International Journal of Research in Engineering and Science (IJRES)
ISSN (Online): 2320-9364, ISSN (Print): 2320-9356
www.ijres.org Volume 10 Issue 12 ǁ December 2022 ǁ PP. 357-366

Unsteady flow and heat transfer through the vertical channel with fluctuating temperature

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Abstract
An analysis is made on unsteady flow and heat transfer through the vertical channel subject to the time dependent periodic suction when the left plate of the channel fluctuates with time. The velocity field, shear stresses, temperature field and rate of heat transfer has been obtained in a closed form. The variations of non-dimensional parameters such as Reynolds number, Prandtl number, frequency parameter and Grashoff number on velocity and temperature field are shown graphically. The variations of non-dimensional parameters on shear stresses and rate of heat transfer are presented in the form of tables.
Keywords: *Three-dimensional, injection, suction, buoyancy, transpiration cooling, free convection.*

Date of Submission: 06-12-2022 Date of acceptance: 18-12-2022

I. Introduction

Viscous fluid flow through vertical channel has applications in transpiration cooling in reentry vehicles. Combined convective flow as well as free convective hydrodynamic flow in a vertical channel were studied by Aung[1] and Aung and Worku[2]. Wang and Skalak[3] studied three dimensional fluid flow through one side of a long vertical channel for Newtonian fluid. An extension of the problem were studied by Sharma and Chaudhary[4] for viscoelastic fluid, Baris[5] for second grade fluid and Baris [6] for Walter's B' Fluid. Sing et. al [7] studied the free convection flow and heat transfer along a porous vertical wall. Chaudhary and Chand [8] studied the effect of injection on flow through vertical channel embedded in porous medium. Berletta et. al [9] studied the mixed convection flows in a vertical channel.

Guria and Jana[10] the studied unsteady flow past a vertical porous plate subject to a periodic suction. Due to periodic suction the flow becomes three dimensional. Guria and Jana[11] also studied the free convection flow and heat transfer through vertical channel subject to the periodic suction. Guria et. al [12] extended this problem by applying transverse magnetic field. Guria et. al [13] also studied the radiation effect on three dimensional vertical channel flow. Guria[14] extended this problem by considering the flow through porous media.

The main object of this paper is to study the effect of time dependent periodic suction and buoyancy force on unsteady flow through vertical channel when the temperature at the left plate fluctuates with time. Our problem presents a non-trivial extension of Guria and Jana[11] by introducing (a) time dependent periodic suction and (b) temperature at the left plate fluctuates with time.

II. Basic Equations

We consider the unsteady flow of viscous, incompressible fluid through the vertical channel at a distance d apart. Here the $x^\star$- axis is chosen along the direction of the flow[see Fig.1]. The temperature at the plate $y^* = d$ is T_∞ and that at the plate $y^* = 0$ fluctuates with time

$$T^* = T_w + \epsilon(T_w - T_\infty)\cos\left(\frac{\pi z^\star}{d} - ct^\star\right), \qquad (2.1)$$

where $\epsilon(\ll 1)$ is the amplitude of the suction velocity.

The plate $y^\star = d$ is subjected to a uniform injection V_0 and the plate $y^\star = 0$ to a time dependent periodic suction of the form

$$v^\star = -V_0\left[1 + \epsilon\cos\left(\frac{\pi z^*}{d} - ct^*\right)\right], \quad (2.2)$$

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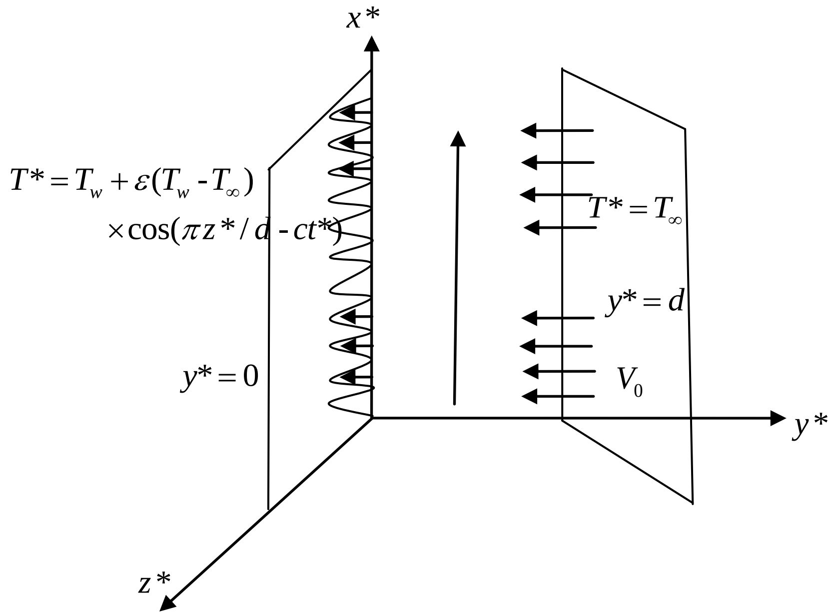

