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A STUDY OF COEFFICIENT OF ACOUSTIC WAVES AT AN INTERFACE BETWEEN TWO MOVING FLUID MEDIA

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Abstract:

The reflection and transmission coefficients of acoustic waves by moving medium are determined. We have considered two fluid medium which are moving parallel to the interface with velocities $V_1$and V_2. It is found that for the angle of incidence α smaller than the critical angle (α_c), no plane travelling waves will penetrate into region II, and total reflection occurs. This shows that the sound pressure in region II decays exponentially with the distance from the interface. It has been found that the reflection and transmitted coefficients depend only upon the component of the velocity of the medium which lies in the plane of incidence.

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Introduction :

The effects of perfectly reflecting moving boundary upon an incident plane acoustic/electromagnetic waves were discussed many years ago by Rayleish, Rudnick, Franken, Fresnel etc.[1-5]. Sommerfeld and Brekhovskikh[6,7] gave a rather comprehensive treatment of this problem in his book. Reflection and transmission of acoustic waves in nonuniform flows is a subject of great interest in many practical problems particularly in transport engineering with automotive exhaust systems, acronautical turbofan engine inlet ducts, etc. [8]. The understanding of this phenomenon is a central feature for the prediction of noise and for esigning components that efficiently attenuate sound. In view of the importance of the above topic make thorough and systematic studies of reflection, scattering and transmission of waves at an interface of relative motion between two fluids. The effects of turbulence upon the transmission of wave in water wave discussed by Dunn[9] and the basic investigation of scattering in the ray theory range were performed by Bergmann .[10] The wave theory treatment of scattering was essentially formulated by Pekeri's and Carstensen. Obukhor[11] obtained and expression for the fluctuation of phase and amplitude, which, in the low frequency range, is equivalent to Mintzer[12] result and, in the ray theory range, is equivalent to Bergmann's solution. However, in most previous work the layer medium is assumed to be stationary with respect to the source as well as the to the observer. If the layered medium moves at a uniform speed with respect to an observer, it is expected that the scattering and transmission characteristics of an incident wave will be affected significantly. An important problem of acoustic radiation is available in the study of the acoustic-vorticity waves in swirling flows[13] and uniformly moving medium.[14] More recently Fabin Treyssede et al[15] studied a mixed finite element method for acoustic wave propagation in moving fluid based on an Eulerian-Lagrangian description. The purpose of this paper is to present the solution to this important problem.

Solution and Discussion:

Suppose two fluid media separated by YZ plane and having the steady velocities $V_1$and V_2 along the Y axis. Let ρ and c are the density and sound speed in region I, ρ' and c' are the corresponding quantities in the region II. Suppose a plane wave originates in region I and travels in a direction which makes an angle α with Y axis. The wave is partially transmitted in region II and makes and angle β with Y axis (Fig1), ω is the angular frequency of the incident wave. Let us first note that only the velocity components that lie in

