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Floquet treatment of time dependent Hamiltonian of a gapless Dirac system

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Abstract
The Dirac systems are characterized by linear dispersion in momentum space. When Dirac system is exposed to the circularly polarized light, the Hamiltonian of the system becomes time dependent. The Schrodinger time dependent equation is used to get dispersion of the system in general. We done the floquet treatment of time dependent Hamiltonian of Dirac two level system, to remove the time dependence and achieve the static effective Hamiltonian. We also observe that the band gap appears in the Dirac system when polarized light is incident on it.
Keyword*: Dirac systems, Time dependent Hamiltonian, Floquet formalism, Band gap.*

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I. Introduction

In quantum mechanics the static potentials are easy to deal with, by finding the solution to the Time independent Schrodinger Equation. The exponential factor vanishes with mode square of wave function in time evolving states. Such states have static eigen energies independent of time[1]. Problem tends to rise when perturbing Hamiltonian is time dependent. The probability coefficient of each states $c_{t(n)}$ becomes time dependent, the probability of each energy states varies with time[2]. Time dependent schrodinger equation(TDSE) is used in general to solve the time varying system. TDSE is a second order differential equation which becomes tedious some time to solve. While, perturbation theories are effective when perturbations are smaller in magnitude than the original Hamiltonian.

The Electromagnetic (EM) pulse of the energy ħω triggers an electron from one to another state. Electron could land in many scattering channels[3]. If the energy given to the system is in periodic manner of frequency ω, then the allowed energy states E for the systems change to $(\mathrm{E} + n\hbar\omega)$ where n is an integer. The periodicity of the pulse reduces the degrees of the freedom for the system. The Hamiltonian of the system becomes time periodic function[3][4]. Remarkable consequence of floquet formalism emerges when apply to such systems. We achieve the static Hamiltonian as similar to the solution of time independent Schrodinger equation. Such Hamiltonian is terms as Floquet Hamiltonian and resultant dispersion energy as Floquet dispersion energy. Floquet formalism is an important tool to simplify the photo-induced periodic systems in time[5][6][7], an analogy of Bloch theorem which is applied to understand band structure of the periodic lattice[8][9].

Semi Dirac systems are characterized by their linear dispersion in one direction and non linear in other, in k space[10][11]. It is reported recently that when semi Dirac systems is exposed to the polarized light, the system becomes time dependent with rise of new topological phase. Floquet formalism is used to get the static effective Hamiltonian of a two level semi Dirac system[12][13][14]. The Dirac system are with linear dispersion in k_x and k_y such as graphene and silicene[16]. The interesting question arises that how can we achieve static Hamiltonian of Dirac systems when exposed to the time dependent potential. Such question is answered affirmatively in this research manuscript. We use the floquet formalism to remove the time dependence of the Hamiltonian of the Dirac gapeless system.

In section II we model the Hamiltonian of the Dirac gapless system. In section III, we couple the time dependent field with the Hamiltonian of system to make the Hamiltonian time dependent, and in section IV we use floquet formalism to formulate the static effective Hamiltonian. In section V we conclude our results.

Model

The generic form of the Hamiltonian of Dirac two level system can be presented in k.p model as following [17] [12],

$$H_0 = m_{(k)} \cdot \sigma \qquad (1)$$

where k represent the momentum with components (k_x, k_y, k_z), while I is 2×2 identity matrix, σ are pauli

pseduspin matrices (i.e., $\sigma = \sigma_x, \sigma_y, \sigma_z$) and the spin vector field $m_{(k)} = (m_1, m_2, m_3) = (vk_x, vk_y, 0)$. The spectrum of the Hamiltonian given in Eq.(1) emerges as follow,

$$E^{\pm}_{(k_x,k_y)} = \pm\sqrt{m_1^2 + m_2^2} \qquad (2)$$

where $\pm$ shows conduction and valence bands, respectively . It can be seen from the Fig.1, the spectrum is linear and gapless in momentum space around gamma point (i.e., $\Gamma_{(x,y)}$).

