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Application of Soham Transform in Newtons Law of Cooling

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ABSTRACT:
Newton's Law of Cooling is stated by Sir Isaac Newton in his work on cooling published in 1701 as "Scala graduum Caloris". This law is very much importance because it tells how an hot object cools in a particular environment. This law can be modelled in the form of differential equation, which is solved by many researchers by using different methods. Some researchers used integral transforms to solve this equation. In this paper we use recently introduced integral transform called as Soham Transform.
KEYWORDS: *Newton's Law of Cooling, Differential equations, Integral Transform, Soham Transform*
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I. INTRODUCTION:

Newton's Law of Cooling is an important law in physics stated by Sir Isaac Newton in 1701. This law states that the rate at which an object cools is proportional to the difference temperature between the object and its surroundings. This law is modelled by the differential equation

$$\frac{dT}{dt} = -K(T - T_0)$$

Where, K is positive constant,

T denotes the temperature of the object,

T_0 is ambient temperature,

t denotes the time.

This differential equation is solved by many researchers by different methods. Some of the researchers used integral transforms. There are various types of integral transforms introduced by many researchers. Recently in October 2021 Khakale and Patil [1] introduce the integral transform called as Soham Transform. For the function f(t), Soham transform denoted and defined as

$S[f(t)] = P(v) = \frac{1}{v}\int_0^\infty e^{-v^\alpha t}\, f(t)\, dt$

Where, α is non-zero real number.

Recently in September 2021 Kushare and Patil [2] introduced Kushare transform, for simplifying the process of obtaining solution of ordinary and partial differential equation in the time domain, many researchers are interested to use newly developed integral transforms in various fiels. Some of them are as follows:

In January 2022 R. S. Sanap and D. P. Patil [3] obtain the solution of Newton's Law of Cooling by Kushare transform. In October 2021 Sawi transform used in Bessel function by D. P. Patil [4]. Further Sawi transform of Error function is used to evaluate improper integral by D. P. Patil [5]. Patil [6] used Laplace and Shehu transform in Chemical sciences, Sawi transform and its convolution theorem is applied to solve wave equation by Patil [7]. Further, D. P. Patil [8] used Mahgoub transform for getting the solution of parabolic boundary value problems. Patil [9] also used double Laplace and double Sumudu transform to solve the wave equation. Dualities between various double integral transforms are obtained by D. P. Patil [10]. Laplace, Elzaki and Mahgoub transforms are used for solving system of first order and first degree differential equations by Kushare and Patil [14]. Dr. Patil [12] also used Aboodh and Mahgoub transform in boundary value problems of system of ordinary differential equations. Recently, in April 2022, Nikam, Shirsath, Aher and Patil [13] used Kushare transform in growth and decay problems. Patil [14] compared Laplace, Sumudu, Aboodh, Elzaki and Mahgoub transform in evaluating boundary value problems. Further Patil [15] used double Mahgoub transform in solving parabolic boundary value problems. D. P. Patil et al [16] used Anuj transform to solve Volterra integral equations of first kind. Soham transform is used to solve same equations by D. P. Patil et al [17] . Rathi sisters and D. P. Patil used Soham transform for system of differential equations [18]. Recently Zankar , Kandekar and D. P. Patil used general integral transform of error function for evaluating improper integrals[19].

In this paper we use Soham transform to obtain the solution for Newton's Law of Cooling. Further, we solved some examples based on Newton's Law of Cooling.

This paper is organised as follows: Some preliminary concepts are stated in second section. Third section is devoted for application of Soham transform for Newton's law of cooling.

II. PRELIMINARIES:

In this section we state some preliminary concepts to understand this paper easily. We write some formulae for soham transform of the required elementary functions. Further inverse Soham transform and properties of transform are stated. Conclusion is drawn in last section.

