

# Modeling and Simulation of Dynamic Systems

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## Abstract:

*Models, particularly mathematical models, are an extremely useful tool in automation, as well as in the analysis and design of control(led) systems. There are strong links not only between the system, the modelling goal, and the modeller, but also between the model and the resulting industrial problem solution. The modelling process itself is examined from a broad perspective, focusing on the aspects that are critical for automation and control. Furthermore, simulation and simulation tools are discussed from the same perspective, including not only topics like model complexity, validation, and verification, but also numerical aspects. This research presents dynamic system modelling and simulation. Modelling and simulation have advanced dramatically in the last few decades. It is now on the desks of all engineers and scientists who require it. The primary emphasis is on thermal dynamic system modelling and simulation. A case study of batch processing in the chemical industry is presented to demonstrate the concepts of modelling, analytical solution of Ordinary Differential Equations (ODEs), and thermal dynamic system simulation.*

**Keywords:** *Model, Modeling, Simulation, Dynamic System, Modeling and Simulation, SIMULINK, Signal generator.*

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## I. Introduction:

Because of the universality of the concepts underlying the terms, both terms, 'model' and 'simulation', are used not only within the control community, but also by engineers of various disciplines and in everyday speech. Both concepts are closely related to what is now referred to as a "system" or "process." Unfortunately, the triple 'system-model-simulation' is not well defined, and thus scientists or engineers using one or both of these terms in a discussion may use them with completely different meanings - a situation that is said to be quite common in philosophical discussions but is usually unknown among scientists or engineers. As a result, the various meanings and applications will be exploited from a control engineering standpoint in the sequel, along with - some - of the answers to 'WHY' has modelling and simulation become so important in control engineering. [1]

There are numerous types of systems and, as a result, numerous ways to model a system. The type of model to be used will depend not only on the system being investigated or designed, but also on the modeler's background and preferences regarding the approach to be taken and tools to be used in order to successfully achieve the given goal. This is easily seen by scanning the table of contents of this *Encyclopedia's* chapter on *Control Systems, Robotics, and Automation*. Modelling in general, and particularly modelling of systems for control or automation purposes, is based primarily on knowledge of the system properties that are relevant to the specific task. Proper identification of important system properties indicates which model classes and simulation tools will be useful. It is well understood that a system with certain properties can be modelled in a variety of ways, and that its mathematical model is not uniquely defined, but can be of various types, such as differential equations or describing functions. However, understanding the various classification concepts for systems and models is beneficial in selecting the right model and tools for analysis and simulation. As a result, the most important classification aspects will be covered briefly. This also provides some insight into the use and abuse of certain terms that are commonly used not only by control engineers but also in a broader context.

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However, demands have risen significantly, and in most cases, improving existing - industrial, engineering, etc. - control systems or designing new ones is only possible when appropriate mathematical models are used. A mathematical model is an abstract, simplified mathematical construct created for a specific purpose that is related to a part of reality. It is adequate if it is adequate for the modeler's intended goal. It is critical to understand that *'a mathematical model is a symbolic representation made up of mathematical symbols. These symbols have precise mathematical meanings, and their manipulation is governed by logic and mathematics rules. By relating its symbols to a system characterization, a mathematical formulation becomes a model'*. [5]

## II. Simulation

There are few words that have changed their meaning and are used in so many different contexts as "simulation." Obviously, its roots are the Latin '*similis*,' and '*to simulate*' means (according to Webster's Collegiate Dictionary) "to feign, to attain the essence of, without the reality." Simulation has long been associated with two areas: control engineering and the investigation of server/queuing systems. For a long time, only two types of models were used—differential equations in one area and models based on the use of probability distributions in the other. Meanwhile, systems, and thus models, vary greatly; events occur in many continuous-time systems as either state-events or (random) stochastic events. Furthermore, continuous-time subsystems exist in discrete-event systems. As a result, boundaries have become weak, and most simulations performed today are 'hybrid' in some way.[6-7]

