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Heat Transfer in Unsteady Power-law Fluid over the Vertical Stretching Cylinder

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Abstract
This paper deals with the analysis of an incompressible flow and heat enhancementin an unsteady fluid by using the power-law model. The similarity transformations areutilized to reform the governing system of partial differential equations into the systemof ordinary differential equations which are then solved by using the BVP4C technique.Further, the temperature profiles are constructed for different physical parameters suchas curvature parameter C, unsteadiness parameter A, Eckert number Ec, Prandtl number Pretc. In addition, the numerical values of skin-friction coefficient (C_f) and Nusseltnumber(Nu)arelistedinTables2-3forn=0.8,1.0,1.2.Finally,theexistingresultsarecomparedwiththeresultsavailableintheliteraturethatrevealthesuperiorityoftheBVP4Cscheme.
Keywords:*Unsteadyflow;Powerlaw;BVP4C;Heattransfer.*

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I. Introduction

Non- Newtonian fluids (don't obey Newton's law of viscosity) have wide applications inengineering and industrial areas as well as exist naturally (lava, honey, blood, mud) and nodoubt have been great uses in our daily life (toothpaste, souses, paints *etc*). In the pastfewyears,theattractionofresearchersincreasestowardthenon-Newtoniantypesoffluidsduetotheirgreatutilizationandmanufacturingininindustries.Differentmodelsproposedby researchers to describe the properties and behavior of these fluids. In these models, wehaveseenmostlyappliedmodelfornon-Newtonianfluidsispower-lawmodel.

Furthermore, in non- Newtonian fluid's important role is played by the boundary layer andcasting of metal,manufacturing of petroleum, die casting, oil and gas refineries, moldingof plastic and rubbers *etc*. So, on very first investigating on pseudo-plastic fluid (power-lawfluid)withboundarylayerequationsispresentedbyShowalter[1].Elahi*etal.*[2]formulatethe heat transfer enhancement in non-Newtonian fluid with boundary conditions. Chamkha*etal.*[3]investigatestheunsteadyheatandmasstransferintheporousmedium.Moreover,Thiagarajan and Sangeetha [4] extract the model in which a numerical analysis of heattransfer over a power-law stretching surface with pressure gradient and viscous dissipationis presented. Elahi *et al.* [5] evaluate the numerical simulation that shows the improvementin heat transfer in nano-fluids over the stretching surface. Additionally, the boundary layerslipflowsalongastretchingcylinderareinvestigatedbyMukhopadhyay[6].Andersson*etal.* [7] analyze the behavior of non- Newtonian power-law fluid along a stretching surface.Ahmed *et al.* [8] investigate the magneto-hydro dynamics axis-symmetric flow of power-lawfluid with convective boundary conditions over the stretching sheet. Further, the model isproposed by Hina*et al.* [9] in which mathematical analysis for fluid flow and heat transfer

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outside a stretching hollow cylinder in a shear thinning fluid is formulated.Moreover,Elbashbeshy*et al.*[10] evaluate the boundary layer flow of stretching surface in a porousmedium.An analysis presents on boundary layer over the stretching surface under thepower-law model by Hassanien*et al.* [11]. Additionally, Ahmed *et al.* [12] investigate theaxi-symmetric flow of power-law fluid over the stretching sheet with convective boundaryconditions.

The aim of this paper is to manipulate the unsteady flow of heat transfer on power-law fluidovertheverticalstretchingcylinder.

II. Nomenclature

u, w	Velocitycomponentsalongr, z directions(ms^{-1})
U_w	Velocityoffluid(ms^{-1})
λ	Convectionparameter
ρ	Densityoffluid(kgm^{-3})
c_p	Specificheatoffluid$(J^{-1}kg^{-1}K)$
T_∞	Ambienttemperature(K)
T_w	Surfacetemperature(K)
T_∞	Slittemperature(K)

III. Mathematical Formulation

Considerthetwo-dimensional unsteadypower-lawfluidflowoveracylinderofradius$R$withuniformmagneticfieldofstrength$B(t) = \frac{B_0}{1-\alpha t}$.