Fig.1: Physical model and Co-ordinates system

Let $u^\star, v^\star, w^\star$ be the velocity components in the directions $x^\star-, y^\star-$ and $z^\star-$ axes respectively. The governing equations are

$$\frac{\partial v^\star}{\partial y^\star}+\frac{\partial w^\star}{\partial z^\star}=0, \quad (2.3)$$

$$\frac{\partial u^\star}{\partial t^\star}+v^\star\frac{\partial u^\star}{\partial y^\star}+w^\star\frac{\partial u^\star}{\partial z^\star}=\nu\left(\frac{\partial^2 u^\star}{\partial y^{\star 2}}+\frac{\partial^2 u^\star}{\partial z^{\star 2}}\right)+g\beta(T^*-T_\infty), \quad (2.4)$$

$$\frac{\partial v^\star}{\partial t^\star}+v^\star\frac{\partial v^\star}{\partial y^\star}+w^\star\frac{\partial v^\star}{\partial z^\star}=-\frac{1}{\rho}\frac{\partial p^\star}{\partial y^\star}+\nu\left(\frac{\partial^2 v^\star}{\partial y^{\star 2}}+\frac{\partial^2 v^\star}{\partial z^{\star 2}}\right), \quad (2.5)$$

$$\frac{\partial w^\star}{\partial t^\star}+v^\star\frac{\partial w^\star}{\partial y^\star}+w^\star\frac{\partial w^\star}{\partial z^\star}=-\frac{1}{\rho}\frac{\partial p^\star}{\partial z^\star}+\nu\left(\frac{\partial^2 w^\star}{\partial y^{\star 2}}+\frac{\partial^2 w^\star}{\partial z^{\star 2}}\right), \quad (2.6)$$

$$\frac{\partial T^\star}{\partial t^\star}+v^\star\frac{\partial T^*}{\partial y^\star}+w^\star\frac{\partial T^*}{\partial z^\star}=\nu\left(\frac{\partial^2 T^*}{\partial y^{\star 2}}+\frac{\partial^2 T^*}{\partial z^{\star 2}}\right), \quad (2.7)$$

where ν is the kinematic coefficient of viscosity, ρ is the density, $p^\star$ is the fluid pressure, g is the acceleration due to gravity, β is the coefficient of thermal expansion.

The boundary conditions are

$$u^\star=0, v^\star=-V_0\left[1+\epsilon\cos\left(\frac{\pi z^\star}{d}-ct^\star\right)\right], w^\star=0,$$

$$T^*=T_w+\epsilon(T_w-T_\infty)\cos\left(\frac{\pi z^\star}{d}-ct^\star\right) \quad \text{at} \quad y^\star=0,$$

$$u^\star=0, v^\star=-V_0, w^\star=0, T^*=T_\infty, p^\star=p_\infty \quad \text{at} \quad y^\star=d. \quad (2.8)$$

Introducing the non dimensional variables

$$y=\frac{y^\star}{d}, z=\frac{z^\star}{d}, p=\frac{p^\star}{\rho V_0^2}, u=\frac{u^\star}{V_0}, v=\frac{v^\star}{V_0}, w=\frac{w^\star}{V_0}, \theta=\frac{(T^*-T_\infty)}{(T_w-T_\infty)}, \quad (2.9)$$

equations (2.3)-(2.7) become

$$\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \quad (2.10)$$

$$\omega\frac{\partial u}{\partial t}+Re\left(v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+ReGr\theta, \quad (2.11)$$

$$\omega\frac{\partial v}{\partial t}+Re\left(v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-Re\frac{\partial p}{\partial y}+\left(\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right), \quad (2.12)$$

$$\omega\frac{\partial w}{\partial t}+Re\left(v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=-Re\frac{\partial p}{\partial z}+\left(\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right), \quad (2.13)$$

