International Journal of Research in Engineering and Science (IJRES)
ISSN (Online): 2320-9364, ISSN (Print): 2320-9356
www.ijres.org Volume 10 Issue 9 ǁ September 2022 ǁ PP. 619-624

Fekete Szego Inequality For A Complicated Class Of Analytic Functions Approaching To A Class In The Limit Form And Other Class Directly

Gurmeet Singh
Khalsa College Patiala-147001, Punjab, India

Abstract
We introduce a class of analytic functions and obtain sharp upper bounds of the functional $|a_3 - \mu a_2^2|$ *for the analytic function* $f(z) = z + \sum_{n=2}^{\infty} a_n z^n, |z| < 1$ *belonging to this class with special character that it tends to the class of convex functions as* $\alpha \to \frac{\pi}{2}$.

Keywords: *Univalent functions, Starlike functions, Close to convex functions and bounded functions*
2020 Mathematical Sciences Classification: 30C50

Date of Submission: 15-09-2022 Date of acceptance: 30-09-2022

I. Introduction

Let $\mathcal{A}$ denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \quad (1.1)$$

which are analytic in the unit disc $\mathbb{E} = \{z: |z| < 1\}$. Let $\mathcal{S}$ be the class of functions of the form (1.1), which are analytic univalent in $\mathbb{E}$.

In 1916, Bieber Bach [1, 2] proved that $|a_2| \leq 2$ for the functions $f(z)\mathcal{S}$. In 1923, Löwner [10] proved that $|a_3| \leq 3$ for the functions $f(z) \in \mathcal{S}$.

With the known estimates $|a_2| \leq 2$ and $|a_3| \leq 3$, it was expected to try to find some relation between a_3 and ${a_2}^2$ for the class $\mathcal{S}$**,** Fekete and Szegö [4] used Löwner's method to prove the following well known result for the class $\mathcal{S}$.

Let $f(z) \in \mathcal{S}$, then

$$|a_3 - \mu a_2^2| = \begin{cases} 3 - 4\mu & if \mu \leq 0 \\ 1 + 2\exp\left(\frac{-2\mu}{1-\mu}\right) & if \ 0 \ \leq \mu \leq 1 \\ 4\mu - 3 & if \mu \geq 1 \end{cases} \quad (1.2)$$

The inequality (1.2) plays a very important role in determining estimates of higher coefficients for some sub classes $\mathcal{S}$ [3, 9, 18-48].

Let us define some subclasses of $\mathcal{S}$.
We denote by $\mathcal{S}^*$, the class of univalent starlike functions

$$g(z) = z + \sum_{n=2}^{\infty} b_n z^n \in \mathcal{A}$$

and satisfying the condition

$$Re \ \left(\frac{zg(z)}{g(z)}\right) > 0, \ z \in \mathbb{E}. \quad (1.3)$$

We denote by $\mathcal{K}$, the class of univalent convex functions

$$h(z) = z + \sum_{n=2}^{\infty} c_n z^n \in \mathcal{A}$$

and satisfying the condition

$$Re\left(\frac{(zh'(z)}{h'(z)}\right) > 0,\ z \in \mathbb{E}. \quad (1.4)$$

A function $f(z) \in \mathcal{A}$ is said to be close to convex if there exists $g(z) \in \mathcal{S}^*$ such that

$$Re\ \left(\frac{zf'(z)}{g(z)}\right) > 0,\ z \in \mathbb{E}. \quad (1.5)$$

The class of close to convex functions is denoted by C and was introduced by Kaplan [7] and it was shown by him that all close to convex functions are univalent.

$$S^*(A,B) = \left\{f(z) \in \mathcal{A}; \frac{zf'(z)}{f(z)} \prec \frac{1+Az}{1+Bz},\ -1 \le B < A \le 1, z \in \mathbb{E}\right\}, \quad (1.6)$$

$$\mathcal{K}(A,B) = \left\{f(z) \in \mathcal{A}; \frac{(zf'(z))}{f'(z)} \prec \frac{1+Az}{1+Bz},\ -1 \le B < A \le 1, z \in \mathbb{E}\right\}. \quad (1.7)$$

It is obvious that $S^*(A,B)$ is a subclass of S^* and $\mathcal{K}(A,B)$ is a subclass of $\mathcal{K}$.
We introduce a new subclass as

$$\left\{f(z) \in \mathcal{A}; \left(\frac{zf'(z)}{f(z)}\right)^{\beta} + \tan\alpha\left(\frac{(zf'(z))'}{f'(z)}\right)^{1-\beta} \prec \frac{1+w(z)}{1-w(z)}; z \in \mathbb{E}\right\}$$

and we shall denote this class as $\mathcal{KS}^*(\alpha,\beta\}$.