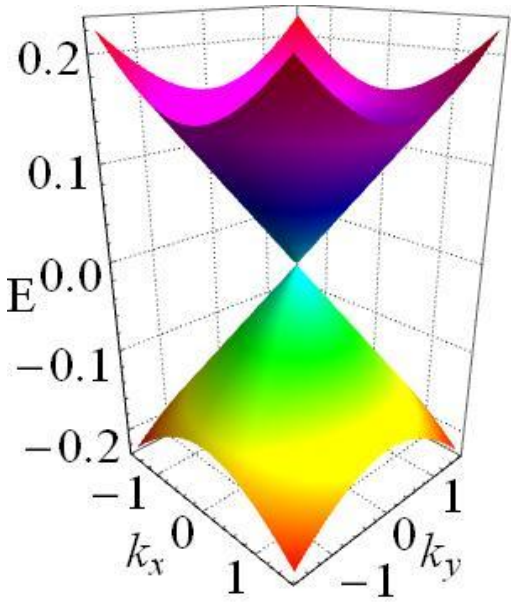


Figure 1 : The dispersion energy of Dirac system in absence of polarized light with gapless single Dirac node around $\Gamma_{(x,y)}$.

Time dependence

Now the circulary polarized light $A_t = (A_{(x,t)}, A_{(y,t)}) = (A_0\sin(\omega t), A_0\sin(\omega t + \phi))$ of amplitude A_o interacts with the Hamiltonian given in Eq.(1) , where ω the frequency, e is charge of electron. This interaction changes the momentum of the system's Hamiltonian from k to $(k + eA_t)$. Notice it, the sample size is much smaller than wavelength used in experiment to rule out spatial dependence of the field. The Hamiltonian given in Eq.(1) becomes time dependent and takes the shape as follow when interacts with light,

$$H_{(k,t)} = H_{0(k)} + H_{1(k,t)}\sigma_x + H_{2(k,t)}\sigma_y \qquad (3)$$

with

$$H_1 = veA_0\sin(\omega t) \qquad (4)$$

and

$$H_2 = veA_0\sin(\omega t + \phi) \qquad (5)$$

Floquet Hamiltonian

Now we apply the floquet formalism to treatl the time dependent Hamiltonian of the Dirac system [3][18]. The floquet technique can only be applied to the periodic function in time (i.e.,$g_{(t+T)} = g_{(t)}$), where T is the time period of the time varying function. The floquet Hamiltonian H_f is a sum of two components upto first order in inverse of the frequency [19], as given below,

$$H_f = H_f^0 + H_f^1 \qquad (6)$$

where

$$H_f^0 = \frac{1}{T}\int_0^T H_t dt = H_0 \qquad (7)$$

and

$$H_f^1 = \frac{1}{2iT\hbar}\int_0^T dt_1 \int_0^{t1} [H_{(k,t_1)}, H_{(k,t_2)}]\, dt_2 \qquad (8)$$

inserting values in Eq.(8) gives first order floquet term as follow,

$$H_f^1 = \left[\mu\{k_x\cos(\phi) - k_y\} + \zeta\sin(\phi)\right]\sigma_z \qquad (9)$$

where $\mu = \frac{2eA_0v^2}{\hbar\omega}$ and $\zeta = \frac{(eA_0v)^2}{\hbar\omega}$. Putting H_f^0 and H_f^1 in Eq.(6) to get the $floquet$ Hamiltonian, in term of new vector field as below,

