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Stochastic Modeling for Using A Cross Cultural Analysis of Salivary Cortisol in Breast Cancer Survivors

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ABSTRACT

Examined whether Chinese and white women with and without a history of breast cancer exhibit differences in physiological and psychological stress profiles. Diurnal and reactive salivary cortisol profiles and physiological stress patterns of 41 breast cancer survivors and 58 healthy women were assessed. Breast cancer survivor displayed a blunted acute cortisol response but there was no main effect of ethnocultural membership using the sharp large deviation for the energy of α - Brownian motion.

Keywords*: Acute cortisol, breast cancer, cultural orientation, diurnal cortisol, physiological stress, reactivity, salivary cortisol.*

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I. INTRODUCTION

Stress is a Universal experience, the product of an interaction between an individual and the environment, namely the individual's appraisal of the environment and coping capability[8]. Thus, stress can be viewed as positive, negative or neural and beneficial toward promoting one's survival[9]. In response to a stressor, the hypothalamic-pituitary-adrenal (HPA) axis is activated to prepare the organism to deal with environmental challenges[10], which initiates the production of glucocorticoids such as cortisol. Cortisol is a hormone that regulates bodily functions, such as homeostasis and immune response, and is commonly measured via plasma, serum, urine or saliva[6].

Cortisol has a relatively stable diurnal pattern in healthy individuals . It reaches its peak approximately 30 to 60 min after waking, thereafter steadily declining throughout the day. Cortisol secretion and basal levels can be affected by numerous factors, such as hormonal status [5], age and sex, circadian rhythm and illnesses such as cancer.

We consider the following α-Brownian bridge:

$$dX_t = -\frac{\alpha}{T-t} X_t dt + dW_t, X_0 = 0$$

Where W is a standard Brownian motion, $t \in [0,T)$, $T \in (0,\infty)$, and the constant $\alpha > 1/2$. Let P_α denote the probability distribution of the solution $\{X_t, t \in [0,T)\}$. The α-Brownian bridge is first used to study the arbitrage profit associated with a given future contract in the absence of transaction costs by [4].

A-Brownian bridge is a time inhomogeneous diffusion process which has been studied by [1,11]. They studied the central limit theorem and the large deviation for parameter estimators and hypothesis testing problem of α-Brownian bridge, while the large deviation is not so helpful in some statistics problems since it only gives a logarithmic equivalent for the deviation probability, overcame this difficulty by the sharp large deviation principle for the empirical mean. Recently, the sharp large deviation principle is widely used in the study of Gaussian quadratic forms in [2], [3].

we consider, the sharp large deviation principle (SLDP) of energy S_t, where

$$S_t = \int_0^t \frac{X_s^2}{(s-T)^2} ds \tag{1}$$

Our main results are the following.

Theorem: 1

Let $\{X_t, t \in [0, t)\}$ be the process given by the stochastic differential equation. Then $\{S_t / \lambda_t, t \in [0, t)\}$ satisfies the large deviation principle with speed λ_t and good rate function (.) defined by the following:

$$I(x) = \begin{cases} \frac{1}{8x}((2\alpha - 1)x - 1)^2, if x > 0 \\ +\infty, if x \leq 0 \end{cases} \quad (2)$$

Where $\lambda_t = \log(T/(T-t))$.

Theorem: 2

$\{S_t / \lambda_t, t \in [0, T)\}$ satisfies SLDP; that is, for any c>1/(2α-1), there exists a sequence $b_{c,k}$ such that, for any p>0, when t approaches T enough,

$$p(S_t \geq c\lambda_t) = \frac{\exp\{-I(c)\lambda_t + H(a_c)\}}{\sqrt{2\pi a_c}\beta_t} \times \left(1 + \sum_{k=1}^{p} \frac{b_c, k}{\lambda_t} + O(\frac{1}{\lambda_t^{p+1}})\right) \quad (3)$$

