Heat Transfer in Unsteady Power-law Fluid over the Vertical Stretching Cylinder

ZafferElahi*, SummiaRani, AzeemShahzad

Abstract
This paper deals with the analysis of an incompressible flow and heat enhancement in an unsteady fluid by using the power-law model. The similarity transformations are utilized to reform the governing system of partial differential equations into the system of ordinary differential equations which are then solved by using the BVP4C technique. In addition, the temperature profiles are constructed for different physical parameters such as curvature parameter \( C \), unsteadiness parameter \( A \), Eckert number \( Ec \), Prandtl number \( Pr \) etc. Finally, the existing results are compared with the results available in the literature that reveal the superiority of the BVP4C scheme.

Keywords: Unsteady flow; Power-law; BVP4C; Heat transfer.

I. Introduction

Non-Newtonian fluids (don’t obey Newton’s law of viscosity) have wide applications in engineering and industrial areas as well as exist naturally (lava, honey, blood, mud) and nodoubt have been great uses in our daily life (toothpaste, souces, paints etc). In the past few years, the attraction of researchers increased towards non-Newtonian types of fluids due to their great utilization and manufacturing in industries. Different models proposed by researchers to describe the properties and behavior of these fluids. In these models, we have seen mostly applied model for non-Newtonian fluids is power-law model. Furthermore, in non-Newtonian fluid’s important role is played by the boundary layer and casting of metal, manufacturing of petroleum, die casting, oil and gas refineries, molding of plastic and rubbers etc. So, on very first investigating on pseudo-plastic fluid (power-law fluid) with boundary layer equations is presented by Showalter [1]. Elahi et al. [2] formulated the heat transfer enhancement in non-Newtonian fluid with boundary conditions. Chamkha et al. [3] investigate the unsteady heat and mass transfer in the porous medium. Moreover, Thiagarajan and Sangeetha [4] extract the model in which a numerical analysis of heat transfer over a power-law stretching surface with pressure gradient and viscous dissipation is presented. Elahi et al. [5] evaluate the numerical simulation that shows the improvement in heat transfer in nano-fluids over the stretching surface. Additionally, the boundary layer slip flows along a stretching cylinder are investigated by Mukhopadhyay [6]. Andersson et al. [7] analyze the behavior of non-Newtonian power-law fluid along a stretching surface. Ahmed et al. [8] investigate the magneto-hydro dynamics axis-symmetric flow of power-law fluid with convective boundary conditions over the stretching sheet. Further, the model is proposed by Hina et al. [9] in which mathematical analysis for fluid flow and heat transfer.

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outside a stretching hollow cylinder in a shear thinning fluid is formulated. Moreover, Elbashbeshy et al. [10] evaluate the boundary layer flow of stretching surface in a porous medium. An analysis presents on boundary layer over the stretching surface under the power-law model by Hassanien et al. [11]. Additionally, Ahmed et al. [12] investigate the axisymmetric flow of power-law fluid over the stretching sheet with convective boundary conditions.

The aim of this paper is to manipulate the unsteady flow of heat transfer on power-law fluid over the vertical stretching cylinder.

II. Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>(u, w)</td>
<td>Velocity components along (r, z) directions ((ms^{-1}))</td>
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<tr>
<td>(U_w)</td>
<td>Velocity of fluid ((ms^{-1}))</td>
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<tr>
<td>(\lambda)</td>
<td>Convection parameter</td>
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<tr>
<td>(\rho)</td>
<td>Density of fluid ((kgm^{-3}))</td>
</tr>
<tr>
<td>(c_p)</td>
<td>Specific heat of fluid ((Jkg^{-1}K))</td>
</tr>
<tr>
<td>(T_\infty)</td>
<td>Ambient temperature ((K))</td>
</tr>
<tr>
<td>(T_w)</td>
<td>Surface temperature ((K))</td>
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<tr>
<td>(T_s)</td>
<td>Slit temperature ((K))</td>
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</table>

III. Mathematical Formulation

Consider the two-dimensional unsteady power-law fluid flow over a cylinder of radius \(R\) with uniform magnetic field of strength \(B(t) = \frac{B_0}{1-\alpha t}\). The fluid movement is due to the stretching cylinder vertically, with uniform velocity illustrated as \(U(z) = \frac{cz}{1-\alpha t}\). Also, assume the cylindrical coordinates \((r, z)\) in radial and axial directions respectively, as shown in Figure 1.