We shall deal with two subclasses of $S^*(f,f',\alpha,\beta)$ defined as follows in our next paper:

$$\mathcal{KS}^*(\alpha,\beta,A,B) = \left\{f(z) \in \mathcal{A}; \left(\frac{zf'(z)}{f(z)}\right)^{\beta} + \tan\alpha\left(\frac{(zf'(z))'}{f'(z)}\right)^{1-\beta} \prec \frac{1+Az}{1+Bz}; z \in \mathbb{E}\right\}, \quad (1.8)$$

$$\mathcal{KS}^*(A,B,\alpha,\beta,\gamma) = \left\{f(z) \in \mathcal{A}; \left(\frac{zf'(z)}{f(z)}\right)^{\beta} + \tan\alpha\left(\frac{(zf'(z))'}{f'(z)}\right)^{1-\beta} \prec \left\{\frac{1+Az}{1+Bz}\right\}^{\gamma}; z \in \mathbb{E}\right\}. \quad (1.9)$$

Symbol $\prec$ stands for subordination, which we define as follows:

Principle of Subordination. *Let $f(z)$ and $F(z)$ be two functions analytic in $\mathbb{E}$. Then $f(z)$ is called subordinate to F(z) in $\mathbb{E}$ if there exists a function $w(z)$ analytic in $\mathbb{E}$ satisfying the conditions $w(0) = 0$and $|w(z)| < 1$ such that $f(z) = F(w(z))$; $z \in \mathbb{E}$ and we write $f(z) \prec F(z)$.*

By $\mathcal{U}$, we denote the class of analytic bounded functions of the form

$$w(z) = \sum_{n=1}^{\infty} d_n z^n,\ w(0) = 0, |w(z)| < 1. \quad (1.10)$$

It is known that

$$|d_1| \le 1, |d_2| \le 1 - |d_1|^2. \quad (1.11)$$

II. Preliminary Lemmas.

For $0 < c < 1$, we write $w(z) = \left(\frac{c+z}{1+cz}\right)$ so that

$$\frac{1+w(z)}{1-w(z)} = 1 + 2cz + 2z^2 + \cdots. \quad (2.1)$$

III. Main Results

Theorem 3.1. Let $f(z) \in \mathcal{KS}^*(\alpha,\beta)$

$$|a_3 - \mu a_2^2| \leq$$

$$\begin{cases} \frac{1}{\{\beta+2(1-\beta)\tan\alpha\}^2}\left[\frac{\{4(1-\beta)(\beta+2)\alpha-\beta(\beta-3)\}\{\beta+2(1-\beta)\tan\alpha\}}{\{\beta+3(1-\beta)\tan\alpha\}} - 4\mu\right] \\ if \quad \mu \leq \frac{\{4(1-\beta)(\beta+2)\tan\alpha-\beta(\beta-3)\}\{\beta+2(1-\beta)\tan\alpha\}-\{(1-\alpha)\beta+2\alpha(1-\beta)\}}{4\{\beta+3(1-\beta)\tan\alpha\}} \quad (3.1) \\ \frac{1}{3\alpha+\beta-4\alpha\beta} \\ if \quad \frac{\{4(1-\beta)(\beta+2)\tan\alpha-\beta(\beta-3)\}\{\beta+2(1-\beta)\tan\alpha\}-\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}{4\{\beta+3(1-\beta)\tan\alpha\}} \leq (3.2) \\ \mu \leq \frac{\{4(1-\beta)(\beta+2)\tan\alpha-\beta(\beta-3)\}\{\beta+2(1-\beta)\tan\alpha\}+\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}{4\{\beta+3(1-\beta)\tan\alpha\}} \\ \frac{1}{\{\beta+2(1-\beta)\tan\alpha\}^2}\left[4\mu - \frac{\{4(1-\beta)(\beta+2)\tan\alpha-\beta(\beta-3)\}\{\beta+2(1-\beta)\tan\alpha\}}{\{\beta+3(1-\beta)\tan\alpha\}}\right] \\ if \quad \mu \geq \frac{\{4(1-\beta)(\beta+2)\tan\alpha-\beta(\beta-3)\}\{\beta+2(1-\beta)\tan\alpha\}+\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}{4\{\beta+3(1-\beta)\tan\alpha\}} \quad (3.3) \end{cases}$$

