytical solution and regression solution in this paper and the existing analytical solution. As can be seen from Fig. 3, the consolidation degree of equal-core gravel pile composite foundation obtained in this paper is very close to the analytical solution calculated by Qi Zhibo et al[15], which considers the radial seepage of gravel shell, and the calculated consolidation degree is slightly smaller. The consolidation degree of short-core TDM pile composite foundation obtained by analytical solution in this paper is very close to that calculated by analytical solution of Yang Tao et al[20]. The numerical example shows that the analytical consolidation solution in this paper has high computational accuracy and strong applicability.

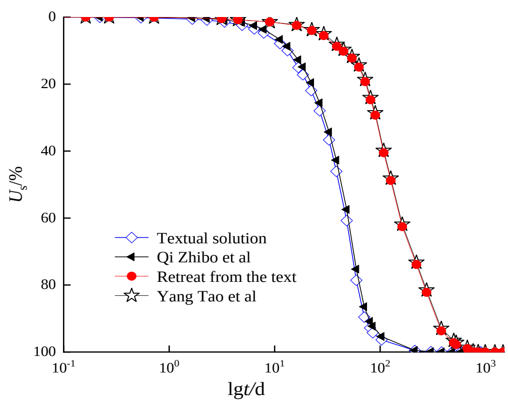


Fig.3 Comparison of the proposed solutions with existing solutions

V. PARAMETER ANALYSIS

In order to study the consolidation characteristics of composite foundation with short core CCSG piles, the influencing factors of composite foundation consolidation are analyzed by using the analytical solution of consolidation. The pile shell is gravel and the radial permeability coefficient of smear area is constant. Reference geometric and mechanical parameters are as follows: H=20m， H_1=14 m， H_2=6m。r_p=0.25m， r_c=0.115m， r_e=1m， r_s=0.5m。E_{s1}=E_{s2}= E_s=3MPa， E_p=60MPa， E_c =20GPa， k_{v1}=k_{v2}=k_v=10^{-7}cm/s， k_{h1}= k_{h2}=2×10^{-7}cm/s， k_p =10^{-2}cm/s， k_{s1}= k_{s2}=0.5×10^{-7} cm/s， Loading time t_c=120d. One parameter value is changed at a time while the other parameter values remain the same.

（1）Parameter analysis

Fig. 4 shows the consolidation curve of the composite foundation of short core CCSG piles with different core length ratio $\beta=H_1/H$. The dimensionless factor $T_h=c_{h1}t/(2r_e)^2$ $c_{h1}=k_{h1}E_{s1}/\gamma_w$ is adopted in the abscissa. The larger the core length ratio is, the longer the core pile is. β=0 indicates the double-layer gravel pile composite foundation. It can be seen from Figure 4 that the consolidation of short-core CCSG pile composite foundation is slightly slower than that of double-layer gravel pile composite foundation. The consolidation rate of composite foundation with short core CCSG piles decreases gradually with the increase of core pile length, but the decreasing range is small, indicating that the change of core pile length has little effect on the consolidation rate of composite foundation.

