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# "Unique Way of Expressing a Math Theorem"

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***Abstract***
*The purpose of this paper is to introduce a unique way of explaining a math theorem that is created by a high school, 10th grade student. This theorem shows a relationship between the angles of a contact triangle and its original triangle. No such explanation or description of this theorem is found in the literature so far. Authors are excited to present this theorem in the present paper.*

***Keywords:*** *Math Theorem, geometry, triangles, contact triangle, corresponding angle, inscribed triangle, algebra.*

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## I. INTRODUCTION

Mathematics is always a subject of interest in terms of counting, solving angles, working with theorems, etc. Some mathematical relationships expressed in references [1-2] shows example of a mathematical procedure that is used in inscribed triangles to solve geometrical questions. The interest to write this paper is to show a unique way to find a relationship between the angles of a contact triangle and the angles of its original triangle.

A contact triangle (also known as the intouch triangle) of another triangle is the triangle formed by the intersection points of the original triangle and its incircle. [3] Given a triangle, △ABC, its contact triangle is △A'B'C'. A' is opposite to A, B' is opposite to B, and C' is opposite to C. Let these pairs be called corresponding vertices. Let the measure of the angle of a vertex be the interior angle of the triangle it is the vertex of. Let the angle of A be corresponding to the angle of A', and similarly for B and B', and C and C'. When an angle is "closer" to another angle than a third angle, it means the difference between the absolute value of the measure of the first angle and second angle is less than the absolute value of the difference of the measure of the third and second angle. For example, if angle X = 30°, angle Y = 60°, and angle Z = 70°, angle Z is "closer" to Y than X. However, if angle Z = 100°, then X would be "closer" to Y than Z.

## II. THE THEOREM

The theorem can be stated as "**The measure of an angle of a contact triangle is equal to the mean of the angles adjacent to the segment the angle is on".**

An extension of the theorem is, **"The difference between 60° (the measure of the interior angles of an equilateral triangle) and the measure of the angles of a contact triangle is (-1/2) times the difference of the measure of the angle of the corresponding vertex of the original triangle and 60°."**

In other words, **"The difference of 60° and one of the angles of the inscribed triangle is negative half times of the difference of 60° and the measure of the angle of the corresponding vertex of the contact triangle that is outside of the circle."**

Or in simple words, if you take any triangle and draw a circle inside it, then connect the intersecting points to make another triangle inside the circle, the angles of the inside triangle are half of the angles of the outside triangle.

What this suggests is that the measure of the contact triangle's angle is "closer" to the measure of an angle in an equilateral triangle than its corresponding angle in the original triangle. If the process of creating a contact triangle is iterated with the original triangle being the previous contact triangle, then the contact triangle will converge to an equilateral triangle.
Here is detailed explanation of the theorem with the hand drawn picture shown in **Figure 1**.

Let's say if α, β, γ are interchangeable angles, and are not absolute. However, once they are defined within a diagram, they cannot be changed. Then, an equation for α can be replaced with β or γ and it will hold true as well.