2.1: SOHAM TRANSFORM OF THE ELEMENTARY FUNCTIONS:

Sr.No.	F(t)	Soham Transform
1	1	$\frac{1}{v^{\alpha+1}}$
2	t	$\frac{1}{v^{2\alpha+1}}$
3	t^n	$\frac{\Gamma(n+1)}{v^{\alpha n+\alpha+1}}$
4	e^{at}	$\frac{1}{v(v^{\alpha}+1)}$

2.2 INVERSE SOHAM TRANSFORM:[1]

If Soham transform of f(t) is P(v) then inverse Soham transform is defined as

$$S^{-1}[P(v)] = f(t)$$

Now we will state some required properties

A. LINEARITY PROPERTY OF SOHAM TRANSFORM:**[1]**

If f_1(t) and $f_2(t)$ be two functions of t and c_1 and c_2 be any two constants then

$$S[c_1 f_1(t) + c_2 f_2(t)] = c_1 S[f_1] + c_2 S[f_2]\ .$$

B. [1] Let P(v) Soham transform of {S[f(t)] = P(v) } then :

$$S[f'(t)] = v^{\alpha} P(v) - \frac{1}{v} f(0)$$

III. APPLICATION OF SOHAM TRANSFORM IN NEWTON'S LAW OF COOLING :

In this section we will solve some problems based on Newton's Law of Cooling by using Soham Integral transform.

Problem .1: A hot coffee with initial temperature of 115°F is kept in a room temperature of 35°F. The rate of change of temperature is 20°F/min. How long it will take coffee to cool to a temperature of 40°F?

Solution: Assuming that a coffee obeys Newton's Law of Cooling, We have

$$\frac{dT}{dt} = -C(T-35)\,, \qquad T(0) = 115, T'(0) = -20$$

First we will find the value of C by using initial condition

$$-20 = -C\,(115-35)$$
$$-20 = -80\,C$$
$$\therefore C = 0.25$$

So, the differential equation can be written as

$$\frac{dT}{dt} = -0.25\,(T-35)$$

Now, By Soham transform, we get

$$\therefore\ v^{\alpha} P(v) - \frac{1}{v} T(0) = -0.25\,P(v) + 0.25\times 35\,S(1)$$
$$\therefore\ v^{\alpha}\,P(v) - \frac{115}{v} = -0.25\,P(v) + 0.25\times 35\left(\frac{1}{v^{\alpha+1}}\right)$$
$$\therefore (v^{\alpha}+0.25)P(v) = 0.25\times 35\,\left(\frac{1}{v^{\alpha+1}}\right) + \frac{115}{v}$$
$$\therefore P(v) = \frac{35\times 0.25}{v}\left[\frac{1}{v^{\alpha}(v^{\alpha}+0.25)}\right] + \frac{115}{v(v^{\alpha}+0.25)} \qquad (1)$$

Then by Partial Fractions, we get

$$\frac{1}{v^{\alpha}(v^{\alpha}+0.25)}=\frac{A}{v^{\alpha}}+\frac{B}{v^{\alpha}+0.25}$$
$$\therefore 1=A(v^{\alpha}+0.25)+Bv^{\alpha}$$
$$A=4 \quad and \quad B=-4$$
$$\frac{1}{v^{\alpha}(v^{\alpha}+0.25)}=\frac{4}{v^{\alpha}}-\frac{4}{v^{\alpha}+0.25}$$

From Equation (1),

$$\therefore P(v)=\frac{35\times 0.25}{v}\left[\frac{4}{v^{\alpha}}-\frac{4}{v^{\alpha}+0.25}\right]+\frac{115}{v(v^{\alpha}+0.25)}$$
$$=\frac{35}{v^{\alpha+1}}-\frac{35}{v(v^{\alpha}+0.25)}+\frac{115}{v(v^{\alpha}+0.25)}$$
$$=\frac{35}{v^{\alpha+1}}+\frac{80}{v(v^{\alpha}+0.25)}$$

Now by taking inverse Soham Transform, we get

$$\therefore T(t)=35S^{-1}\left(\frac{1}{v^{\alpha+1}}\right)+80S^{-1}\left(\frac{1}{v(v^{\alpha}+0.25)}\right)$$

$$\therefore 40=35(1)+80\,e^{-0.25t}$$

$$\therefore e^{-0.25t}=\frac{1}{16}$$

$$\therefore e^{0.25t}=16$$

$$\therefore 0.25t=\ln(16)$$

$$\therefore 0.25t=2.772588722$$

$$t=11.090354888\,min.$$

Thus, Coffee will take 11.09 minutes for cooling to a temperature of 40°F.