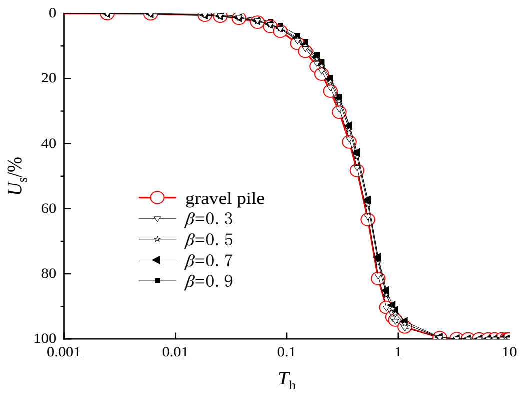
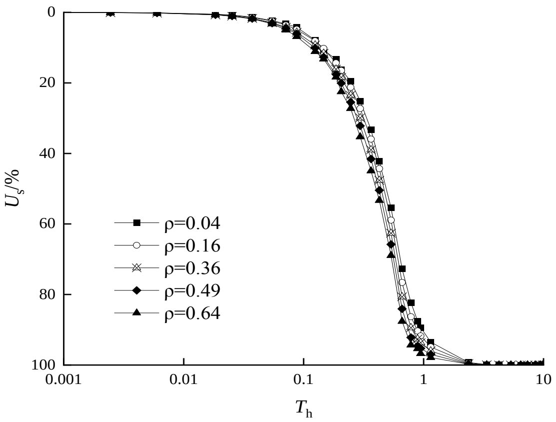
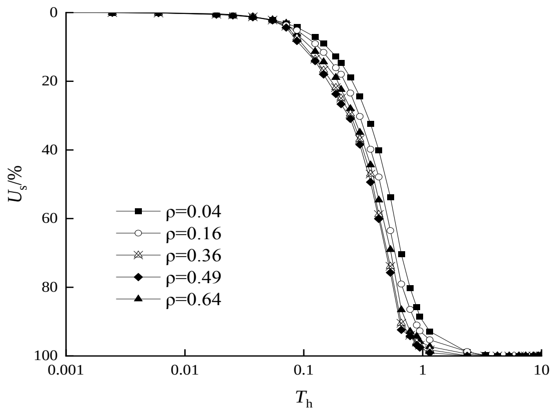


Fig. 4 Influence of core- column length ratio on consolidation rate

Fig. 5 shows the comparison of consolidation degree of short core CCSG pile composite foundation with different core ratio of β=0.3 and β=0.85. The radius of the rubble shell (pile) remains unchanged, the greater the value of ρ, the greater the diameter of the core pile. As can be seen from FIG. 5, the consolidation rate of composite foundation as a function of sectional heart rate is related to core length ratio β : when the core length ratio is small (β=0.3), the consolidation rate of composite foundation gradually increases with the increase of sectional core content, but the increase is small. When the core length ratio is large (β=0.85), the consolidation rate of composite foundation firstly increases and then decreases with the increase of the core pile diameter: the consolidation rate increases with the increase of $\rho\leqslant0.36$, but decreases with the increase of ρ > 0.36. Shi Beixiao et al[12] found that the consolidation rate of the composite foundation of equal-core CCSG piles first increases and then decreases with the increase of the diameter of the core pile. It can be seen that the consolidation characteristics of the composite foundation of short core CCSG piles are close to that of the composite foundation only when the core length ratio is large enough.