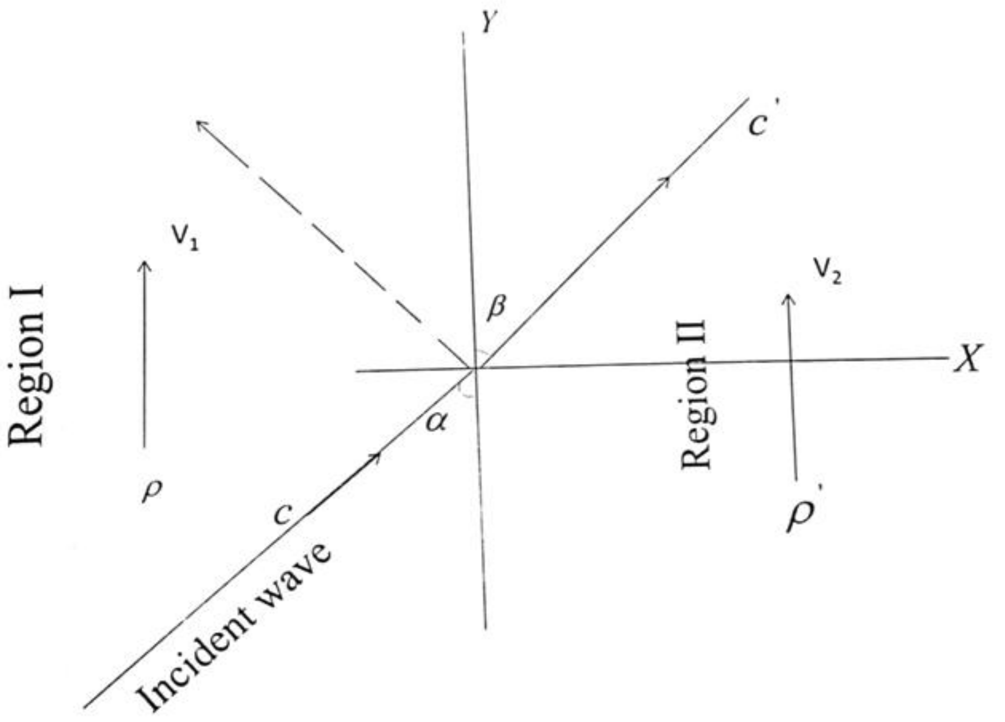

Fig.1 : The incident, reflected and transmitted waves at an interface between two moving fluid media.

The plane of the incident wave will affect the propagation and refraction of acoustic wave. The wave equation for sound propagation in moving medium may bee expressed as

$$(\frac{\partial}{\partial t} + V.\nabla)^2 \, p = c^2 \nabla^2 p \qquad \text{---------------- (1)}$$

The acoustic pressure p and the velocity component U_2 of the moving medium in the plane of incidence is given by

$$p = P\, e^{i(k.r-\omega t)} \qquad \text{----------------- (2)}$$

and $u_{i=} U\ e^{i(k.r-\omega t)}$ -------------------(3)

where P and U are constants. The vector k is called the propagation vector.

We have $V.\nabla = i\,V.k = i\,V_{11}.k$ ------------------ (4)

Where V_{11} is the component in the plane of the wave. We have supposed that V_1 and V_2 be placed along the Y axis in the plane of incidence of the wave. Then k = (k_x,k_y 0)V=0,V,0)and V.k = Vk $\cos\alpha$ = Vk_y ------------------(5)

Where α is the angle between V and k.With k_y= k Cosα and k_x = k Sinα, it follows from eqn (1) that, for either medium,

$$k = \frac{\omega}{C + V\cos\alpha.}$$

$$\left.\begin{array}{l} k_x = k \sin\alpha. \\ \text{And } k_y = k \cos\alpha. \end{array}\right\} \qquad \text{---------------- (6)}$$

Using this expression for k_y in the two region I and II together with the boundary condition that $k_{1y} = k_{2y}$.

$$\frac{C}{\cos\alpha} + V_1 = \frac{C'}{\cos\beta} + V_2 \qquad \text{--------------- (7)}$$

$$\text{Or } \cos\beta = \frac{C' \cos\alpha}{C - (V_2 - V_1)\cos\alpha} \qquad \text{--------------- (8)}$$

If $V_2 > V_1$, so that (V_2- V_1) is positive, with includes the case when region I is at rest and region II in motion, we note that $\beta < \propto$. The critical angle of incidence α_c , corresponding a refracted wave travelling along the boundary in region II ($\beta = 0$) is

$$\text{Cos}\,\alpha = \frac{C}{C+V} \qquad \text{----------------(9)}$$

For angle α smaller than this critical angle, no plane travelling waves will penetrate in region II, and total reflection occurs. Then the sound pressure in region II decays exponential with the distance from the boundary.