Where $\sigma_c^2 = 4c^2$, $\beta_t = \sigma_c \sqrt{\lambda_t}$,

$$H(a_c) = -\frac{1}{2}\log\left(\frac{1-(1-2\alpha)c}{2}\right) \quad (4)$$

The coefficients $b_{c,k}$ be explicitly computed as functions of the derivatives of L and H(defined in Lemma 3) at point a_c. For example, $b_{c,1}$ is given by

$$b_{c,1} = \frac{1}{\sigma_c^2}\left(-\frac{h_2}{2} - \frac{h_1^2}{2} + \frac{l_4}{8\sigma_c^2} + \frac{l_3 h_1}{2\sigma_c^2} - \frac{5l_3^2}{24\sigma_c^4} + \frac{h_1}{a_c} - \frac{l_3}{2a_c\sigma_c^2} - \frac{1}{a_c^2}\right) \quad (5)$$

With $l_k = L^{(k)}(a_c)$, and $h_k = H^{(k)}(a_c)$.

2. Large Deviation for Energy:

Given α>1/2, we first consider the following logarithmic moment generating function of S_t; that is,

$$L_t(u) := \log \mathrm{E}_\alpha \exp\left\{u\int_0^t \frac{X_s^t}{(s-T)^2} ds\right\}, \quad \forall \lambda \in R. \quad (6)$$

and let $\mathcal{D} L_t := \{u \in R, L_t(u) < +\infty\}$ (7)

be the effective Domaine of L_t, by the same method as in [11], we have the following lemma.

Lemma:3

Let $\mathcal{D}_L$ be the effective domain of the limit L of L_t; then for all $u \in \mathcal{D}_L$,

one has

$$\frac{L_t(u)}{\lambda_t} = L(u) + \frac{H(u)}{\lambda_t} + \frac{R(u)}{\lambda_t} \tag{8}$$

With

$$L(u) = -\frac{1 - 2\alpha - \varphi(u)}{4}$$

$$H(\lambda) = -\frac{1}{2}\log\left\{\frac{1}{2}(1 + h(u))\right\}, \tag{9}$$

$$R(u) = -\frac{1}{2}\log\left\{1 + \frac{1 - h(u)}{1 + h(u)}\exp\{2\varphi(u)\lambda_t\}\right\},$$

Where $\phi(u) = -\sqrt{(1-2\alpha)^2 - 8u}$ and $h(u) = (1-2\alpha)/\varphi(u)$.

Furthermore, the remainder R(u) satisfies

$$R(u) = O_{t\to T}(\exp\{2\varphi(u)\lambda_t\}) \tag{10}$$

Proof:

BY Itô's formula and Girsanov's formula [7], for all

$u \in \mathcal{D}_L$ and $t \in [0, T)$,

$$\log\frac{dp_\alpha}{dp_\beta}|_{[0,t]}$$

$$= (\alpha - \beta)\int_0^t \frac{X_s}{s - T}dX_s - \frac{\alpha^2 - \beta^2}{2}\int_0^t \frac{X_s^2}{(s-T)^2}ds, \tag{11}$$

$$\int_0^t \frac{X_s}{s - T}dX_s = \frac{1}{2}\left(\frac{X_t^2}{(t-T)} + \int_0^t \frac{X_s^2}{(s-T)^2}ds - \log\left(1 - \frac{t}{T}\right)\right).$$

Proof:

BY Itô's formula and Girsanov's formula [7], for all

$u \in \mathcal{D}_L$ and $t \in [0,T)$,

$$\log \frac{dp_\alpha}{dp_\beta}\Big|_{[0,t]}$$

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$$\int_0^t \frac{X_s}{s-T}\,dX_s = \frac{1}{2}\left(\frac{X_t^2}{(t-T)} + \int_0^t \frac{X_s^2}{(s-T)^2}ds - \log\left(1-\frac{t}{T}\right)\right).$$