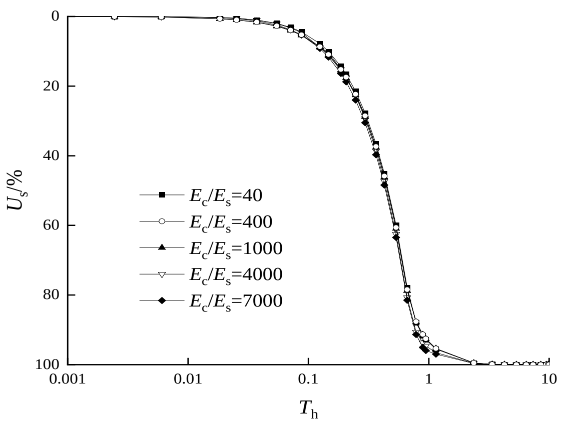
（a）β=0.3



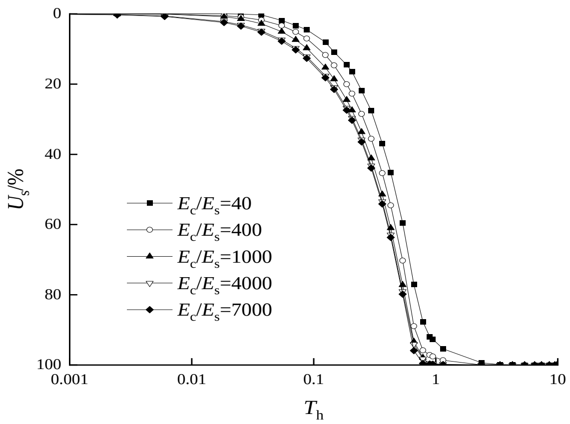
（b）β=0.85

Fig.5 Influence of area core ratio on consolidation rate

Fig. 6 shows the consolidation velocity curves of composite foundation with different core pile-soil compression modulus ratios E_c/E_s when β=0.3 and β=0.85. E_s remains unchanged, and the core pile material is considered from cement to concrete. The greater the E_c/E_s, the greater the stiffness of the core pile. As can be seen from Figure 6, the influence of core pile stiffness on the consolidation rate of composite foundation is also related to the core length ratio: the consolidation rate of composite foundation with short core pile is almost not affected by the change of core pile stiffness, while the consolidation rate of composite foundation with long core pile gradually increases with the increase of core pile stiffness. Yu Jin et al[13] pointed out that the consolidation rate of equal-core CCSG pile composite foundation increases significantly with the increase of core pile stiffness. It can be seen that the consolidation characteristics of short core CCSG pile composite foundation are close to that of equal core CCSG pile composite foundation only when the core length ratio is large enough.



（a）β=0.3



（*b*）β=0.3

Fig. 6 Influence of constrained modulus of core pile on consolidation rate

As can be seen from Figures 4 to 6, under the condition of meeting the requirements of bearing capacity and settlement control, the length of core pile should be as short as possible, with large stiffness and small diameter, and the heart rate of cross section should not exceed 0.36.

（2）The influence of rubble shell

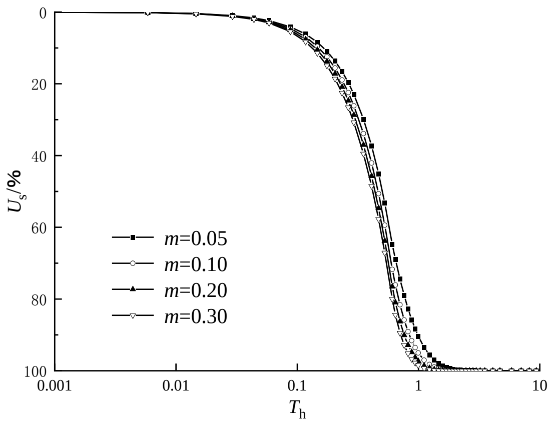


Fig.7 Influence of replacement ratio of CCSG column with short core on consolidation rate

Fig. 7 shows the consolidation degree curve of composite foundation under different replacement rates m of short core CCSG piles. r_c core pile radius remains unchanged in calculation. The smearing effect was not considered due to the single factor analysis. The larger the value of displacement rate m, the larger the thickness of the upper gravel shell and the diameter of the lower gravel pile. It can be seen from Figure 7 that the consolidation rate of the composite foundation with short core CCSG piles increases with the increase of the thickness of the gravel shell. This is because with the increase of the thickness of the rubble shell, the drainage area of the rubble shell and the pile below gradually increases.

Fig. 8 shows the influence of the compression modulus E_p of crushed stone shell (pile) on the consolidation rate of composite foundation. As can be seen from FIG. 8, the consolidation rate of composite foundation with short core CCSG piles increases with the increase of the compression modulus E_p of gravel shell (pile), but the consolidation rate of composite foundation does not increase much because the compression modulus of gravel shell (pile) itself does not change much.