Furthermore, simulation is closely related to the concept of a model in this context. This is also true for two other statements. The first was delivered by McLeod at the launch of the journal '*Simulation*': '*... as Editor of the Journal, I proclaim....*

The act of representing some aspects of the real world by numbers or symbols that can be easily manipulated to facilitate their study is referred to as simulation.' The second statement comes from VDI-Richtlinie 3633, which states, 'Simulation is the imitation of a dynamical process in a model in order to derive knowledge that can be transferred to reality.' (From the original German text: '*Simulation is the representation of a dynamic process in a model in order to arrive at conclusions that are transferable to reality*!') Again, simulation serves a purpose, which (as stated by Mezencev) can be '*to draw conclusions about process properties*' by '*driving a model of a system with suitable inputs and observing the corresponding outputs*'. This is similar to Cellier's definition: '*An experiment is the process of extracting data from a system by exerting it through its inputs*' as background. This applies to many simulation applications, such as analysis of an existing system (such as the plant) or a proposed system (e.g. a newly designed controller) or design of a new system using a guess-and-test method or prediction of what will happen under certain - normal or extreme - conditions or information on what to do to allow the system to run smoothly throughout its entire life cycle.

The final question is more difficult to answer because it requires the inclusion of the design process. This has resulted in a broader definition of the term experiment: '*An experiment is the application of a method to a model*.' As a result, experiments include not only time histories of inputs and corresponding outputs, but also methods such as (optimal) controller design, stability analysis, linearization, statistical analysis of specific events, and so on.[8]

### ➤ Classification of Systems and Models

As previously stated, a system is a collection of one or more related objects, which are typically physical entities with specific characteristics or attributes and can interact but are not required to do so. Furthermore, entities and the system may evolve over time. Aspects like these, as well as the various types of changes, are characteristics that lead to system and model classifications. Unfortunately, these characteristics are used differently in different scientific and engineering fields. The property 'discrete', in particular, is frequently used without specifying which variable it applies to. It can denote discrete-time, discrete-event, discrete state, or output variables depending on the area and author. As a result, an overview of some of the most important characteristic

features or, more precisely, characteristic pairs used for classification is provided, along with a brief discussion of their various applications.

### **Properties of Systems and Models**

The first set of property pairs is related to both the system and/or the model (see General Models of Dynamic Systems):

*dynamic - static:* When the relationships between all relevant system elements do not change over time, a system or model is said to be static. As shown below, there are only a few types of possible and interesting models for static systems. Independent and dependent variables are used in dynamic formulation. At least one independent variable is usually related to time, either as a sequence of instants (discrete-time, discrete-event) or as values in an interval (continuous-time). Depending on the system's other properties, a variety of models emerge. At the end of this (incomplete) list of classification properties, there is a brief discussion. [9-11]

*deterministic - stochastic:* Many real-world problems involve demands (events) whose occurrence and lengths can be specified only probabilistically. Computer systems, communication networks, inspection, maintenance, and repair operations, industrial production processes, and inventory systems are examples of such systems. The system is a stochastic system because it is random. Among them, Markov processes are particularly interesting and well understood. They can be either discrete or continuous in terms of state or time. Exact prediction of future states (i.e. computation if all data is provided) is not possible. Furthermore, a process can be stationary, which means that its distribution function remains constant as time passes. Deterministic systems, if precisely modelled, exhibit predictable behaviour; future behaviour can be computed from its current state and known influences on it. However, it should be noted that being deterministic does not always result in '*predictable*' behaviour in everyday life. Deterministic systems and, as a result, models (even very simple ones) may exhibit behaviour in which very small changes in one parameter result in large and unexpected changes in the system's future behaviour. Although the various behaviours are in principle calculable (assuming that all computations are exact), ad hoc prediction of long-term outcomes of small parameter changes is impossible. This is known as chaotic behaviour of variables in technical terms. Examples include the beating of aeroplane wings and the expansion of certain insect populations. It should be noted that some systems, such as weather forecasting, can be modelled in both ways for the same goal (for a detailed discussion of models for stochastic systems, see *Models of Stochastic Systems*).