The results are sharp.

***Proof*.** By definition of $\mathcal{KS}^*(\alpha,\beta)$, we have

$$f(z) \in \mathcal{A}; \left(\frac{zf'(z)}{f(z)}\right)^{\beta} + \tan\alpha\left(\frac{(zf'(z))'}{f'(z)}\right)^{1-\beta} = \frac{1+w(z)}{1-w(z)}; w(z) \in \mathcal{U} \quad (3.4)$$

Expanding the series (3.1), we get

$$\left\{1 + \beta a_2 z + \left(2\beta a_3 + \frac{\beta(\beta-3)}{2}a_2^2\right)z^2 + \cdots\right\} + \tan\alpha\ \{1 + 2(1-\beta)a_2 z + 2(1-\beta)(3a_3 - (\beta+2)a_2^2)z^2 + \cdots\}$$
$$= (1 + 2c_1 z + 2(c_2 + {c_1}^2)z^2 + \cdots). \quad (3.5)$$

Identifying terms in 3.2, we get

$$a_2 = \frac{2}{\beta+2(1-\beta)\tan\alpha}c_1. \quad (3.6)$$

$$a_3 = \frac{1}{\beta+3(1-\beta)\tan\alpha}c_2 + \frac{4(1-\beta)(\beta+2)\tan\alpha - \beta(\beta-3)}{\{\beta+3(1-\beta)\tan\alpha\}\{\beta+2(1-\beta)\tan\alpha\}}{c_1}^2. \quad (3.7)$$

From (3.3) and (3.4), we obtain

$$a_3 - \mu a_2^2 = \frac{c_2}{\beta+3(1-\beta)\tan\alpha} + \left[\frac{4(1-\beta)(\beta+2)\tan\alpha - \beta(\beta-3)}{\{\beta+3(1-\beta)\tan\alpha\}\{\beta+2(1-\beta)\tan\alpha\}\}} - \frac{4\mu}{\{\beta+2(1-\beta)\tan\alpha\}^2}\right]c_1^2 \quad (3.8)$$

Taking absolute value and using Triangular inequality, (3.5) can be rewritten as

$$|a_3 - \mu a_2^2| \leq \frac{|c_2|}{\beta+3(1-\beta)\tan\alpha} + \frac{1}{\{\beta+2(1-\beta)\tan\alpha\}^2}\left|\frac{\{4(1-\beta)(\beta+2)\tan\alpha - \beta(\beta-3)\}\{\beta+2(1-\beta)\tan\alpha\}}{\{\beta+3(1-\beta)\tan\alpha\}} - 4\mu\right||c_1^2| \quad (3.9)$$

Using (1.9) in (3.6), simple calculations yield

$$|a_3 - \mu a_2^2| \leq \frac{1}{\beta+3(1-\beta)\tan\alpha} + \frac{1}{\{\beta+2(1-\beta)\tan\alpha\}^2}\left[\left|\frac{\{4(1-\beta)(\beta+2)\tan\alpha - \beta(\beta-3)\}\{\beta+2(1-\beta)\tan\alpha\}}{\{\beta+3(1-\beta)\tan\alpha\}} - 4\mu\right| - \frac{\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}{\beta+3(1-\beta)\tan\alpha}\right]|c_1|^2 \quad (3.10)$$