Therefore,

$$L_t(u) = \log \mathrm{E}_\beta\left(\exp\left\{u\int_0^t \frac{X_s^2}{(s-T)^2}ds\right\}\frac{dp_\alpha}{dp_\beta}\Big|_{[0,t]}\right)$$

$$= \log E_\beta \exp\left\{\frac{\alpha-\beta}{2(t-T)}X_t^2 - \frac{\alpha-\beta}{2}\log\left(1-\frac{t}{T}\right) + \frac{1}{2}\left(\beta^2-\alpha^2+\alpha-\beta+2u\right)\times\int_0^t \frac{X_s^2}{(s-T)^2}ds\right\}$$

(12)

If $4u \le (1-2\alpha)^2$, we can choose β such that $(\beta-1/2)^2-(\alpha-1/2)^2+2u=0$. Then

$$\begin{aligned} L_t(u) = &-\frac{1-2\alpha-\varphi(\lambda)}{4}\lambda_t \\ &-\frac{1}{2}\log\left\{\frac{1}{2}(1+h(u))\right\} \\ &-\frac{1}{2}\log\left\{1+\frac{1-h(u)}{1+h(u)}\exp\{2\varphi(u)\lambda_t\}\right\}, \end{aligned} \quad (13)$$

Where $\varphi(u) = -\sqrt{(1-2\alpha)^2-8u}$, and $h(u) = (1-2\alpha)/\varphi(u)$.

Therefore,

$$\begin{aligned} \frac{L_t(u)}{\lambda_t} = &\frac{1-2\alpha-\varphi(u)}{4} \\ &-\frac{1}{2\lambda_t}\log\left\{\frac{1}{2}(1+h(u))\right\} \\ &-\frac{1}{2\lambda_t}\log\left\{1+\frac{1-h(u)}{1+h(u)}\exp\{2\varphi(u)\lambda_t\}\right\} \\ &= L(u)+\frac{H(u)}{\lambda_t}+\frac{R(u)}{\lambda_t}. \end{aligned} \quad (14)$$

Proof of the theorem 1. From Lemma 3, we have

$$L(u) = \lim_{t \to T} \frac{L_t(u)}{\lambda_t} = \frac{1 - 2\alpha - \varphi(u)}{4} \tag{15}$$

and L(·) is steep; by the Gartner-Ellis theorem , S_t / λ_t satisfies the large deviation principle with speed λ_t and good rate function I(·) defined by the following:

$$I(x) = \begin{cases} \frac{1}{8x}((2\alpha - 1)x - 1)^2, if x > 0; \\ +\infty, if + x \le 0 \end{cases} \tag{16}$$

3. Sharp Large Deviation for Energy

For c> 1/(2 α − 1), let

$$a_c = \frac{(1-2\alpha)^2 c^2 - 1}{8c^2}, \quad \sigma_c^{\ 2} = L''(a_c) = 4c^3, \tag{17}$$

$$H(a_c) = -\frac{1}{2}\log(1-(1-2\alpha)c).$$

Then

P($S_t \ge c\lambda_t$)

$$= \int_{S_t \ge c\lambda_t} \exp\{L(a_c) - ca_c\lambda_t + ca_c\lambda_t - a_c S_t\} dQ_t$$

$$= \exp\{L(a_c) - ca_c\lambda_t\} E_Q \exp\{-a_c\beta_t U_t I_{\{U_t \ge 0\}}\} = A_t B_t, \tag{18}$$

Where E $_Q$ is the expectation after the change of measure

$$\frac{dQ_t}{dP} = \exp\{a_c S_t - L_t(a_c)\},$$

$$U_t = \frac{S_t - c\lambda_t}{\beta_t}, \quad \beta_t = \sigma_c\sqrt{\lambda_t}. \tag{19}$$

By lemma 3, we have the following expression of A $_t$.

Lemma 5.

For all c>1/(2α-1), when t approaches T enough,

$$A_t = \exp\{-I(c)\lambda_t + H(a_c)\}(1+O((T-t)^c)). \qquad (20)$$

For B_t, one gets the following.

Lemma 6.