$$Pr\omega\frac{\partial \theta}{\partial t}+RePr\left(v\frac{\partial \theta}{\partial y}+w\frac{\partial \theta}{\partial z}\right)=\left(\frac{\partial^2 \theta}{\partial y^2}+\frac{\partial^2 \theta}{\partial z^2}\right), \quad (2.14)$$

where $Re=V_0 d/\nu$, the Reynolds number, $Pr=\nu/\rho$, the Prandtl number and $Gr=dg\beta(T_w-T_\infty)/V_0^2$, the Grashof number, $\omega=cd^2/\nu$, the frequency parameter. Using (2.9), the boundary conditions (2.8) become

$$u=0, v=-[1+\epsilon\cos(\pi z-t)], w=0, \theta=1+\epsilon\cos(\pi z-t), \quad \text{at} \quad y=0,$$

$$u=0, v=-1, w=0, \theta=0, p=\frac{p_\infty}{\rho V_0^2} \quad \text{at} \quad y=1. \quad (2.15)$$

III. Solution of the Problem

To solve the differential equations (2.10)-(2.14), we assume

$$u(y,z,t) = u_0(y) + \epsilon u_1(y,z,t) + \epsilon^2 u_2(y,z,t) + \cdots,$$
$$v(y,z,t) = v_0(y) + \epsilon v_1(y,z,t) + \epsilon^2 v_2(y,z,t) + \cdots,$$
$$w(y,z,t) = w_0(y) + \epsilon w_1(y,z,t) + \epsilon^2 w_2(y,z,t) + \cdots, \qquad (3.1)$$
$$p(y,z,t) = p_0(y) + \epsilon p_1(y,z,t) + \epsilon^2 p_2(y,z,t) + \cdots,$$
$$\theta(y,z,t) = \theta_0(y) + \epsilon \theta_1(y,z,t) + \epsilon^2 \theta_2(y,z,t) + \cdots.$$

On substituting (3.1) in equations (2.10)-(2.14) , we get the term free from ϵ

$$v_0' = 0, \quad (3.2)$$
$$u_0'' - Rev_0 u_0' = -ReGr\theta_0, \qquad (3.3)$$
$$\theta_0'' - RePrv_0\theta_0' = 0. \qquad (3.4)$$

The corresponding boundary conditions become

$$u_0 = 0, v_0 = -1, \theta_0 = 1 \text{ at } y = 0, \text{ and } u_0 = 0, v_0 = -1, \theta_0 = 0 \text{ at } y = 1. \qquad (3.5)$$

The solution of the equations (3.2) to (3.4), subject to the boundary conditions (3.5) are

$$v_0(y) = -1, \qquad (3.6)$$

$$\theta_0(y) = \frac{(e^{-RePr} - e^{-RePry})}{(e^{-RePr} - 1)}, \quad (3.7)$$
$$u_0(y) = A_1 y + A_2(e^{-RePry} - 1) + A_3(e^{-Rey} - 1), \qquad \text{for } Pr \neq 1, \quad (3.8)$$
$$u_0(y) = \frac{-2\mathrm{Gr}\,\mathrm{e}^{-\mathrm{Re}}}{(1-\mathrm{e}^{-\mathrm{Re}})^2}(1 - e^{-Rey}) + \frac{Gr\ \ y}{(1-e^{-Re})}(e^{-Re} + e^{-Rey}), \text{ for } Pr = 1, \quad (3.9)$$

where

$$A_1 = \frac{-Gr\,e^{-RePr}}{(e^{-RePr} - 1)}$$
$$A_2 = \frac{Gr}{RePr\,(Pr-1)(e^{-RePr} - 1)}$$
$$A_3 = \frac{A_1 + A_2(e^{-RePr} - 1)}{(1-e^{-Re})}. \qquad (3.10)$$

The coefficient of ϵ is

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0, \quad (3.11)$$
$$\omega\frac{\partial u_1}{\partial t} + Re\left(v_0\frac{\partial u_1}{\partial y} + v_1\frac{\partial u_0}{\partial y}\right) = \left(\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2}\right) + ReGr\theta_1, \qquad (3.12)$$
$$\omega\frac{\partial v_1}{\partial t} + Rev_0\frac{\partial v_1}{\partial y} = -Re\frac{\partial p_1}{\partial y} + \left(\frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2}\right), \qquad (3.13)$$
$$\omega\frac{\partial w_1}{\partial t} + Rev_0\frac{\partial w_1}{\partial y} = -Re\frac{\partial p_1}{\partial z} + \left(\frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2}\right), \qquad (3.14)$$
$$Pr\omega\frac{\partial \theta_1}{\partial t} + RePr\left(v_0\frac{\partial \theta_1}{\partial y} + v_1\frac{\partial \theta_0}{\partial y}\right) = \left(\frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 \theta_1}{\partial z^2}\right). \quad (3.15)$$