***Case* I.** $\mu \leq \frac{\{4(1-\beta)(\beta+2)\tan\alpha - \beta(\beta-3)\}\{\beta+2(1-\beta)\tan\alpha\}\}}{4\{\beta+3(1-\beta)\tan\alpha\}}$. In this case, (3.10) can be rewritten as

$$|a_3-\mu a_2^2| \le \frac{1}{\beta+3(1-\beta)\tan\alpha} + \frac{1}{\{\beta+2(1-\beta)\tan\alpha\ \}^2} \left[\frac{\{4(1-\beta)(\beta+2)\tan\alpha\ -\beta(\beta-3)\}\{\beta+2(1-\beta)\tan\alpha\ \}-\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}{\{\beta+3(1-\beta)\tan\alpha\ \}} - 4\mu\right]|c_1|^2. \quad (3.11)$$

***Subcase* I (a).** $\mu \le \frac{\{4(1-\beta)(\beta+2)\tan\alpha\ -\beta(\beta-3)\}\{\beta+2(1-\beta)\tan\alpha\ \}-\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}{4\{\beta+3(1-\beta)\tan\alpha\ \}}$.

Using (1.9), (3.8) becomes

$$|a_3-\mu a_2^2| \le \frac{1}{\{\beta+2(1-\beta)\tan\alpha\}^2}\left[\frac{\{4(1-\beta)(\beta+2)\tan\alpha\ -\beta(\beta-3)\}\{\beta+2(1-\beta)\tan\alpha\}}{\{\beta+3(1-\beta)\tan\alpha\}} - 4\mu\right]. \quad (3.12)$$

***Subcase* I(b).** $\mu \ge \frac{\{4(1-\beta)(\beta+2)\tan\alpha\ -\beta(\beta-3)\}\{\beta+2(1-\beta)\tan\alpha\ \}-\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}{4\{\beta+3(1-\beta)\tan\alpha\}}$.

We obtain from (3.8)

$$|a_3-\mu a_2^2| \le \frac{1}{\beta+3(1-\beta)\tan\alpha}. \quad (3.13)$$

***Case* II.** $\mu \ge \frac{\{4(1-\beta)(\beta+2)\tan\alpha\ -\beta(\beta-3)\}\{\beta+2(1-\beta)\tan\alpha\}}{4\{\beta+3(1-\beta)\tan\alpha\}}$

Preceding as in case I, we get

$$|a_3-\mu a_2^2| \le \frac{1}{3\alpha+\beta-4\alpha\beta} + \frac{1}{\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}\left[4\mu - \frac{\{4(1-\beta)(\beta+2)\tan\alpha\ -\beta(\beta-3)\}\{\beta+2(1-\beta)\tan\alpha\ \}+\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}{\{\beta+3(1-\beta)\tan\alpha\}}\right]|c_1|^2 \quad (3.14)$$

***Subcase* II(a).** $\mu \le \frac{\{4(1-\beta)(\beta+2)\tan\alpha\ -\beta(\beta-3)\}\{\beta+2(1-\beta)\tan\alpha\}+\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}{4\{\beta+3(1-\beta)\tan\alpha\ \}}$

(3.11) takes the form

$$|a_3-\mu a_2^2| \le \frac{1}{\beta+3(1-\beta)\tan\alpha} \quad (3.15)$$

Combining subcase I (b) and subcase II (a), we obtain

$$|a_3-\mu a_2^2| \le \frac{1}{\beta+3(1-\beta)\tan\alpha}\ \text{if}\ \frac{\{4(1-\beta)(\beta+2)\tan\alpha\ -\beta(\beta-3)\}\{\beta+2(1-\beta)\tan\alpha\}-\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}{4\{\beta+3(1-\beta)\tan\alpha\}} \le \mu \le \frac{\{4(1-\beta)(\beta+2)\tan\alpha\ -\beta(\beta-3)\}\{\beta+2(1-\beta)\tan\alpha\}+\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}{4\{\beta+3(1-\beta)\tan\alpha\}} \quad (3.16)$$

***Subcase* II (b).** $\mu \ge \frac{\{4(1-\beta)(\beta+2)\tan\alpha\ -\beta(\beta-3)\}\{\beta+2(1-\beta)\tan\alpha\}+\{(1-\alpha)\beta+2\alpha(1-\beta)\}^2}{4\{\beta+3(1-\beta)\tan\alpha\}}$