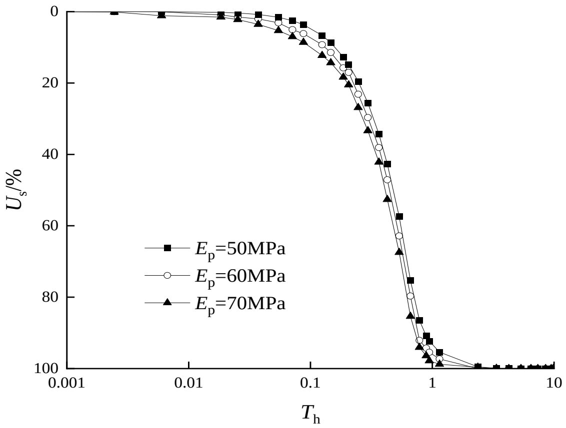


Fig. 8 Influence of constrained modulus of stone shell（column） on consolidation rate

Fig. 9 shows the effect of the ratio of crushed stone husk-soil permeability to k_p/k_v on the consolidation rate of composite foundation. The greater the k_p/k_v, the stronger the permeability of the rubble shell and the pile below. As can be seen from Figure 9, the consolidation rate of composite foundation with short core CCSG piles increases with the increase of permeability coefficient of gravel shell (pile), but the increase rate decreases gradually, or it decreases with the increase of well resistance of gravel shell (pile), and the decrease rate increases gradually.

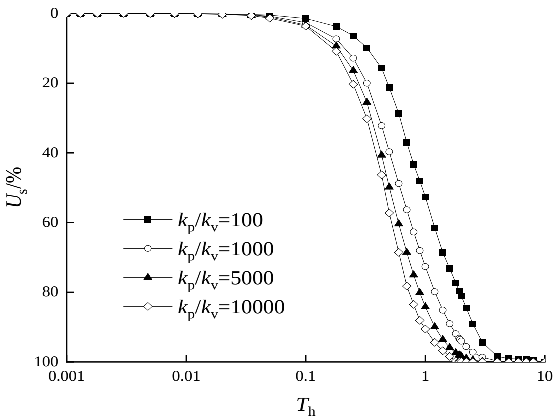


Fig.9 Influence of permeability coefficient of stone shell (column)on consolidation rate

VI. CONCLUSION

The analytical solution of consolidation of composite foundation with short core CCSG pile is established, and the consolidation characteristics of composite foundation are obtained. The main conclusions are as follows:

(1) The proposed consolidation analytical solution can be used to analyze the consolidation of the composite foundation of equal-core CCSG piles, and is a common analytical solution for the consolidation of the composite foundation of short core-equal-core CCSG piles.

(2) The consolidation rate of short core CCSG pile composite foundation is slightly less than that of gravel pile composite foundation.

(3) The consolidation rate of short core CCSG pile composite foundation is less affected by the length of core pile.

(4) The change of core pile diameter and stiffness is shorter than that of small core length

The effect of consolidation rate on the composite foundation with CCSG piles is small. When the core length ratio is large, the consolidation rate of composite foundation increases with the increase of core pile stiffness, and increases first and then decreases with the increase of core pile diameter.

(5) The consolidation rate of short core CCSG pile composite foundation increases with the increase of sand shell thickness and stiffness, but decreases with the increase of well resistance.

(6) When the core length ratio is large, the consolidation characteristics of short-core CCSG pile composite foundation are close to those of long-core CCSG pile composite foundation.