The boundary conditions become

$$u_1 = 0, v_1 = -\cos(\pi z - t), w_1 = 0, \theta_1 = \cos(\pi z - t) \quad \text{at } y = 0,$$
$$u_1 = 0, \ v_1 = 0, \quad w_1 = 0, \quad \theta_1 = 0 \quad \text{at } y = 1. \quad (3.16)$$

We assume

$$u_1(y,z,t) = u_{11}(y)e^{i(\pi z - t)},$$
$$v_1(y,z,t) = v_{11}(y)e^{i(\pi z - t)},$$
$$w_1(y,z,t) = \frac{\mathrm{i}}{\pi}v_{11}'(y)e^{i(\pi z - t)}, \qquad (3.17)$$
$$p_1(y,z,t) = p_{11}(y)e^{i(\pi z - t)},$$
$$\theta_1(y,z,t) = \theta_{11}(y)e^{i(\pi z - t)}.$$

Substituting (3.17) in (3.12)-(3.15), we obtain

$$v_{11}'' + Rev_{11}' - (\pi^2 - i\omega)v_{11} = Rep_{11}', \qquad (3.18)$$
$$v_{11}''' + Rev_{11}'' - (\pi^2 - i\omega)v_{11}' = Re\pi^2 p_{11}, \quad (3.19)$$
$$\theta_{11}'' + RePr\theta_{11}' - (\pi^2 - iPr\omega)\theta_{11} = RePrv_{11}\theta_0', \quad (3.20)$$
$$u_{11}'' + Reu_{11}' - (\pi^2 - i\omega)u_{11} = Rev_{11}u_0' - GrRe\theta_{11}. \qquad (3.21)$$

The corresponding boundary conditions are

$$u_{11} = 0, v_{11} = -1, v_{11}' = 0, \theta_{11} = 1 \quad \text{at } y = 0,$$
$$u_{11} = 0, v_{11} = 0, \ v_{11}' = 0, \ \theta_{11} = 0 \quad \text{at } y = 1. \quad (3.22)$$

The Solutions are

$$v_1(y,z,t) = \left[Ae^{\pi y} + Be^{-\pi y} + Ce^{-\lambda_1 y} + De^{-\lambda_2 y}\right]e^{i(\pi z - t)}, \quad (3.23)$$
$$w_1(y,z,t) = i\left[Ae^{\pi y} - Be^{-\pi y} - \frac{C\lambda_1}{\pi}e^{-\lambda_1 y} - \frac{D\lambda_2}{\pi}e^{-\lambda_2 y}\right]e^{i(\pi z - t)}, \qquad (3.24)$$

$$\theta_1(y,z,t) = \Big[C_1 e^{-\mu_1 y} + C_2 e^{-\mu_2 y} + \frac{AK}{i\omega - \pi Re} e^{(\pi - RePr)y} + \frac{BK}{i\omega + \pi Re} e^{-(\pi + RePr)y} + \frac{CKPr\, e^{-(\lambda_1 + RePr)y}}{\lambda_1 Re(Pr+1) + i\omega(Pr-1)} + \frac{DKPr\, e^{-(\lambda_2 + RePr)y}}{\lambda_2 Re(Pr+1) + i\omega(Pr-1)}\Big] e^{i(\pi z - t)}, \qquad (3.25)$$