Thefluidmovementisduetothe stretching cylindervertically,withuniformvelocityillustratedas $U(z) = \frac{cz}{1-\alpha t}$. Also, assumethecylindricalcoordinates(r,z)inradialandaxialdirectionsrespectively, asshownin Figure 1.

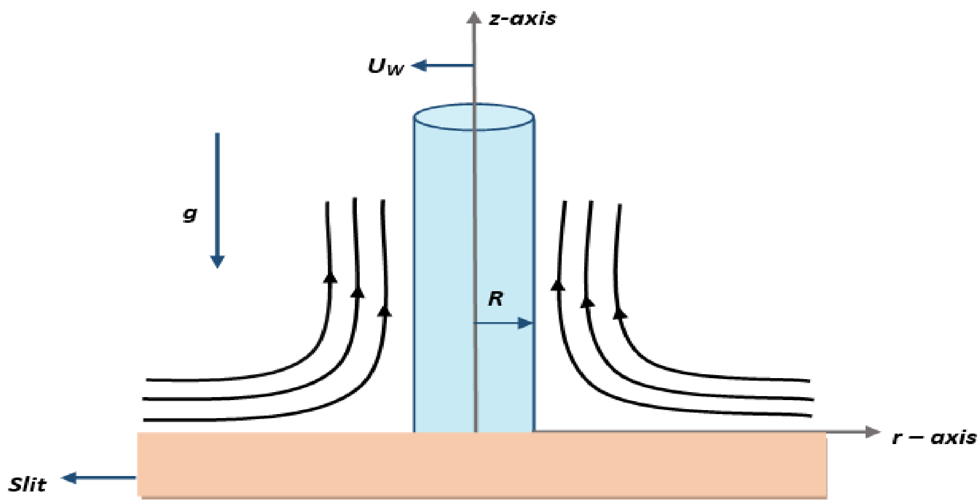


Figure1: Physical model with the coordinate system of the problem

Therelationshipbetweenthesurfacetemperatureandambienttemperaturewrittenas$T_w > T_\infty$and

thermalconductivityoffluidistakenintheform$K(T) = K_\infty\left[1 + \varepsilon\left(\frac{T - T_\infty}{T_w - T_\infty}\right)\right]$.

Inaddition,the velocityandtemperaturecomponentsaretakenintheform of$\left(u(r, z, t), 0, w(r, z, t)\right)andT = T\ (r, z, t)$ andunder theseassumptions,thegoverningequationsofcontinuity,momentumandenergyofthepowerlawmodelaretypified,asunder

$$\frac{1}{r}\frac{\partial}{\partial r}(ur) + \frac{\partial w}{\partial z} = 0, \qquad (1)$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = \frac{1}{\rho r}\frac{\partial}{\partial r}kr\left|\frac{\partial w}{\partial r}\right|^{n-1}\frac{\partial w}{\partial r} + g\beta(T - T_\infty) - \frac{\sigma\left(B(t)\right)^2 w}{\rho}, \qquad (2)$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}=\frac{k}{\rho c_p}\left(-\frac{\partial w}{\partial r}\right)^{n+1}+\frac{K(T)}{\rho c_p}\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}, \quad (3)$$

subjectedto[13,14]

$u=0, w=U_w, T=T_w,$ at $r=R$

$w\to 0,\quad T\to T_\infty,\quad as\quad r\to 0 \quad (4)$

where, n indicatesthepower-lawindex.Additionally,Similarityvariablesareexpressedas

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\ \eta=\frac{r^2-R^2}{2Rz}Re^{\frac{1}{n+1}},\quad \psi=RUzRe^{\frac{-1}{n+1}}f(\eta). \quad (5)$$

The equation(1)issatisfieddirectlyfromthecontactofvelocitycomponentsandstreamfunctions whosemathematicalpatternis

$$u=-\frac{1}{r}\frac{\partial\psi}{\partial z}\ ,\ w=\frac{1}{r}\frac{\partial\psi}{\partial r}\quad . \quad (6)$$