$$H_f = g(k) \cdot \sigma \qquad (10)$$

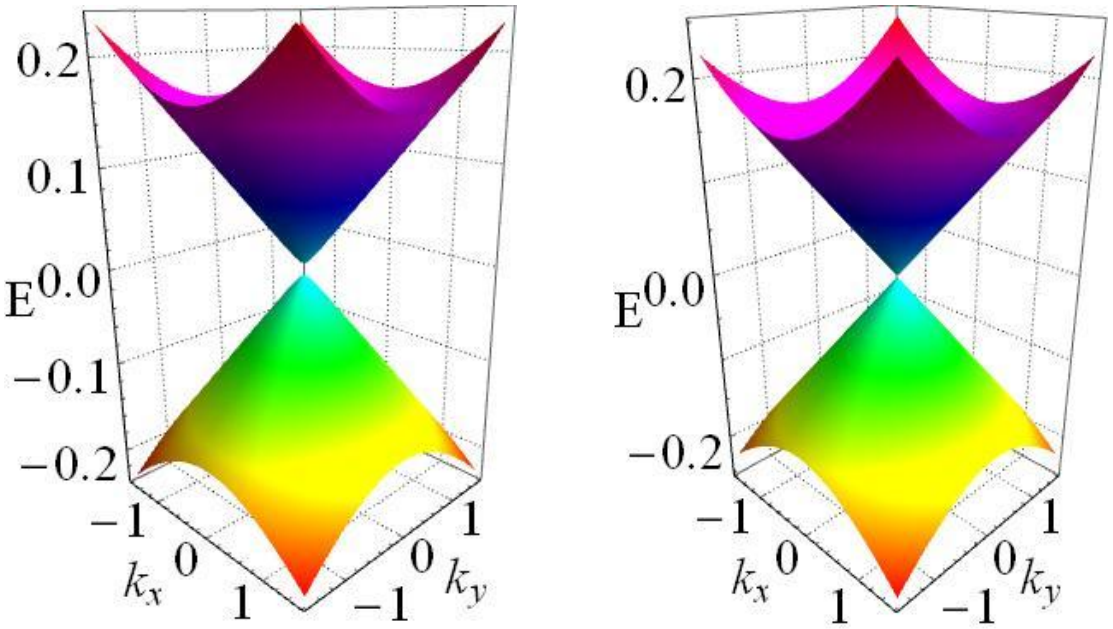
where

$$\begin{aligned} g_1 &= \nu k_x \\ g_2 &= \nu k_y \\ g_3 &= \mu\{k_x \cos(\phi) - k_y\} + \zeta \sin(\phi) \end{aligned} \qquad (11)$$

It is clear from Eq.(11) that the time dependence of the Hamiltonian has been removed. Now, the floquet energy spectrum of the Eq.(10) appears as follow,

$$E_f^{\pm} = \pm\sqrt{g_1^2 + g_2^2 + g_3^2} \qquad (12)$$

It is clearly visible that the dispersion of Dirac system is static in time. The dispersion is plotted in Fig.2(a), with phase angle half of π. It is interesting to note from Eq.(12) that the band gap emerges as polarized light incidents on the graphene to the $2\zeta\sin(\phi)$ and the band gap can be controlled with field strength eA_0 and angle of polarization. In Fig.2(b), the dispersion energy given in Eq.(12) is plotted keeping the phase angle 0 (i.e., linearly polarized light) with again gapless Dirac node similar to original system's dispersion energy.



(a) (b)

Figure 2: (a) The dispersion energy of Dirac system in presence of circularly polarized light with gapped Dirac node keeping phase angle half of π. (b) The dispersion energy of Dirac system in presence of linearly polarized light with gapless Dirac node keeping phase angle 0.

The state vectors in term of new vector field $g(k)$ can be described as below,

$$|\phi_+\rangle = N^+ \begin{pmatrix} 1 \\ \frac{E_f^+ - g_3}{g_1 - \iota g_2} \end{pmatrix} \qquad (13)$$

and

$$|\phi_-\rangle = N^- \begin{pmatrix} 1 \\ \frac{E_f^- - g_3}{g_1 - \iota g_2} \end{pmatrix} \qquad (14)$$

where, $\pm$ represents conduction and valence band, respectively and $N^{\pm}$ is a normalization constant,

$$N^{\pm} = \left[\frac{g_1^2 + g_2^2}{2E_f^{\pm}[E_f^{\pm} - g_3]}\right]^{\frac{1}{2}} \qquad (15)$$

For Dirac system fermi velocity $\nu = 2.5 \times 10^6 m/s$. The single photon of the energy $0.25\ eV$ is used during the experiment and the amplitude of circularly polarized light (i.e.,eA_0) is kept $\sim 0.3\ eV\text{Å}^{-1}$.

II. Conclusion

The non Dirac two level gapped system, becomes time dependent when exposed to the polarized light. It is concluded that floquet treatment done to the non Dirac system returns a static effective Hamiltonian of the system the static energy dispersion. The band gap further enhances when linearly polarized incidents on the non

Dirac gapped system. The band gap can be controlled with field strength and and angle of polarization.

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