Figure 1: Physical model with the coordinate system of the problem

The relationship between the surface temperature and ambient temperature written as \(T_w > T_s\), and

thermal conductivity of fluid is taken in the form \(K(T) = K_\infty \left[1 + \varepsilon \left(\frac{T - T_s}{T_w - T_s}\right)\right]\).

In addition, the velocity and temperature components are taken in the form of \((u(r, z, t), 0, w(r, z, t))\) and \(T = T_r(z, t)\) under these assumptions, the governing equations of continuity, momentum, and energy of the power-law model are typified as

\[
\frac{1}{r} \frac{\partial}{\partial r} (ru) + \frac{\partial w}{\partial z} = 0, \quad (1)
\]

\[
\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} = \frac{1}{\rho \rho_r} kr \left| \frac{\partial w}{\partial r} \right| n-1 \frac{\partial w}{\partial r} + g \beta (T - T_s) - \frac{\sigma (B(t))^2 w}{\rho}, \quad (2)
\]
subjected to \[ u = 0, w = U_w, T = T_w, \text{ at } r = R \]
\[ w \rightarrow 0, \hspace{1cm} T \rightarrow T_e, \hspace{1cm} \text{as } r \rightarrow 0 \]  
where \( n \) indicates the power-law index. Additionally, similarity variables are expressed as
\[
\eta = \frac{r - R_e}{R_w - T_e}, \hspace{1cm} \xi = \frac{r}{R_e} \hspace{1cm} \psi = R乌Re\frac{1}{\eta}f(\eta).
\]
The equation (1) is satisfied directly from the contact of velocity components and streamfunctions whosemathematical patterns
\[ u = -\frac{1}{r} \frac{\partial \phi}{\partial r}, \hspace{1cm} w = \frac{1}{r} \frac{\partial \phi}{\partial r}. \]
Solving the above equations (2-3) with (5) the resulting non-linear ordinary-differential equations for the given power-law model are
\[
A \left( f'' + \frac{2-2n}{n+1} \eta f' \right) - \frac{2n}{n+1} f' f'' + f''^2 + (1 + n)C(1 + 2C\eta)^{n+1} \frac{r}{r^2} (-f' f)^n - n(1 + 2C\eta)^{n+1} \frac{r}{r^2} (-f' f)^n + Mf' - \lambda f = 0,
\]
\[ Pr \cdot A \left( 0 + \eta \frac{2-n}{n+1} \theta' \right) + Pr \left( \theta f' - \frac{2n}{n+1} f' \theta' \right) - (1 + 2C\eta)^{n+1} \frac{r}{r^2} (-f' f)^n + 2C(1 + \varepsilon \theta) \theta' - (1 + \varepsilon \theta) (1 + 2C\eta) \theta'' = 0. \]
subject to boundary conditions
\[ f(\eta) = 0, \hspace{1cm} f' = 1, \hspace{1cm} \theta = 1 \hspace{1cm} \text{at } \eta = 0 \]
\[ f'(\eta) \rightarrow 0, \hspace{1cm} \theta(\eta) \rightarrow 0, \hspace{1cm} \text{as } \eta \rightarrow \infty. \]

The dimensionless parameters such as unsteadiness parameter, curvature parameter, magnetic parameter, and thermal slip parameter are introduced as
\[ A = \frac{a}{c}, \hspace{1cm} C = \frac{2x}{R Re^{\frac{1}{n+1}}}, \hspace{1cm} M = \frac{\sigma B_0^2 z}{\rho U}, \hspace{1cm} \lambda = \frac{g \beta z (T_w - T_o)}{U^2}. \]
In addition, Prandtl and Eckert numbers are defined as follows
\[ Pr = \frac{\frac{\nu_2 \eta}{K_w Re^{\frac{1}{n+1}}}}{\frac{\nu_1 \eta}{c_p(T_w - T_e)}}, \hspace{1cm} Ec = \frac{U^2}{\frac{\nu_1 \eta}{c_p(T_w - T_e)}}. \]
The beneficial engineering physical quantities like skin friction coefficient and Nusselt numbers are as follows
\[ C_f = \frac{\tau_w}{\nu_2 U_w}, \hspace{1cm} Nu = \frac{q_w}{\frac{\kappa w}{k(T_w - T_e)}}, \]
where
\[ \tau_w = k \left( \left| \frac{\partial u}{\partial r} \right|^n \right) \tau, \hspace{1cm} q_w = -k \left( \frac{\partial \theta}{\partial r} \right) \tau \chi. \]
The dimensionalless form of (11-12) using (5) is given as
\[ \frac{1}{2} Re^{\frac{1}{n+1}} C_f = -(-f''(\eta))^n, \hspace{1cm} Re = -\theta'(0). \]