Solvingtheabove equations(2−3) with(5) the resultingnon-linear ordinary-differentialequationsfor thegivenpowerlawmodelare

$$A\left(f'+\frac{2-n}{1+n}\eta f''\right)-\frac{2n}{n+1}ff''+f'^2+(1+n)C(1+2C\eta)^{\frac{n-1}{2}}(-f'')^n-n(1+2C\eta)^{\frac{n+1}{2}}(-f'')^{n-1}f'''+Mf'-\lambda\theta\ =0, \quad (7)$$

$$PrA\left(\theta+\eta\frac{2-n}{n+1}\theta'\right)+\Pr\left(\theta f'-\frac{2n}{n+1}f\theta'\right)-(1+2C\eta)^{\frac{n+1}{2}}(-f'')^{n+1}-2C(1+\varepsilon\theta)\theta'-(1+\varepsilon\theta)(1+2C\eta\theta''\ =0. \quad (8)$$

subjecttoboundaryconditions

$f=0,\quad f'=1,\quad \theta=1 \qquad$ at $\eta=0$

$f'(\eta)\to 0,\quad \theta(\eta)\to 0, \qquad$ as $\eta\to\infty$ ..(9)

Thedimensionlessparameterssuchasunsteadinessparameter,curvatureparameter,magnetic parameter,andthermalslipparameterareintroducedas

$$A=\frac{\alpha}{c},\ C=\frac{2z}{R}Re^{\frac{-1}{n+1}},\quad M=\frac{\sigma {B_o}^2 z}{\rho U},\quad \lambda=\frac{g\beta z(T_w-T_\infty)}{U^2}.$$

Inaddition,PrandtlandEckertnumbersaredefinedasfollows

$$Pr=\frac{Uz\rho c_p}{K_\infty Re^{\frac{2}{n+1}}}\quad ,\quad Ec=\frac{U^2}{c_p(T_w-T_\infty)}. \quad (10)$$

Thebeneficialengineeringphysicalquantitieslikeskinfrictioncoefficientand Nusseltnumbers areasfollows

$$C_f=\frac{\tau_w}{\frac{1}{2}\rho {U_w}^2},\quad Nu=\frac{zq_w}{k(T_w-T_\infty)}, \quad (11)$$

where

$$\tau_w=k\left(\ |\frac{\partial w}{\partial r}\ |^n\right)_{r=R}\ ,\ q_w=-k\left(\frac{\partial T}{\partial r}\right)_{r=R}. \quad (12)$$

Thedimensionlessformof(11-12)using(5)isgivenas

$$\frac{1}{2}Re^{\frac{1}{n+1}}C_f=-[-f''(0)]^n\quad ,\quad Re=\ -\theta'(0). \quad (13)$$

IV. Method of Solution

Several techniques are adopted by researchers to determine the solutions of non-linearordinarydifferentialequations. Hence, themoreconvergentandlesserroragreeableBVP4Cschemeisfoundin[15]whichisusedtosolvetheboundaryvalueproblem(7-8)alongwithboundaryconditions(9).ThefirstordersystemofODE'sisobtainedbyconsidering,

$y_1=f \quad (14)$

$y_1'=y_2$ (15)

$y_2'=y_3 \quad (16)$

$y_3'=\xi \quad (17)$

$\theta=y_4 \quad (18)$

$y_4'=y_5 \quad (19)$

$y_5'=\zeta \quad (20)$

$y_1(0)=0,\quad y_2(0)=1,\quad y_4(0)=\ 1, \quad (21)$

$y_2(\infty)=0,\quad y_4(\infty)=\ 0, \quad (22)$

where

$$\xi = \frac{(-\lambda y_4 + My_2 + {y_2}^2 + (1+n)C(1+2C\eta)^{\frac{n-1}{2}}(-y_3)^n + A\left(y_2 + \frac{2-n}{1+n}\eta y_3\right) - \frac{2n}{n+1}y_1 y_3}{n(1+2C\eta)^{\frac{n+1}{2}}(-y_3)^{n-1}},$$

and

$$\zeta = \frac{PrA\left(y_4 + \frac{2-n}{1+n}\eta y_5\right) + \Pr\left(y_2 y_4 - \frac{2n}{n+1}y_1 y_5\right) - PrEc(1+2C\eta)^{\frac{n+1}{2}}(-y_3)^{n+1} - 2Cy_5(1+\varepsilon y_4)}{(1+\varepsilon y_4)(1+2C\eta)}.$$

To solve the system of equations (14-22), BVP4C scheme is used on MATLAB.Moreover,thegraphicalaswellasnumericalcomputationshavebeendoneforskin-frictioncoefficient and Nusselt number at different values of physical parameters, are listed in Tables 3-4. In addition, thevalidity and accuracy of scheme has been verified through comparison of the results availableintheliterature,asshown inTable1-2.