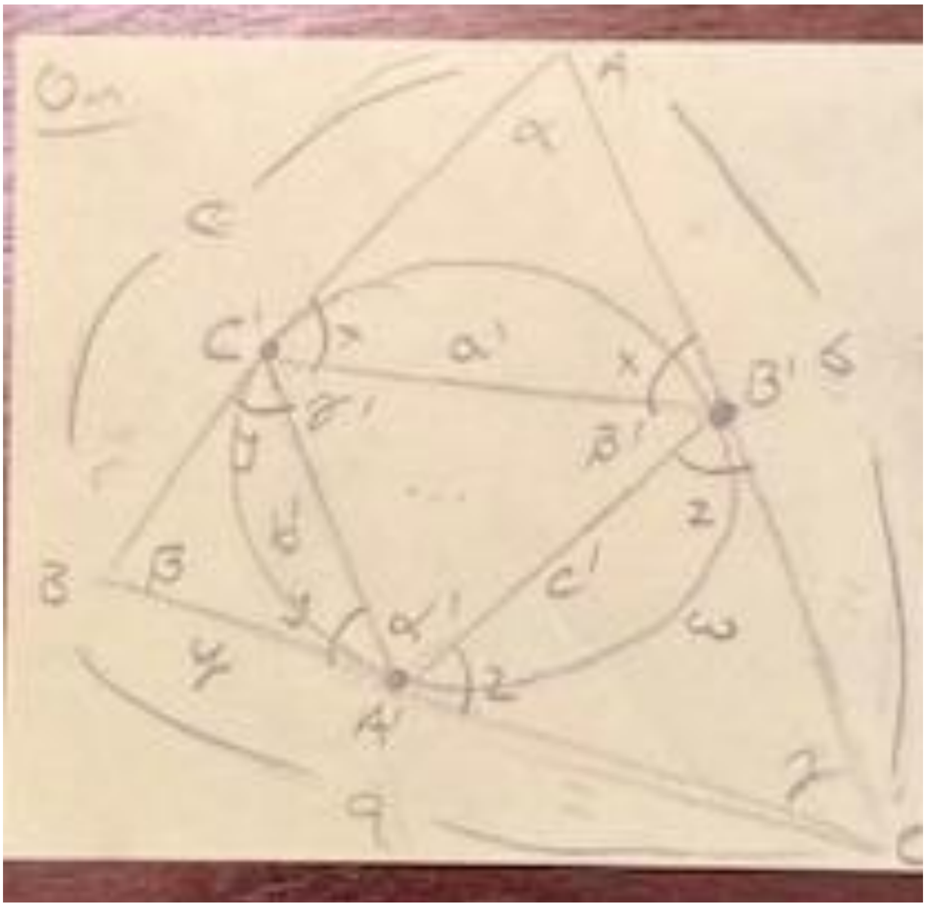
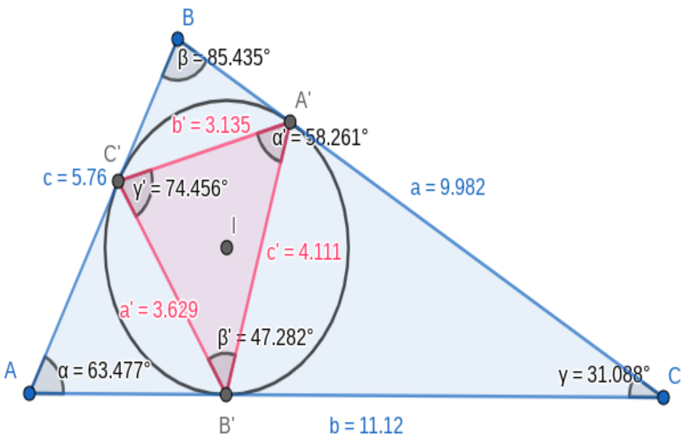


**Figure 1:** The hand drawn picture of theorem made by first author (Om)

The hand-made picture shown Figure 1, is now remade using the online GeoGebra tool and shown below in Figure 2. [4]




As you can see, the corresponding angles (α and α', etc.) in the new triangle (inner) are closer to 60° than the outer angles

**Figure 2:** Picture of the triangle drawn using GeoGebra for a geometrical simulation. [4]

## III. PROOF OF THE THEOREM

This theorem can be proved to show that it is true and hold right relationship. It is valid for any type of triangles that is used for this theorem example an acute or obtuse triangles. The steps and the ways to describe this theorem in detail is given below.

Let us consider that $\alpha$ , $\beta$and $\gamma$ are to be the angles of the original triangle, △ABC as shown in the Figure 1 and Figure 2 with side-lengths a, b, and c, and angles $\alpha$ , $\beta$ and $\gamma$. Let △A'B'C' be its contact triangle. This ABC triangle is formed by the contact angles of a triangle △A'B'C' that is inscribed in a circle. The side-lengths of A'B'C' are a', b', and c', and its angles of the triangle △A'B'C' are $\alpha'$ , $\beta'$ and $\gamma'$. The sides of triangle △ABC are a, b, c respectively and the sides of triangle △A'B'C' are a', b' and c' respectively. The relationship between angles can be seen in the form of equations below.