For all c> 1/(2 α − 1), the distribution of U_t under Q_t converges to N(0, 1) distribution. Furthermore, there exists a sequence ψ_k such that, for any p>0 when t approaches T enough,

$$B_t = \frac{1}{a_c\sigma_c\sqrt{2\pi\lambda_t}}\left(1+\sum_{k=1}^{p}\frac{\psi_k}{\lambda_t^k}+O\left(\lambda_t^{-(p+1)}\right)\right). \qquad (21)$$

Proof of Theorem 2. The theorem follows from Lemma 5 and Lemma 6.

It only remains to prove Lemma 6. Let $\phi_t(.)$ be the characteristic function of U_t under Q_t; then we have the following.

Lemma 7.

When t approaches T, ϕ_t belong to $L^2(R)$ and, for all $u \in R$,

$$\phi_t(u) = \exp\left\{-\frac{iu\sqrt{\lambda_t c}}{\sigma_c}\right\}\times\exp\left\{\left(L_t\left(a_c+\frac{iu}{\beta_t}\right)-L_t(a_c)\right)\right\}. \qquad (22)$$

Moreover,

$$B_t = E_Q\exp\{-a_c\beta_t U_t I_{\{U_t\geq 0\}}\} = C_t + D_t, \qquad (23)$$

With

$$C_t = \frac{1}{2\pi a_c\beta_t}\int_{|u|\leq s_t}\left(1+\frac{iu}{a_c\beta_t}\right)^{-1}\phi_t(u)du$$

$$D_t = \frac{1}{2\pi a_c\beta_t}\int_{|u|>s_t}\left(1+\frac{iu}{a_c\beta_t}\right)^{-1}\phi_t(u)du \qquad (24)$$

$$|D_t| = O\left(\exp\left\{-D\lambda_t^{1/3}\right\}\right),$$

Where

$$s_t = s\left(\log\left(\frac{T}{T-t}\right)\right)^{1/6}, \qquad (25)$$

For some positive constant s, and D is some positive constant.

Proof. For any $u \in R$

$$\phi_t(u) = E(\exp\{iuU_t\}\exp\{a_c S_t - L_t(a_c)\})$$

$$= \exp\left\{-\frac{iu\sqrt{\lambda_t c}}{\sigma_c}\right\}$$

$$\times \exp\left\{\left(L_t\left(a_c + \frac{iu}{\beta_t}\right) - L_t(a_c)\right)\right\} \quad (26)$$

By the same method as in the proof of Lemma 2.2 in [2], there exist two positive constants τ and k such that

$$|\phi_t(u)|^2 \le \left(1 + \frac{\tau u^2}{\lambda_t}\right)^{-(k/2)\lambda_t}, \quad (27)$$

therefore $\phi_t(.)$ belongs to $L^2(R)$, and by Parseval's formula, for some positive constant s, let

$$s_t = s\left(\log\left(\frac{T}{T-t}\right)\right)^{1/6} \quad (28)$$

We get

$$B_t = \frac{1}{2\pi a_c \beta_t} \int_{|u| \le s_t} \left(1 + \frac{iu}{a_c \beta_t}\right)^{-1} \phi_t(u)du + \frac{1}{2\pi a_c \beta_t} \times \int_{|u| > s_t} \left(1 + \frac{iu}{a_c \beta_t}\right)^{-1} \phi_t(u)du \quad (29)$$

$$=: C_t + D_t \quad (30)$$

$$|D_t| = O(\exp\{-D\lambda_t^{1/3}\}) \quad (31)$$

Where D is some positive constant.

Proof of Lemma 6. By Lemma 3, we have

$$\frac{L_t^{(k)}(a_c)}{\lambda_t} = L^{(k)}(a_c) + \frac{H^{(k)}(a_c)}{\lambda_t} + \frac{O(\lambda_t^k (T-t)^{-2c})}{\lambda_t}. \quad (32)$$

Noting that $L'(a_c)=0, L''(a_c)=\sigma_c^2$ and

$$\frac{L''(a_c)}{2}\left(\frac{iu}{\beta_t}\right)^2\lambda_t=-\frac{u^2}{2}, \tag{33}$$