Table1:Comparisonofnumerical valuesof $f''(0)$at $\lambda = C = A = 0$.

M	kandelousi [16]	Ganesh et al. [17]	Present result
0.5	−1.1180340	−1.11803399	−1.11804
1.0	−1.4142135	−1.41421356	−1.41421
1.5	−1.802775638	−1.80277564	−1.80277

Table2:Comparisonofnumericalvaluesof$-\theta'(0)$at$Ec=\epsilon=A=C=0$.

Pr	Grubka and Bobba [18]	Elbashbeshy and Bazid [19]	Sharidan et al. [20]	Present result
1.0	0.99999	1.00000	0.99999	1.0000

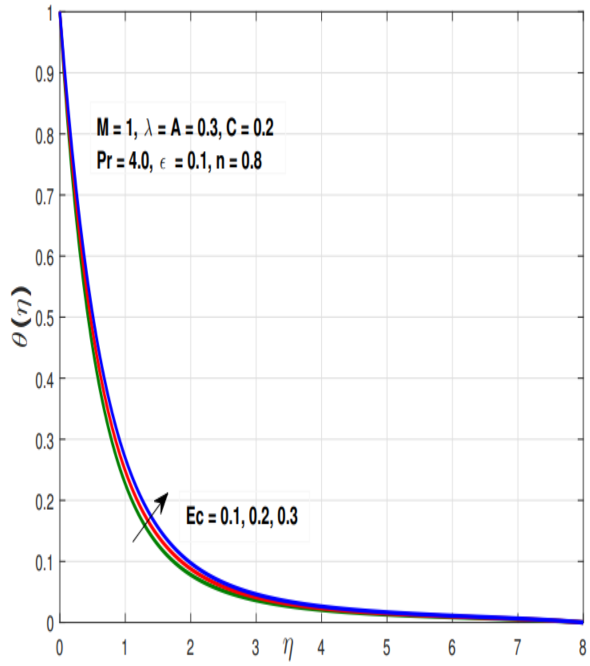
V. Results and Discussion

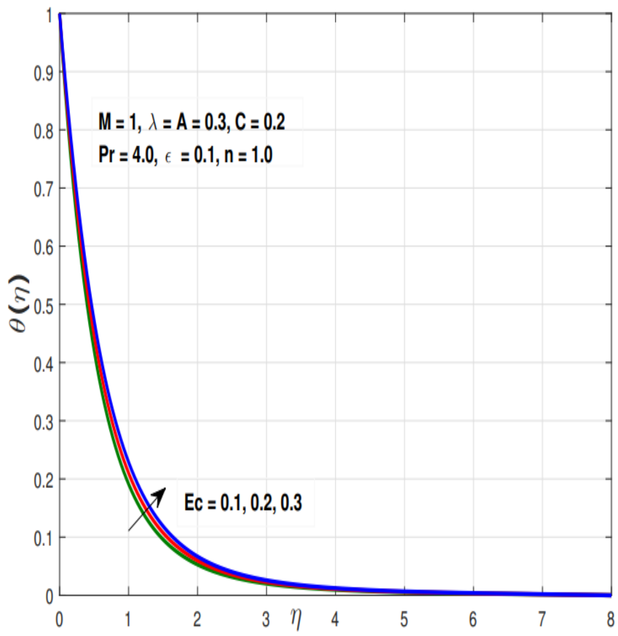
This portion is conferred for exploring the numerically computed results and graphical simulation of the proposed model.For this reason, the BVP4C technique on MATLAB isutilized to solve the velocity and temperature profile under boundary conditions. Further,computed the effects of all physical parameters like Curvature parameter, Eckert number,Prandtl number, thermal conductivity parameterson both skin friction and Nusselt number and have been estimated in the Tables (3-4). Additionally, physical insight (graphicalpattern) is taken for more understanding of the temperature profile which shows the heattransfer in the power-law fluid. Moreover, the heat transfer effects are computed for threecases;pseudo-plasticfluid,dilatantfluid,andgeneralizedNewtonianfluid.