Some of the proof texts are written in LaTeX. [5]

$$\begin{cases} \alpha + \beta + \gamma = 180 \\ \alpha\prime + \beta\prime + \gamma\prime = 180 \\ \alpha + 2x = 180 \\ \beta + 2y = 180 \\ \gamma + 2z = 180 \\ x + y + \gamma\prime = 180 \\ \alpha\prime + y + z = 180 \\ x + \beta\prime + z = 180 \end{cases}$$

$$\therefore\ x + y + z = 180$$
$$\therefore\ \alpha' = x, \beta' = y, \gamma' = z$$

For α′ and α′:

$$\alpha + 2\alpha' = 180$$
$$\alpha' = \frac{180 - \alpha}{2}$$
$$\therefore\ \alpha' = \frac{\beta + \gamma}{2}$$

Similarly, for β′ and γ′.

If the theorem is true, then:

$$60 - \alpha' = -\frac{1}{2}(60 - \alpha)$$
$$\frac{\alpha + \beta + \gamma}{3} - \alpha' = -\frac{1}{2}\left(\frac{\alpha + \beta + \gamma}{3} - \alpha\right)$$
$$2\alpha' - \frac{2\alpha + 2\beta + 2\gamma}{3} = \frac{\alpha + \beta + \gamma}{3} - \alpha$$
$$2 \cdot \frac{\beta + \gamma}{2} - \frac{2\alpha + 2\beta + 2\gamma}{3} = \frac{\alpha + \beta + \gamma}{3} - \alpha$$
$$\frac{3\beta + 3\gamma - 2\alpha - 2\beta - 2\gamma}{3} = \frac{\alpha + \beta + \gamma - 3\alpha}{3}$$
$$3\beta + 3\gamma - 2\alpha - 2\beta - 2\gamma = \alpha + \beta + \gamma - 3\alpha$$
$$-2\alpha + \beta + \gamma = -2\alpha + \beta + \gamma$$
$$0 = 0$$

The expression is always true, so the theorem is correct. $\alpha$ represents any angle of the original triangle, and $\alpha'$ represents the corresponding angle in the contact triangle. $\alpha$ and $\alpha'$ can be interchanged with $\beta$ and $\beta'$ or $\gamma$ and $\gamma'$, as the theorem will still hold.

## IV. CHECKING THE PROOF

This theorem can also be checked by flipping this relationship to show that the formula is accurate. This relationship can also be written as 60 - a' = -1/2(60 - a) which simplifies algebraically to: a = 180 - 2a'. This is another way of checking that the theorem is and right and accurate.

## V. TEST CASES

Some test cases can be provided to show this theorem in action as shown below and can be considered as examples of using types of conditions to see if theorem is right. GeoGebra is rounding the decimals to the 3rd decimal place, so the values given will be approximates.