*lumped parameter - distributed parameter:* Both properties are related to dynamic systems. Its elements may change only with time or may change with time and space, such as beam oscillations or BOD- and DO-values (BOD and DO stand for biological oxygen demand and dissolved oxygen, respectively, which describe water quality and are thus important for its control) along a river. Ordinary differential equations describe lumped parameter systems in continuous time, whereas partial differential equations model distributed parameter systems. The simplest models for the latter in the time-discrete case are known as 2D- or 3D-systems. Infinite-dimensional systems are sometimes used to describe distributed parameter systems (see *Some Basics in Modelling of Mechatronic Systems and Modelling and Simulation of Distributed Systems*). The latter mathematical term, on the other hand, includes systems with (finite or distributed) dead lag, such as differential-difference equations and integro-differential equations. [12]

*stationary (time-invariant) - time varying:* When a system remains invariant under arbitrary time shifts, it is said to be stationary. In the deterministic case, such systems are modelled by differential (difference) equations that do not explicitly depend on time ( $t$ ,  $t_k$  resp. ), whereas in the stochastic case, their distribution function is time invariant.

*continuous-time and discrete-time:* This is not a precise distinction because systems can change at discrete points in time in a deterministic manner or stochastically. As a result, it is preferable to distinguish three properties as follows:

*continuous-time and discrete-time - discrete-event - hybrid:* Systems that change over time frequently include a plant to be controlled. Differential equations and bond graphs, or, for linear systems only, transfer functions and frequency domain descriptions (operator models), are among the most commonly used models. Other systems only change at specific points in time or at very short intervals. As a result, models (such as difference equations) are used to account for this fact. Furthermore, certain operations (e.g., maintenance and repair, grinding, path welding) in industrial production processes appear at random and/or have a random duration. These systems are subjected to a sequence of countable events in which nothing of interest occurs between them, giving rise to the terms discrete-event system or model, respectively. It should be noted that deterministic discrete-events are also known. More specifically, one speaks of state-events, which typically appear in continuous-time systems as a

switching between behaviours, such as a car switching between rolling and sliding when aquaplaning occurs. [13-14]

*continuous - discrete:* This distinction stems from the debate over analogue (= continuous) versus digital (= discrete) simulation. Meanwhile, using these two attributes without further precision is discouraged because it leads to confusion. Both dynamic systems and models have dependent and independent variables that can change continuously or discretely. It should be noted that these characteristics can also be used with a very different meaning, such as in problems arising in continuum physics, where discrete models may refer to models consisting of ordinary differential equations and continuous models refer to partial differential equations. [15]

*linear - nonlinear:* Linear systems have the advantage of obeying the superposition principle. As a result, they are relatively simple to understand and handle (the latter only in the time-invariant case where several simple descriptions are available). However, it is well understood that real-world systems have essentially nonlinear behaviour, and linear models are always approximations that can only be used in a specific neighbourhood of a working point, and so on. *'Our preoccupation with linear time-invariant systems is not a reflection of a belief in a linear time-invariant real world, but rather a reflection of the current state of the art of describing the real world.'* Pindyck wrote in 1972. [16-17]

**Simulation:** Now we use the **SIMULINK** Environment (SimuLink User's Guide) to create a block diagram in **MATLAB** using the modelling equations developed for the thermal system.