Figure2depictsthatbygrowingthevaluesofEckertnumber,temperatureprofile$\theta(\eta)$goesenlarged.ThereasonisthatEchasadirectrelationwithkineticenergy(temperature is also known as the average kinetic energy of all substances) and when Ecincreasesthe kinetic energy also grows and hence the temperature goes large. Correspondingly, theEckert number has a significant impact on the enhancement of heat transfer. The behaviorof the temperature profile for different values of the Prandtl number shows in Figure 3. ItimpliesthatvariationinthePrandtlnumber(Pr)causesdecreaseintemperatureprofileandboundary-layerthickness.As,thefluidmovementisslowernearthewall, will be able less heat movement with it. Hence thicker the boundary layer less theheatistransferred.So,inheattransferproblem'simportantroleisplayedbyPr,utilizedtocontrolthemomentumandboundarylayerthickness.

Moreover, Table 3 is constructed for recording the numerical values of skin-friction coefficient under the different values of Eckert number, Prandtl number, thermal conductivityparametersfor power-law index *n*. Table provides the decreasing trend in the numericalresults of skin-friction coefficient (C_f) for the upward moving values of Curvature as well asPrandtl number.While the trend has been getting opposite for the thermal-conductivityparameter(ϵ)andEckertnumber(Ec)under$0.1 \leq Ec \leq 0.3$.

Further,theinfluencedofdifferentphysicalparametersonlocalNusseltnumbersiscalculatedandlistedinTable4.ItdemonstratedthattheEckertnumberandthermalconductivityparameterreducedthenumericalcalculationsoftheNusseltnumber.Butimportantisto be noticethatwhentheCurvatureparametergettinglargeNu becomesdown.More,thevaluesofNusseltnumberriseswheneverweimprovethePrandtlnumber.


M = 1, λ = A = 0.3, C = 0.2
Pr = 4.0, ϵ = 0.1, n = 0.8
Ec = 0.1, 0.2, 0.3
θ(η)
η


M = 1, λ = A = 0.3, C = 0.2
Pr = 4.0, ϵ = 0.1, n = 1.0
Ec = 0.1, 0.2, 0.3
θ(η)
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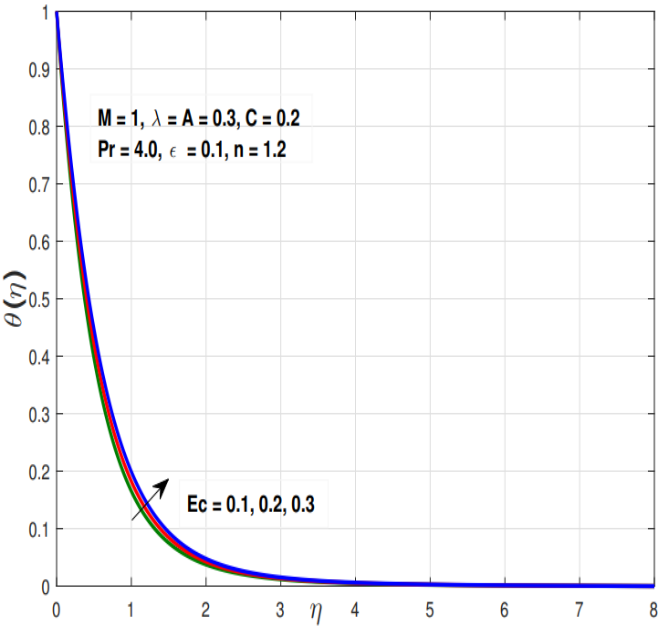

Figure2:ChangeintemperatureprofilefordifferentvaluesofEckertnumber(Ec)