For any p > 0, by Taylor expansion, we obtain

$$\begin{aligned}\log\phi_t(u)=-\frac{u^2}{2}+\lambda_t\sum_{k=3}^{2p+3}\left(\frac{iu}{\beta_t}\right)^k\frac{L^{(k)}(a_c)}{k!}\\ +\sum_{k=1}^{2p+1}\left(\frac{iu}{\beta_t}\right)^k\frac{H^{(k)}(a_c)}{k!}\\ +O\left(\frac{\max\left(1,|u|^{2p+4}\right)}{\lambda_t^{p+1}}\right);\end{aligned} \tag{34}$$

therefore, there exist integers q(p), r(p) and a sequence $\phi_{k,l}$ independent of p; when t approaches T, we get

$$\phi_t(u)=\exp\left\{-\frac{u^2}{2}\right\}\left(1+\frac{1}{\sqrt{\lambda_t}}\sum_{k=0}^{2p}\sum_{l=k+1}^{q(p)}\frac{\varphi_{k,l}u^l}{\lambda_t^{k/2}}+O\left(\frac{\max\left(1,|u|^{r(p)}\right)}{\lambda_t^{p+1}}\right)\right) \tag{35}$$

where O is uniform as soon as $|u|\le S_t$.

Finally, we get the proof of Lemma 6 by Lemma 7 together with standard calculations on the N(0, 1) distribution.

4 Example:

Participants were deemed non-compliant if they ingested alcohol within 24h of saliva collection, or if they ingested the large meal, exercised or brushed their teeth within 1h of saliva collection. Furthermore, Participants were marked as non-compliant if samples were not collected within 30min of the collection time. Specifically mean cortisol of concentration (fig 1) compared with collection time points like Chinese control participants, white control participants, Chinese breast cancer survivor, White breast cancer survivor.