Preceding as in subcase I (a), we get

$$|a_3-\mu a_2^2| \le \frac{1}{\{\beta+2(1-\beta)\tan\alpha\}^2}\left[4\mu - \frac{\{4(1-\beta)(\beta+2)\tan\alpha\ -\beta(\beta-3)\}\{\beta+2(1-\beta)\tan\alpha\ \}}{\{\beta+3(1-\beta)\tan\alpha\}}\right]. \quad (3.17)$$

Combining (3.9), (3.13) and (3.14), the theorem is proved.

Extremal function for (3.1) and (3.3) is defined by

$$f_1(z) = (1+az)^b,$$

where

$$a = \frac{2\{\beta+3(1-\beta)\tan\alpha\}}{\{4(1-\beta)(\beta+2)\tan\alpha-\beta(\beta-3)\}\{\beta+2(1-\beta)\tan\alpha\}^3-2}$$

and

$$b = \frac{\{4(1-\beta)(\beta+2)\tan\alpha \;-\beta(\beta-3)\}\{\beta+2(1-\beta)\tan\alpha\}^3-2}{\{\beta+3(1-\beta)\tan\alpha\;\}\{\beta+2(1-\beta)\tan\alpha\}}.$$

Extremal function for (3.2) is defined by $f_2(z) = z(1+cz^2)^d$,

where $c = \frac{\tan\alpha}{\beta+3(1-\beta)\tan\alpha}$ and $d = \frac{1}{\tan\alpha}$.

Corollary 3.2.*Putting* $\beta = 0$, *and applying limit as* $\alpha \to \frac{\pi}{2}$ *in the theorem, we get*

$$|a_3 - \mu a_2^2| \leq \begin{cases} 1-\mu, \; if \;\; \mu \leq \;\; 1; \\ \frac{1}{3} \quad if \quad 1 \leq \mu \leq \frac{4}{3}; \\ \mu - 1, \; if \;\; \mu \geq \;\; \frac{4}{3}. \end{cases}$$

These estimates were derived by Keogh and Merkes [8] and are results for the class of univalent convex functions.

Corollary 3.3.*Putting* $\alpha = 0, \beta = 1$,*in the theorem, we get*

$$|a_3 - \mu a_2^2| \leq \begin{cases} 3-4\mu \;\; if \;\; \mu \leq \;\; \frac{1}{2}; \\ 1 \quad if \quad \frac{1}{2} \leq \mu \leq 1; \\ 4\mu - 3, \; if \;\; \mu \geq \;\; 1. \end{cases}$$

These estimates were derived by Keogh and Merkes [8] and are results for the class of univalent starlike functions.

References

[1]. Alexander, J.W *Function which map the interior of unitcircle upon simple regions*, Ann. Of Math., **17** (1995),12-22.

[2]. Bieberbach, L. Uber die Koeffizientem derjenigen Potenzreihen, welche eine schlichte Abbildung des Einheitskrsises vermitteln , *S. – B . Preuss. Akad. Wiss.* **38** (1916), 940-955.

[3]. De Branges L., A proof of Bieberbach Conjecture, *Acta. Math*., **154** (1985),137-152.

[4]. Duren, P.L., Coefficient of univalent functions, Bull. *Amer. Math. Soc.*, **83** (1977), 891-911.

[5]. Fekete, M. and Szegö, G, Eine Bemerkung uber ungerade schlichte funktionen, *J.London Math. Soc*., 8 (1933), 85-89.

[6]. Garabedian, P.R. And Schiffer, M., AProof for the Bieberbach Conjecture forthe fourth coefficient, *Arch. Rational Mech. Anal*., 4 (1955), 427-465.

[7]. Kaur, C. and Singh, G., Approach To Coefficient Inequality For A New Subclass Of Starlike Functions With Extremals , *International Journal Of Research In Advent Technology*, **5**(2017) ,

[8]. Kaur, C. and Singh, G., Coefficient Problem For A New Subclass Of Analytic Functions Using Subordination , *International Journal Of Research In Advent Technology*, **5**(2017) ,

[9]. Keogh, F.R. and Merkes , E.P., A coefficient inequality for certain classes of analytic functions , *Proc. Of Amer. Math. Soc.*, **20** (1989), 8-12.