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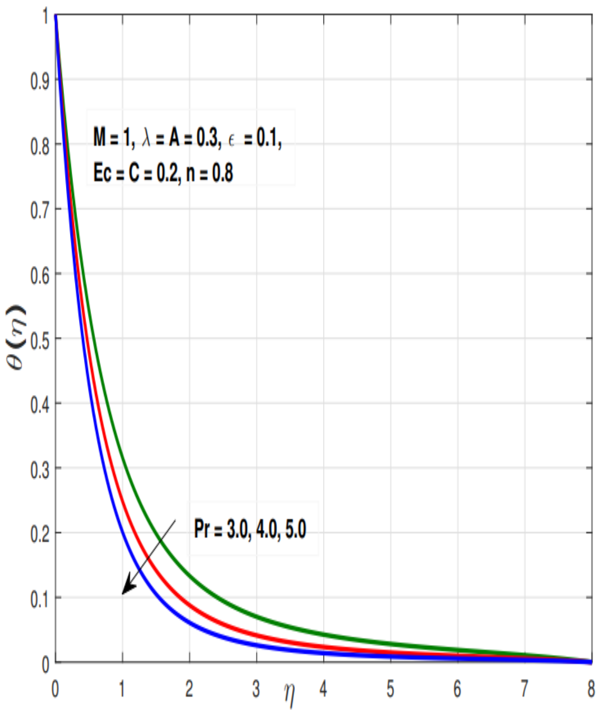

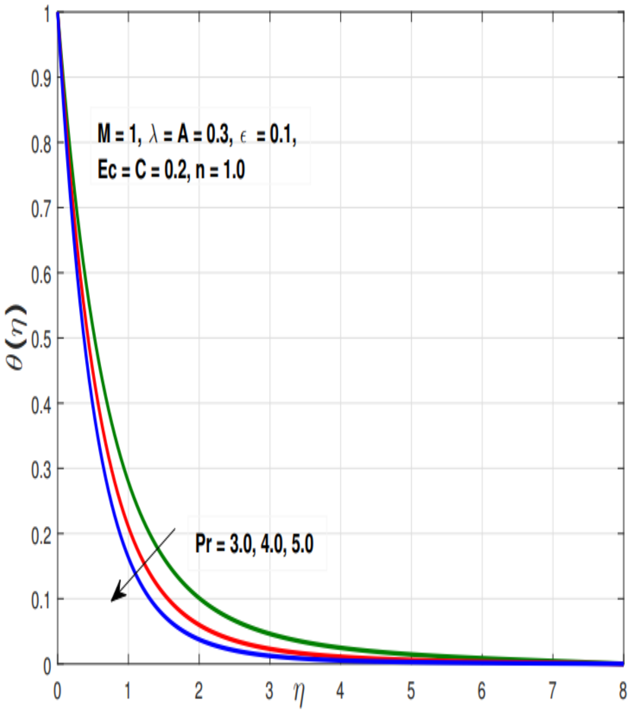

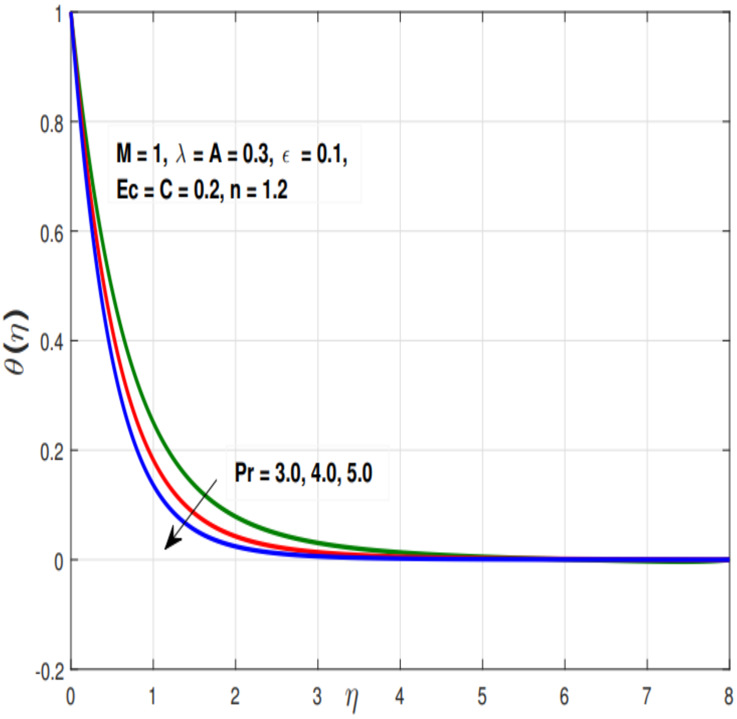

Figure3:ChangeintemperatureprofilefordifferentvaluesofPrandtlnumber(Pr)