REFERENCES

[1]. HAN J. Recent research and development of ground column technologies[J]. Proceedings of the Institution Civil Engineers -Ground Improvement, 2015, 68 (4):246-264.
[2]. TANG Tongzhi, ZHAO Weibing, CHEN Junsheng, et al. Experimental study on deformation and bearing capacity of composite foundation with concrete-cored sand-gravel piles under high embankment load[J].Chinese Journal of Geotechnical Engineering, 2010, 32(12):1829-1836.
[3]. JIANG Y, ZHENG G, HAN J. Numerical evaluation of consolidation of soft foundations improved by sand -deep -mixed composite columns[J].International Journal of Geomechanics, 2017,17(8): 04017034.
[4]. WENG Jiawei,YANG Tao. Consolidation behavior of composite ground with short concrete-core stone columns under embankment loading[J].Journal of Water Resources and Architectural Engineering, 2020,18(4):111-116.
[5]. HAN J, YE S L. A theoretical solution for consolidation rates of stone column-reinforced foundations accounting for smear and well resistance effects[J]. International Journal of Geomechanics, 2002, 2(2): 135-151.
[6]. GUO Biao, CHEN He, LI Ya-jun. Analytical study on consolidation of composite ground considering two-dimensional well resistance under time and depth dependent stresses[J].Chinese Journal of Rock Mechanics and Engineering, 2017, 36(3):754-761.
[7]. ZHOU Y, ZHOU G Q, LU M M, et al. Analytical solution for the consolidation process of a stone-column reinforced foundation under partially drained boundaries[J]. International Journal of Geomechanics, 2018, 18(1): 06017023.
[8]. GUO Biao,YANG Yuan-zhou, DU Feng-bin,et al. Nonlinear consolidation solution for composite foundation by stone columns [J].Journal of Xi' an University of Architecture and Technology (Natural Science Edition), 2021, 53(5):652-658.
[9]. WANG L, ZHOU A N, XU Y F, et al. Consolidation of unsaturated composite ground reinforced by permeable columns[J]. Computers and Geotechnics, 2020,125:103706
[10]. DOAN S, FATAHI B. Analytical solution for free strain consolidation of stone column-reinforced soft ground considering spatial variation of total stress and drain resistance[J]. Computers and Geotechnics, 2020, 118:1-17.
[11]. TAI P, BUDDHIMA I. Consolidation analysis of soft ground improved by stone columns incorporating foundation stiffness[J]. International Journal of Geomechanics, 2020, 20(6):04020067.
[12]. SHI Beixiao,YANG Yanwei, XIE Rongxing. Analytical solution for consolidation of composite foundation with concrete-cored sand-gravel piles[J]. Rock and Soil Mechanics,2013, 34(10):2835-2840, +2856.
[13]. YU Jin, QI Zhibo, CAI Yanyan, et al. Study on the general solution for the consolidation of low-grade concrete-core sand-gravel pile composite foundation [J]. Engineering Mechanics, 2013, 30(11):127-135.
[14]. YANG Yanwei, GUAN Yunfei, LI Jintao. A calculation method for consolidation of concrete core sand-gravel piles composite foundation under embankment[J]. Hydro-science and Engineering, 2016, 3:46-52.
[15]. QI Zhibo, YU Jin, CAI Yanyan, et al. Analysis on consolidation of CCSG pile composite foundation under time-dependent loading[J]. Chinese Journal of Underground Space and Engineering, 2016, 12(2):336-342.
[16]. CAI Yanyan, WANG Ha, QI Zhibo, et al. Analytical solution for consolidation of CCSG pile composite foundation by considering nonlinearity of soil[J]. Rock and Soil Mechanics, 2015, 36(Suppl.1) :639-645.
[17]. LU M M, ZHANG Q, JIANG H W, et al .Analytical solutions for consolidation of composite ground improved by composite columns with circular and non-circular cross sections[J].European Journal of Environmental and Civil Engineering, 2022, 26(7):2780-2796.
[18]. YE Guanbao, ZHANG Qingwen, ZHANG Zhen. Consolidation analysis of concrete-cored sand-gravel piles improved composite foundation under combined vacuum and surcharge preloading [J]. Rock and Soil Mechanics, 2016, 37(12) : 3356-3364.
[19]. XIE K H, LU M M, LIU G B. Equal strain consolidation for stone columns reinforced foundation[J]. International Journal for Numerical and Analytical Methods in Geomechanics, 2009, 33:1721-1735.
[20]. YANG Tao, TANG Feng. Consolidation analysis of composite ground with stiffened deep cement mixing piles under Time-dependent loading[J].Journal of Building Structures, 2017, 38(9): 160-166.