Let us suppose that $u_i = 0$ for x >0 and $u_i = 0$ for 0< x. Then the plane x separates the first medium which is to be left and from the second medium which is to right. If the incident, reflected and transmitted pressure field are p_{inc}, p_{ref} and p_{trans} respectively the pressure is given by

$$p_{inc} = P_1\, e^{i}(k_x x + k_y y) - i\omega t$$

$$= P_1\, e^{ik_1(x\,\text{Cos}\,\alpha + y_1\,\text{Sin}\,\alpha - ct)},\ y_1 = y - V_1 t \qquad \text{---------------(10)}$$

We expect that this wave will give rise to reflected and transmitted plane waves which must have the form.

$$p_{ref} = R\,P1\, e^{i}(-k_{1x}x + k_{1y}y) - i\omega t$$

$$= R\,P_1\, e^{ik_1}(-x\,\text{Cos}\,\alpha + y_1\,\text{Sin}\,\alpha - ct),\ y_1 = y - V_1 t$$

$$= R\,P_1\, e^{ik_1}(-x\,\text{Cos}\,\alpha + (y - V_1 t) - ct \qquad \text{-------------------(11)}$$

and

$$p_{trans} = T\,P_1\, e^{i(-k_2 x x + k_2 y y) - i\omega t}$$

$$= T P_1\, e^{ik(x\,\text{Cos}\,\beta + (y - V_2 t)\,\text{Sin}\,\beta - ct} \qquad \text{------------------(12)}$$

where R and T are reflection and transmission coefficients respectively. In order to determine R and T, we make use of the boundary condition on the plane x=0. This given.

$$p_{inc} + p_{ref} = p_{trans}$$

If ρ_1° denotes the density and u_2° denotes the Y –component of velocity in the moving medium the normal component of u_i in terms of p by means of equation (3) and equation

$u_{it} + u_j^{\circ}(u_i)x_j + \frac{1}{\rho^{\circ}} px^i \; X = 0$, we obtain

$$\frac{\cos\alpha}{\rho^{\circ} C}\left(p_{inc} - p_{ref}\right) = \frac{\frac{\omega}{c'}\cos\beta}{\rho_1^{\circ}\left(\omega - u_2^{\circ}\frac{\omega}{c'}\sin\beta\right)} p_{trans} \qquad \text{------------------ (14)}$$

Solving equation (10), (12), (13) and (14) we obtain.

$$\frac{Sin\alpha}{C} = \frac{Sin\beta}{C'} \qquad \text{------------------ (15)}$$

$$1+R = T \qquad \text{------------------ (16)}$$

$$\frac{Cos\alpha}{\rho^{\circ} C}(1-R)_i = \frac{Cos\beta}{\rho_0' C'\left(1 - \frac{u_2^{\circ}}{c'} Sin\,\beta\right)} T \qquad \text{------------------(17)}$$

Equation (15) which embodies the law of reflection is a consequence of the fact that the y dependence on the two sides of equation (13) and (14) must be the same. After solving the equation (16) and (17) , we obtain.

$$R = \frac{1 - \frac{\rho C\, Cos\beta}{\rho' C' Cos\alpha}}{1 + \frac{\rho\, C\, Cos\beta}{\rho' C' Cos\alpha}}$$

$$= \frac{\rho' C' Cos\alpha - \rho\, C\, Cos\beta}{\rho' C' Cos\alpha + \rho\, C\, Cos\beta} \qquad \text{------------------ (18)}$$

and $$T = \frac{2}{1 + \frac{\rho\, C\, Cos\beta}{\rho' C' Cos\alpha}}$$

$$= \frac{2\,\rho' C' Cos\alpha}{\rho' C' Cos\alpha + \rho\, C\, Cos\beta} \qquad \text{------------------(19)}$$

For normal incidence, α=β =90°, this relation reduces to the familiar relation $(\rho' c' - \rho c)/(\rho' c' + \rho c)$ because it follows from equation (7) that, for normal incidence have Sin 2α/Sin2β= c/c'.

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