Table3:Variationofskin-frictioncoefficient(C_f)at$M = 1.0, A = \lambda = 0.3, Ec = C$=0.2, ϵ=0.1,Pr=4.0

Physical parameters				$\frac{1}{2}Re^{\frac{1}{n+1}}C_f$		
C	Ec	Pr	ϵ	$n = 0.8$	$n = 1.0$	$n = 1.2$
0.1	-	-	-	-1.4357678	-1.4452603	-1.4529104
0.3	-	-	-	-1.5076609	-1.5186532	-1.5268007
0.6	-	-	-	-1.6102774	-1.6237608	-1.6327372
-	0.1	-	-	-1.4757363	-1.4861547	-1.4941528
-	0.2	-	-	-1.4721759	-1.4824095	-1.4902976
-	0.3	-	-	-1.4686611	-1.4787088	-1.4864856
-	-	3.0	-	-1.4628454	-1.4713067	-1.4780533
-	-	4.0	-	-1.4721759	-1.4824095	-1.4902976
-	-	5.0	-	-1.4790528	-1.4904	-1.4991044
-	-	-	0.1	-1.4721759	-1.4824095	-1.4902976
-	-	-	0.5	-1.4674839	-1.4773486	-1.4850065
-	-	-	1.0	-1.4622621	-1.4716504	-1.4790062

Table4:VariationofNusseltnumber(Nu)at$M = 1.0, A = \lambda = 0.3, Ec = C = 0.2,$ $\epsilon = 0.1, Pr = 4.0$

Physical parameters				$Re^{\frac{-1}{n+1}}Nu$		
C	Ec	Pr	ϵ	$n = 0.8$	$n = 1.0$	$n = 1.2$
0.1	-	-	-	1.6779654	1.7746371	1.8495676
0.3	-	-	-	1.3873574	1.475415	1.5439783
0.6	-	-	-	1.0729383	1.1491917	1.2078628
-	0.1	-	-	1.6885964	1.7750733	1.8431008
-	0.2	-	-	1.5226514	1.6151571	1.6871535
-	0.3	-	-	1.3589373	1.4572447	1.5330505
-	-	3.0	-	1.2780208	1.3532436	1.4119815
-	-	4.0	-	1.5226514	1.6151571	1.6871535
-	-	5.0	-	1.7372389	1.8448212	1.9282364
-	-	-	0.1	1.5226514	1.6151571	1.6871535
-	-	-	0.5	1.3312828	1.4173101	1.4849154
-	-	-	1.0	1.1620694	1.2418008	1.305112

VI. Conclusion

The model is presented to analyze the fluid flow and heat transfer enhancement over a ver-tical stretching cylinder by using the power-law model in the presence of a magnetic field.Further,thenumericalaswellasgraphicalsimulationsareperformedbyusingBVP4Ctechnique.Theobtained resultsaregivenas

- The skin friction coefficient (Cf) is getting increased for Eckert number and thermalconductivity parameter while the reverse trend is seen for curvature and PrandtlnumberasshowninTable3.
- Nusselt number (Nu) predicted enhanced by increasing Prandtl number.But get-ting decrease for other parameters like curvature (C), Eckert (Ec), and conductivityparameter(ϵ)asshowninTable4.
- Temperature profile $\theta(\eta)$ found growing for the values of the Eckert number in in-creasing order as in Figure 2 while Prandtl number under $0.3 \leq Pr \leq 5.0$ shows theoppositetrendunderFigure3.

The above results show great improvement in temperature distribution which shows theenhancementofheatinthepower-lawfluidflow.

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