**IV. Method of Solution**

Several techniques are adopted by researchers to determine the solutions of nonlinear ordinary differential equations. Hence, the more convergent and less error prone BVP4C scheme is found in [15] which is used to solve the boundary value problem (7-8) along with boundary conditions (9). The first order system of ODE’s is obtained by considering
\[ y_1 = f \]
\[ y_1' = y_2 \]
\[ y_2' = y_3 \]
\[ y_3' = \xi \]
\[ \theta = y_4 \]
\[ y_4' = y_5 \]
\[ y_5' = \xi \]
\[ y_1(0) = 0, \hspace{1cm} y_2(0) = 1, \hspace{1cm} y_3(0) = 1, \hspace{1cm} y_4(\infty) = 0 \]
where
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\[ \xi = \frac{(-\lambda y_4 + My_2 + y_2^2 + (1 + n)C(1 + 2\eta)^{n-1}(-y_3)^n + A \left( y_2 + \frac{2-n}{n+1} \eta y_3 \right) - \frac{2n}{n+1} y_1 y_3}{n(1 + 2\eta)^{n-1}(-y_3)^{n-1}}, \]

and

\[ \zeta = \frac{PrA \left( y_4 + \frac{2-n}{n+1} \eta y_5 \right) + Pr \left( y_2 y_4 - \frac{2n}{n+1} y_1 y_5 \right) - PrEc(1 + 2\eta)^{n+1}(-y_3)^{n+1} - 2Cy_5(1 + \epsilon y_4)}{(1 + \epsilon y_4)(1 + 2\eta)}. \]

To solve the system of equations (14-22), BVP4C scheme is used on MATLAB. Moreover, the graphical as well as numerical computations have been done for skin-friction coefficient and Nusselt number at different values of physical parameters, as listed in Tables 3-4. In addition, the validity and accuracy of scheme has been verified through comparison of the results available in the literature, as shown in Table 1-2.

Table 1: Comparison of numerical values of \( f''(0) \) at \( \lambda = C = A = 0 \).

<table>
<thead>
<tr>
<th>M</th>
<th>kandelous [16]</th>
<th>Ganesh et al. [17]</th>
<th>Present result</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>-1.1180340</td>
<td>-1.11803399</td>
<td>-1.11804</td>
</tr>
<tr>
<td>1.0</td>
<td>-1.4142135</td>
<td>-1.41421356</td>
<td>-1.41421</td>
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<tr>
<td>1.5</td>
<td>-1.802775638</td>
<td>-1.80277564</td>
<td>-1.80277</td>
</tr>
</tbody>
</table>

Table 2: Comparison of numerical values of \( -\theta'(0) \) at \( Ec = \epsilon = A = C = 0 \).

<table>
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<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.99999</td>
<td>1.00000</td>
<td>0.99999</td>
<td>1.00000</td>
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</tbody>
</table>

V. Results and Discussion

This portion is referred for exploring the numerically computed results and graphical simulation of the proposed model. For this reason, the BVP4C technique on MATLAB is utilized to solve the velocity and temperature profiles under boundary conditions. Further, computed the effects of all physical parameters like Curvature parameter, Eckert number, Prandtl number, thermal conductivity parameter on both skin friction and Nusselt number and have been estimated in the Tables (3-4). Additionally, physical insight (graphical pattern) is taken for more understanding of the temperature profile which shows the heat transfer in the power-law fluid. Moreover, the heat transfer effects are computed for three cases: pseudo-plastic fluid, dilatant fluid, and generalized Newtonian fluid.