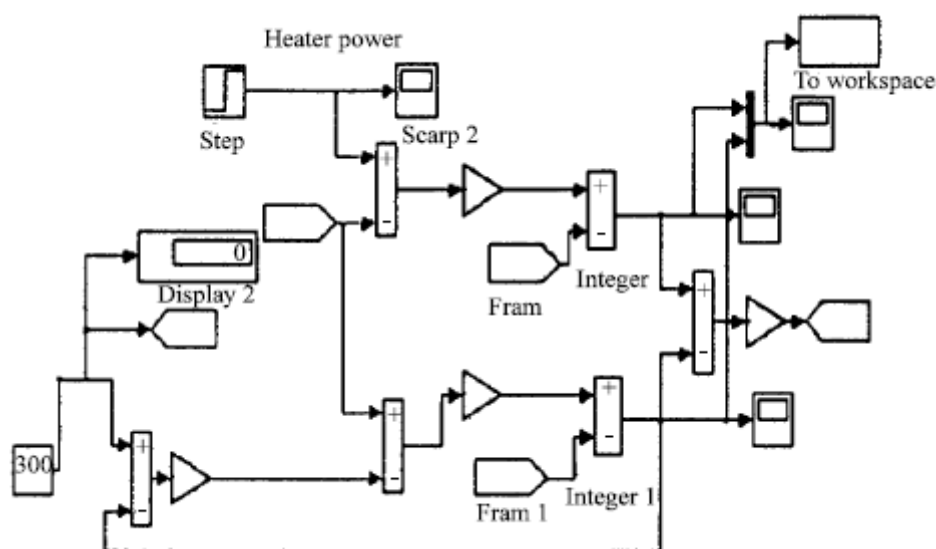


Figure 1: Block diagram developed in SIMULINK

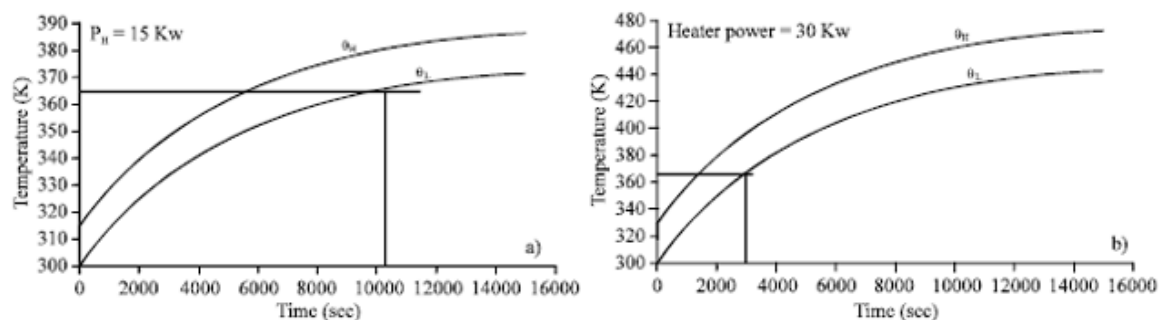


Figure 2: Graphs of (a) and (b) system using step function as input

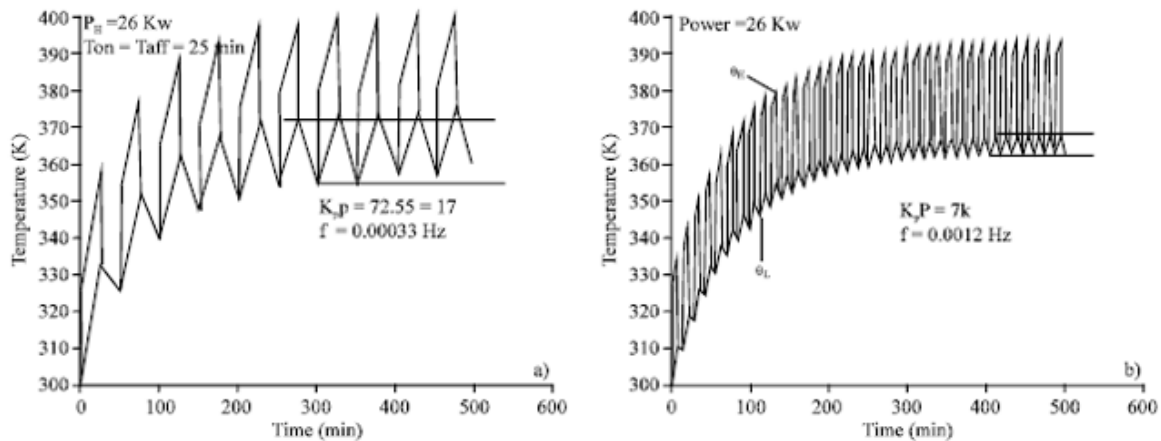


Figure 3: Graphs (a) and (b) of system using signal generator as input

Figure 1 depicts the block diagram produced by implementing the modelling equations. The system's response has been analysed using the three different inputs listed below: [18-20]

**Step function:** We observed changes in the behaviour of the heater temperature and, as a result, liquid temperature using the step function as a constant heating source of 15 and 30 K Joule sec 1, as shown in Figure 2.

The time required to reach the desired temperature is shown in Figure 2.

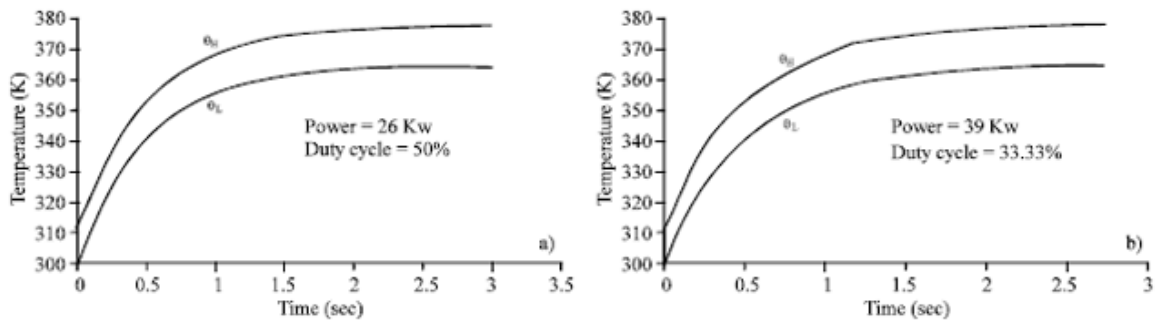


Figure 4: Graphs (a) and (b) of system using pulse generator as input

The rise in liquid temperature is proportional to the rate of heating.

**Signal generator:** In this case, we looked at another scenario that is commonly required by industry in order to maintain the desired temperature. We used a signal generator (square wave) for this purpose. The heater's power is approximately 26 KW. The signal period is 50 minutes. As a result, the frequency is 0.00033 Hz. After implementing the data in the SIMULINK environment, we discovered that the liquid temperature reaches the desired temperature (365K) during the fourth cycle. We can also see that after 300 minutes, the steady state response has been achieved, but the temperature fluctuation from peak to peak is approximately 17 K. We increased the frequency of the signal generator to reduce peak-to-peak temperature fluctuations. When the frequency is increased from 0.00033 to 0.0012 Hz, the graph shows that the fluctuation range is only about 7 K. Figure 3 depicts the graphs of these two frequencies. [21-22]

**Pulse generator:** We used a Pulse Generator as the system's input source. The heater has a power rating of 26 KW and a duty cycle of 50%. Figure 4 depicts the output response of the system. We increased the heater power to 39 KW and then reduced the duty cycle to 33.33% to achieve the same results as in the first case of using a pulse generator.



### III. Conclusion:

The thermal system's response has been observed using the step function, signal generator, and pulse generator. The time required by the step function to reach the desired liquid temperature agrees well with that calculated analytically. Graphs also show that the response of the liquid temperature has a linear relationship with the heater temperature. Peak to peak temperature fluctuations (also known as ripple) in liquid temperature have been reduced by controlling the frequency of the input signal. So, this article provides an overview of dynamic system modelling and simulation, with a focus on thermal systems, as well as a solid understanding of how to model, solve, and simulate dynamic thermal systems.

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