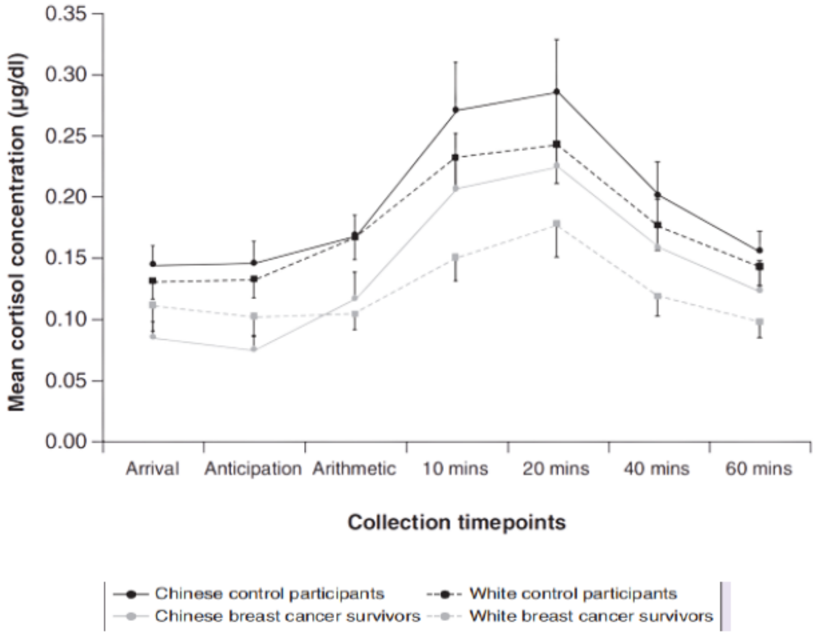

Fig 1

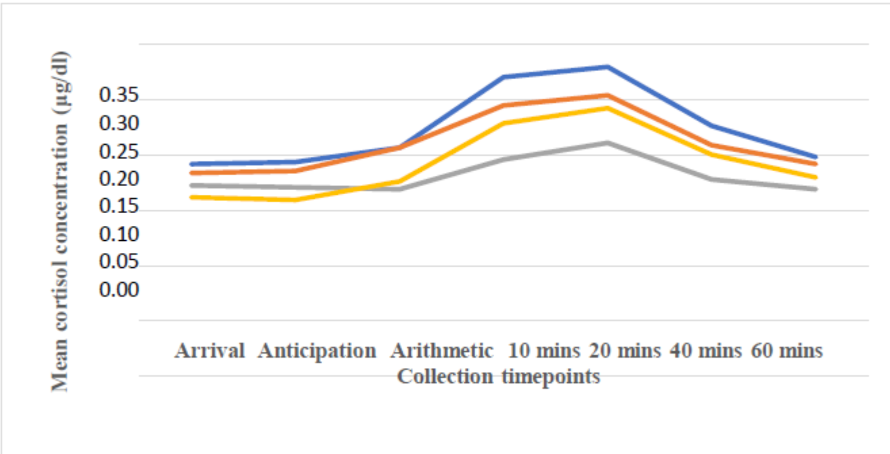

Fig 2

IV. Conclusion

The present study investigated whether chinese breast cancer survivor exhibit different diurnal and acute cortisol patterns in comparision to that of healthy Chinese women with previous finding, diurnal profiles did not differ between groups. But atypical acute cortisol patterns were found, suggesting changes in the HPA stress mechanism of breast cancer survivors, possibly due to the chronic activation of the HPA axis throughout the disease trajectory. All groups indicate similar patterns of subjective stress appraisal during the TSST protocol. They showed increasingly higher levels of psychological stress levels during the Anticipation and Arithmetic phase of the protocol and a gradual disease in the psychologiacal stress levels during the recovery period (10 to 60 min after) with some minor difference across the groups. Using sharp large deviation for the energy of α-Brownian bridge. Finally, from fig 2, we conclude that the results coincide with the mathematical and medical report.

References

[1]. Barczy.M and Pap.G, “Asymptotic behavior of maximum like-lihood estimator for time inhomogeneous diffusion processes,” Journal of Statistical Planning and Inference, vol. 140, no. 6, pp. 1576–1593, 2010.
[2]. Bercu.B and Rouault.A, “Sharp large deviations for the Ornstein-Uhlenbeck process,” Rossiˇıskaya Akademiya Nauk. Teoriya Veroyatnosteˇı i ee Primeneniya, vol. 46, no. 1, pp. 74–93, 2001.
[3]. Bercu.B, Gamboa. F, and Lavielle. M, “Sharp large deviations for Gaussian quadratic forms with applications,” European Series in Applied and Industrial Mathematics. Probability and Statistics, vol. 4, pp. 1–24, 2000.
[4]. Brennan.M. J and Schwartz.E. S, “Arbitrage in stock index futures,” The Journal of Business, vol. 63, pp. 7–31, 1990.
[5]. Huang Y, Zhou R, Wu M, Wang Q, Zhao Y. Premenstrual syndrome is associated with blunted cortisol reactivity to the TSST. Stress 18(2), 160–168 (2015).
[6]. Hellhammer DH, Wust S, Kudielka BM. Salivary cortisol as a biomarker in stress research. Psychoneuroendocrinology 34(2), 163–171 (2009)
[7]. Jacod. J and Shiryaev A. N, Limit Theorems for Stochastic Processes, Springer Press, Berlin, Germany, 1987.
[8]. Lazarus RS, Folkman S. Transactional theory and research on emotions and coping. Eur. J. Personal. 1(3), 141–169 (1987).
[9]. Nesse RM, Bhatnagar S, Ellis B. Evolutionary Origins and Functions of the Stress Response System. In: Stress: Concepts, Cognition, Emotion, and Behavior. Fink G. (Ed.), Academic Press, CA, USA, 95–101 (2016)
[10]. Thiel KJ, Dretsch MN. The basics of the stress response: A historical context and introduction. Conrad CD. (Ed.), Wiley–Blackwell, Oxford, UK, 1–28 (2011).
[11]. Zhao.S and Liu.Q, “Large deviations for parameter estimators of α-Brownian bridge,” Journal of Statistical Planning and Inference, vol. 142, no. 3, pp. 695–707, 2012.