Problem 2: A heated beam cools at the rate of 3°C per minute when its temperature is 50°C. Find the time taken to cool at 36°C if the temperature of the surroundings is 27°C.

Solution: Assuming that heated metal beam obeys Newton's Law of Cooling, We have

$$\frac{dT}{dt}=-C(T-27), \qquad T(0)=50, T'(0)=-3$$

First we will find the value of C by using initial condition

$$-3=-C(50-27)$$
$$-3=-23C$$
$$C=0.13$$

So, the differential equation can be written as

$$\frac{dT}{dt}=-0.13\,(T-27)$$

Now by Soham Transform, we get

$$\therefore v^{\alpha}P(v)-\frac{1}{v}T(0)=-0.13\,P(v)+0.13\times 27\,S(1)$$
$$\therefore v^{\alpha}P(v)-\frac{50}{v}=-0.13\,P(v)+0.13\times 27\left(\frac{1}{v^{\alpha+1}}\right)$$
$$\therefore (v^{\alpha}+0.13)P(v)=0.13\times 27\left(\frac{1}{v^{\alpha+1}}\right)+\frac{50}{v}$$

$$\therefore P(v) = \frac{27}{v}\left[\frac{0.13}{v^{\alpha}(v^{\alpha}+0.13)}\right] + \frac{50}{v(v^{\alpha}+0.13)} \qquad (2)$$

By the Partial Fraction, We get

$$\frac{0.13}{v^{\alpha}(v^{\alpha}+0.13)} = \frac{A}{v^{\alpha}} + \frac{B}{v^{\alpha}+0.13}$$

$$\therefore 0.13 = A(v^{\alpha}+0.13) + Bv^{\alpha}$$

$$A = 1 \quad and \quad B = -1$$

$$\therefore \frac{0.13}{v^{\alpha}(v^{\alpha}+0.13)} = \frac{1}{v^{\alpha}} - \frac{1}{v^{\alpha}+0.13}$$

From Equation (2),

$$\therefore P(v) = \frac{27}{v}\left[\frac{1}{v^{\alpha}} - \frac{1}{v^{\alpha}+0.13}\right] + \frac{50}{v(v^{\alpha}+0.13)}$$

$$= \frac{27}{v^{\alpha+1}} - \frac{27}{v(v^{\alpha}+0.13)} + \frac{50}{v(v^{\alpha}+0.13)}$$

$$= \frac{27}{v^{\alpha+1}} + \frac{23}{v(v^{\alpha}+0.13)}$$

By taking Inverse Soham transform, we get

$$\therefore T(t) = 27S^{-1}\left(\frac{1}{v^{\alpha+1}}\right) + 23S^{-1}\left(\frac{1}{v(v^{\alpha}+0.13)}\right)$$

$$\therefore 36 = 27(1) + 23\, e^{-0.13t}$$

$$\therefore e^{-0.13t} = \frac{9}{23}$$

$$\therefore e^{0.13t} = 2.5556$$

$$\therefore 0.13t = 0.9383$$

$$\therefore t = 7.2175\,\text{min}$$

Thus, A heated metal beam will take 7.2175 minutes for cooling to a temperature of 36°C.

IV. Conclusion:

In this paper, we have successful used SOHAM Integral Transform For solving Newton's Law of Cooling. We get the solutions of the problems based on Newton's Law of Cooling easily and accurate.

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