Figure 2 depicts that by growing the value of Eckert number, temperature profile \( \theta'(\eta) \) goes enlarged. There is an observation that the higher value of Eckert number, temperature profile \( \theta'(\eta) \) increases. The Eckert number has a significant impact on the enhancement of heat transfer. The behavior of the temperature profile for different values of the Prandtl number shows in Figure 3. It implies that variation in the Prandtl number \( (Pr) \) causes a decrease in temperature profile and boundary-layer thickness. As the fluid movement is slower near the wall, it will be able less heat movement with it. Hence thicker the boundary layer less the heat is transferred. So, in heat transfer problem important role is played by \( Pr \), utilized to control the momentum boundary layer thickness.

Moreover, Table 3 is constructed for recording the numerical results of skin-friction coefficient under different values of Eckert number, Prandtl number, thermal conductivity parameters for power-law index \( n \). Table provides the decreasing trend in the numerical results of skin-friction coefficient \( (C_f) \) for the upward moving values of Curvature as well as Prandtl number. While the trend has been getting opposite for the thermal-conductivity parameter \( (\epsilon) \) and Eckert number \( (Ec) \) under \( 0.1 \leq Ec \leq 0.3 \). Further, the influence of different physical parameters on the local Nusselt number is calculated and listed in Table 4. It demonstrated that the Eckert number and thermal conductivity parameter reduced the numerical calculations of the Nusselt number. But, it is important to be noted that when the Curvature parameter getting large \( Nu \) becomes less. More, the values of Nusselt number rises whenever we improve the Prandtl number.
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\[ \theta(\lambda) \]

- \( M = 1, \, \lambda = A = 0.3, \, C = 0.2 \)
- \( Pr = 4.0, \, \epsilon = 0.1, \, n = 0.8 \)

- \( Ec = 0.1, 0.2, 0.3 \)

\[ \theta(\lambda) \]

- \( M = 1, \, \lambda = A = 0.3, \, C = 0.2 \)
- \( Pr = 4.0, \, \epsilon = 0.1, \, n = 1.0 \)

- \( Ec = 0.1, 0.2, 0.3 \)
Figure 2: Change in temperature profile for different values of Eckert number ($E_c$)
Figure 3: Change in temperature profile for different values of Prandtl number ($Pr$).
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Table 3: Variation of skin-friction coefficient ($C_f$) at $M = 1.0, A = \lambda = 0.3, Ec = C = 0.2, \epsilon = 0.1, Pr = 4.0$

<table>
<thead>
<tr>
<th>Physical parameters</th>
<th>$\frac{1}{2}\pi nC_f$</th>
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<tbody>
<tr>
<td>$C$</td>
<td>$Ec$</td>
</tr>
<tr>
<td>0.1</td>
<td>-</td>
</tr>
<tr>
<td>0.3</td>
<td>-</td>
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<tr>
<td>0.6</td>
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Table 4: Variation of Nusselt number ($Nu$) at $M = 1.0, A = \lambda = 0.3, Ec = C = 0.2, \epsilon = 0.1, Pr = 4.0$

<table>
<thead>
<tr>
<th>Physical parameters</th>
<th>$Re^{1/2}Nu$</th>
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<tbody>
<tr>
<td>$C$</td>
<td>$Ec$</td>
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<tr>
<td>0.1</td>
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<tr>
<td>0.3</td>
<td>-</td>
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VI. Conclusion

The model is presented to analyze the fluid flow and heat transfer enhancement over a ver-tical stretching cylinder by using the power-law model in the presence of a magnetic field. Further, thenumerical as well as graphs simulations are performed by using BVP4C technique. The obtained results are given as follows:

- The skin friction coefficient ($C_f$) is getting increased for Eckert number and thermal conductivity parameter while the reverse trend is seen for curvature and Prandtl number as shown in Table 3.
- Nusselt number ($Nu$) predicted enhanced by increasing Prandtl number. But get-ting decrease for other parameters like curvature ($C$), Eckert ($Ec$), and conductivity parameter ($\epsilon$) as shown in Table 4.
- Temperature profile $\theta(\eta)$ found growing for the values of the Eckert number in in-creasing order as in Figure 2 while Prandtl number under $0.3 \leq Pr \leq 5.0$ shows the oppo-site trend as under Figure 3.

The above results show great improvement in temperature distribution which shows the enhancement of heat in the power-law fluid flow.
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References