[10]. Koebe, P., Uber Die uniformisiesrung beliebiger analyischeer Kurven, *Nach. Ges. Wiss.Gottingen* (1907), 633-669.

[11]. Lindelof,E., Memoire sur certaines inegalities dans la theorie des functions monogenes et sur quelques proprieties nouvellles de ces fontions dans la voisinage d'un point singulier essential, *Acta Soc. Sci. Fenn*., **23** (1909) , 481-519.

[12]. Ma,W. and Minda , D. unified treatment of some special classes of univalent functions , *In Proceedings of the Conference on Complex Analysis* ,Z. Li, F. Ren , I. Yang and S.Zhang (Eds),Int. Press Tianjin (1994) , 157-169.

[13]. Miller, S.S., Mocanu, P.T. And Reade, M.O., All convex functions are univalent and starlike, *Proc. of Amer. Math. Soc.*, 37 (1973), 553-554.

[14]. Nehari, Z. (1952), *Conformal Mappings*, Mc Graw- Hill, New York.

[15]. Nevanlinna, R., Uber die Eigenshaften einer analytischen funkion in der umgebung einer singularen stele order Linte, *Acta Soc. Sci. Fenn*.,**50** (1922) , 1-46.

[16]. Pederson, R., A proof for the Bieberbach conjecture for the sixth coefficient,*Arch. Rational Mech. Anal*., 31 (1968-69), 331-351.

[17]. Pederson, R. and Schiffer, M., A proof for the Bieberbach conjecture for the fifth coefficient, *Arch. Rational Mech. Anal.*, 45 (1972), 161-193.

[18]. Rani, M., Singh, G., Some Classes Of Schwarzian Functions And Its Coefficient Inequality That Is Sharp, *Turk. Jour. Of Computer and Mathematics Education*, **11** (2020), 1366-1372.

[19]. Rathore, G. S., Singh , G. and Kumawat, L. et.al., Some Subclaases Of A New Class Of Analytic Functions under Fekete-Szego Inequality, *International Journal Of Research In Advent Technology*, **7**(2019) ,

[20]. Rathore, G. S., Singh , G., Fekete – Szego Inequality for certain subclasses of analytic functions , *Journal Of Chemical , Biological And Physical Sciences,* **5**(2015) ,

[21]. Singh, G, Fekete – Szego Inequality for a new class and its certain subclasses of analytic functions , *General Mathematical Notes,* **21** (2014),

[22]. Singh, G, Fekete – Szego Inequality for a new class of analytic functions and its subclass, *MathematicalSciences International Research Journal*, 3 (2014),

[23]. Singh, G., Construction of Coefficient Inequality For a new Subclass of Class of Starlike Analytic Functions, *Russian Journal of Mathematical Research Series,* **1** (2015), 9-13.

[24]. Singh, G., Introduction of a new class of analytic functions with its Fekete – Szegö Inequality, *International Journal of Mathematical Archive,* **5** (2014), 30-35.

[25]. Singh, G, An Inequality Of Second and Third Coefficients For A Subclass Of Starlike Functions Constrcted Using Nth Derivative, *Kaav InternationalJournal Of Science, Engineering And Technology*, **4** (2017), 206-210.

[26]. Singh, G, Fekete – Szego Inequality for asymptotic subclasses of family of analytic functions, *Stochastic Modelling And Applications*, 26 (2022),

[27]. Singh, G, Coefficient Inequality For Close To Starlike Functions Constrcted Using Inverse Starlike Classes , *Kaav InternationalJournal Of Science, Engineering And Technology* , **4** (2017) , 177-182.

[28]. Singh, G, Coefficient Inequality For A Subclass Of Starlike Functions That Is Constrcted Using Nth Derivative Of The Functions In The Class , *Kaav InternationalJournal Of Science, Engineering And Technology* , **4** (2017) , 199-202.