***(a) Example of an acute triangle:***

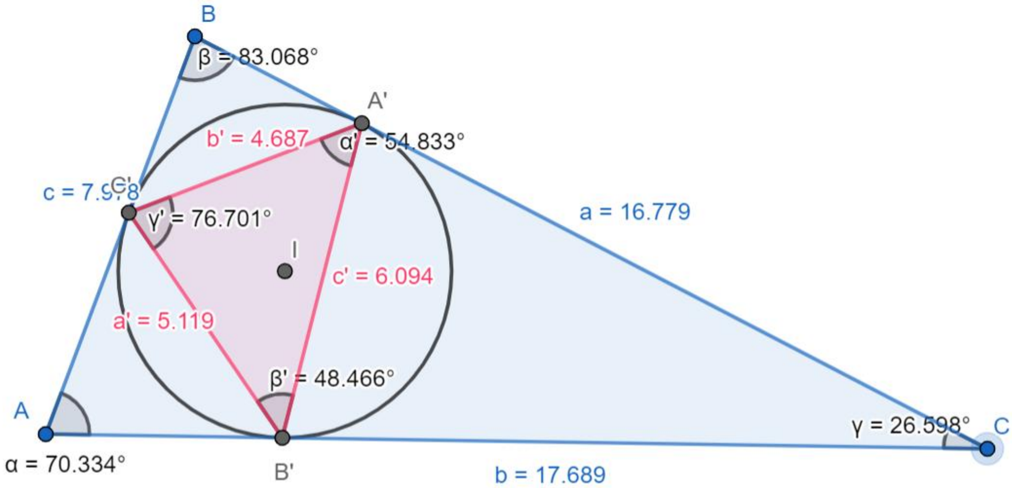



**Figure 3:** Picture of an acute triangle in GeoGebra for a geometrical simulation. [4]

According to the results, $\alpha'$ should equal $\frac{\beta+\gamma}{2}$. $\alpha' = 54.833°$ according to GeoGebra, and $\frac{\beta+\gamma}{2} = \frac{83.068°+26.598°}{2} = 54.833°$. $\beta' = 48.466°$ according to GeoGebra, and $\frac{\alpha+\gamma}{2} = \frac{70.334°+26.598°}{2} = 48.466°$. $\gamma' = 76.701°$ according to GeoGebra, and $\frac{\alpha+\beta}{2} = \frac{70.334°+83.068°}{2} = 76.701°$. $\alpha\beta\gamma°$

***(b) Example of an obtuse triangle:***

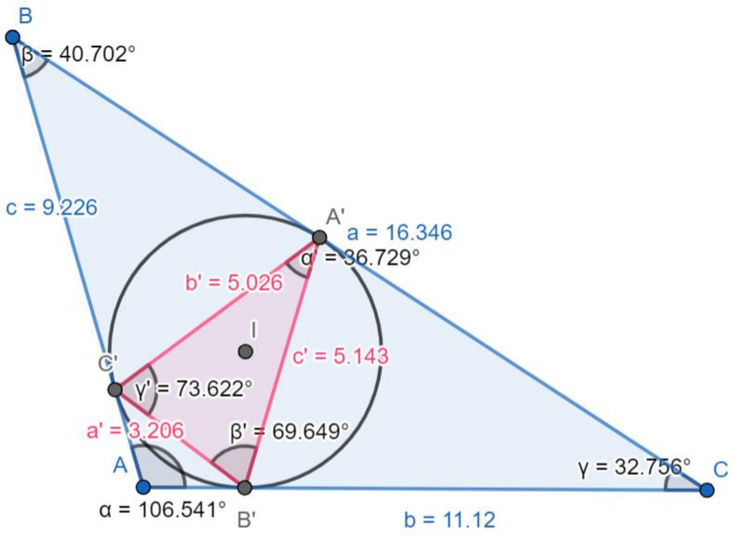



**Figure 4:** Picture of an obtuse triangle in GeoGebra for a geometrical simulation. [4]

In this triangle, $\alpha' = 36.729°$, and $\frac{\beta+\gamma}{2} = \frac{40.702°+32.756°}{2} = 36.729°$. $\beta' = 69.649°$, and $\frac{\alpha+\gamma}{2} = \frac{106.541°+32.756°}{2} = 69.6485°$. Since GeoGebra rounds to the nearest thousandth, it says that $\beta'$=69.649 ° instead of $\beta = 69.6485°$. $\gamma' = 73.622°$, $\frac{\alpha+\beta}{2} = \frac{106.541°+40.702°}{2} = 73.6215°$. Again, due to rounding by GeoGebra, there is a slight error. [4]

## VI. CONCLUSION

As a summary, I like to state that the theorem discovered and expressed in this paper is right and can be used for any types of triangles. The way of expressing this theorem is unique and was not found in the literature so far at all. Some altered form of expression can be seen as question and answer as mathematical problems. [1-2]. As a conclusion, this paper shows a unique way of representing and expressing a mathematical theorem developed by a high school author. This theorem is explained with its accuracy, and it can be used to explain or solve mathematical problems that includes any type of triangles such as acute, obtuse, right angle or equilateral triangles and inscribe triangles in geometry.

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Some of our recent publications can be seen in these references that show some experimental work done by the same high school student during COVID 19 time at home that gives a good insight how student can be thoughtful to do some experimental work at home without going outside based on COVID 19 issues and maintain research work. [6-7]

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