[29]. Singh, G., Fekete – Szegö Inequality For Functions Approaching to A Class In The Limit Form and another Class directly, *Journal Of Information And Computational Sciences*, 12 (4), 2022, 181-186

[30]. Singh,G. and Garg, J., Coefficient Inequality For A New Subclass Of Analytic Functions, *Mathematical SciencesInternational Research Journal,* **4**(2015)

[31]. Singh, G, Singh, Gagan, Fekete – Szegö Inequality For Subclasses Of A New Class Of Analytic Functions , *Proceedings Of The World Congress On Engineering* , (2014) , .

[32]. Singh, G, Sarao , M. S. , and Mehrok , B. S., Fekete – Szegö Inequality For A New Class Of Analytic Functions , *Conference Of Information And Mathematical Sciences* , (2013) , .

[33]. Singh, G, Singh, Gagan, Sarao , M. S. , Fekete – Szegö Inequality For A New Class Of Convex Starlike Analytic Functions , *Conference Of Information And Mathematical Sciences* , (2013) ,

[34]. Singh, G., Kaur, G., Coefficient Inequality for a Subclass of Starlike Function generated by symmetric points, *Ganita*, **70** (2020), 17-24.

[35]. Singh ,G., Kaur, G., Coefficient Inequality For A New Subclass Of Starlike Functions, *International Journal Of Research In Advent Technology,* **5**(2017) ,

[36]. Singh, G., Kaur, G., Fekete-Szegö Inequality For A New Subclass Of Starlike Functions, *International Journal Of Research In Advent Technology,* **5**(2017) ,

[37]. Singh,G., Kaur, G., Fekete-Szegö Inequality For Subclass Of Analytic Function Based On Generalized Derivative, *Aryabhatta Journal Of Mathematics And Informatics,* **9**(2017) ,

[38]. Singh,G., Kaur, G., Coefficient Inequality For A Subclass Of Analytic Function using Subordination Method With Extremal Function, *International Journal Of Advance Research In Science And Engineering* , **7** (2018) , .

[39]. Singh, G., Kaur, G., Arif, M., Chinram R, Iqbal J, A study of third and fourth Hankel determinant problem for a particular class of bounded turning functions, *Mathematical Problems in Engineering*, 2021

[40]. Singh, G. and Kaur, G., 4th Hankel determinant for α bounded turning function, *Advances in Mathematics: Scientific Journal*, 9 (12), 10563-10567

[41]. Singh,G., Kaur, N., Fekete-Szegö Inequality For Certain Subclasses Of Analytic Functions, *Mathematical SciencesInternational Research Journal,* **4**(2015)

[42]. Singh, G, Singh, B, Fekete Szego Coefficient Inequality of Regular Functions for A Special Class, International Journal of Research in Engineering and Science (IJRES), 10 (8), 2022, 556-560

[43]. Singh, G, Singh, P.,Fekete – Szegö Inequality For Functions Belonging To A Certain Class Of Analytic Functions Introduced Using Linear Combination Of Variational Powers Of Starlike And Convex Functions, *Journal Of Positive School Psychology*, **6** (2022), 8387-8391.

[44]. Singh. G, Rani M, An advance subclass of Analytic Functions having a unique coefficient inequality, International Journal of Research in Engineering and Science (IJRES), 10 (8), 2022, 474-476

[45]. Singh, G., Singh, G.,Singh, G.,A subclass of bi-univalent functions defined by generalized Sãlãgean operator related to shell-like curves connected with Fibonacci numbers, *International Journal of Mathematics and Mathematical Sciences*, 2019

[46]. Singh, G., Singh, G., Singh, G., Certain subclasses of univalent and biunivalent functions related to shell-like curves connected with Fibonacci numbers, *General Mathematics*, 28 (1), 125-140 2020

[47]. Singh, G., Singh, G., Singh, G., Certain subclasses of Sakaguchi-type bi-univalent functions, *Ganita*, 69 (2), 45-55, 2019

[48]. Singh, G., Singh, G., Singh, G., Fourth Hankel determinant for a subclass of analytic functions defined by generalized S˘al˘agean operator,*Creat. Math. Inform.*, 31 (2), 229-240, 2022