$$u_1(y,z,t) = \Big[D_1 e^{-\lambda_1 y} + D_2 e^{-\lambda_2 y} - \frac{ReGr}{(Pr-1)}\left\{\frac{C_1 \quad e^{-\mu_1 y}}{(Re\mu_1 - i\omega)} + \frac{C_2 \quad e^{-\mu_2 y}}{(Re\mu_2 - i\omega)}\right\} + D_3 e^{(\pi - RePr)y} + D_4 e^{-(\pi + RePr)y} + \frac{D_5 e^{-(\lambda_1 + RePr)y}}{RePr(2\lambda_1 + RePr - Re)} + \frac{D_6 e^{-(\lambda_2 + RePr)y}}{RePr(2\lambda_2 + RePr - Re)} + \frac{D_7 e^{\pi y}}{(\pi Re + i\omega)} + \frac{D_8 e^{-\pi y}}{(-\pi Re + i\omega)} + \frac{D_9 y \quad e^{-\lambda_1 y}}{(Re - 2\lambda_1)} + \frac{D_{10} y \quad e^{-\lambda_2 y}}{(Re - 2\lambda_2)} + \frac{D_{11} \quad e^{-(\lambda_1 + Re)y}}{\lambda_1} + \frac{D_{12} e^{-(\lambda_2 + Re)y}}{\lambda_2} + \frac{D_{13} \quad e^{(\pi - Re)y}}{(-\pi Re + i\omega)} + \frac{D_{14} \quad e^{-(\pi + Re)y}}{(\pi Re + i\omega)}\Big] e^{i(\pi z - t)}, \qquad \text{for} \quad Pr \neq 1. \qquad (3.26)$$

$$u_1(y,z,t) = \big[E_1 e^{-\lambda_1 y} + E_2 e^{-\lambda_2 y} + E_3 e^{(\pi - Re)y} + E_4 e^{-(\pi + Re)y} + E_5 e^{-(\lambda_1 + Re)y} + E_6 e^{-(\lambda_2 + Re)y} + E_7 y e^{(\pi - Re)y} + E_8 y e^{-(\pi + Re)y} + E_9 y\, e^{-(\lambda_1 + Re)y} + E_{10} y\, e^{-(\lambda_2 + Re)y}\big] e^{i(\pi z - t)}, \qquad \text{for} \quad Pr = 1 \qquad (3.27)$$

where

$$\lambda_{1,2} = \frac{1}{2}\left\{Re \pm \sqrt{Re^2 + 4(\pi^2 - i\omega)}\right\},$$
$$\mu_{1,2} = \frac{1}{2}\left\{RePr \pm \sqrt{Re^2 Pr^2 + 4(\pi^2 - iPr\omega)}\right\},$$
$$A = -\frac{1}{2\pi}[\pi + C(\pi - \lambda_1) + D(\pi - \lambda_2)],$$
$$B = -\frac{1}{2\pi}[\pi + C(\pi + \lambda_1) + D(\pi + \lambda_2)],$$
$$C = [\pi r_2(e^{\pi} - e^{-\pi}) + r_4(e^{\pi} + e^{-\pi})]/2(r_1 r_4 - r_2 r_3),$$
$$D = -[\pi r_1(e^{\pi} - e^{-\pi}) + r_3(e^{\pi} + e^{-\pi})]/2(r_1 r_4 - r_2 r_3),$$
$$r_1 = e^{-\lambda_1} - \frac{1}{2\pi}[e^{\pi}(\pi - \lambda_1) + e^{-\pi}(\pi + \lambda_1)],$$
$$r_2 = e^{-\lambda_2} - \frac{1}{2\pi}[e^{\pi}(\pi - \lambda_2) + e^{-\pi}(\pi + \lambda_2)],$$
$$r_3 = \lambda_1 e^{-\lambda_1} + \frac{1}{2}[e^{\pi}(\pi - \lambda_1) - e^{-\pi}(\pi + \lambda_1)],$$
$$r_4 = \lambda_2 e^{-\lambda_2} + \frac{1}{2}[e^{\pi}(\pi - \lambda_2) - e^{-\pi}(\pi + \lambda_2)]. \qquad (3.28)$$

We omite the other constants to save space.

IV. Results and discussion

We have plotted the velocity field, temperature field, shear stresses and rate of heat transfer for several values of nondimensional parameters. We have plotted the primary velocity u in Figs.2 and 3 for different values of Grashof number and Reynolds number. It is found that primary velocity u increases with increase in Gr. It is also found that u increases near the left plate and decreases away from the plate with increase in Re. Variations of secondary velocity w for several values of Reynolds number is shown in Fig.4. It is observed that w increases near the left plate and decreases away from the plate with increase in Reynolds number.

The shear stress at the plate $y^* = 0$ due to the primary flow is given by

$$\tau_x^* = \mu\left(\frac{\partial u^*}{\partial y^*}\right)_{y^*=0} = \frac{\mu V_0}{d}\left(\frac{\partial u}{\partial y}\right)_{y=0} \qquad (4.1)$$

In non-dimensional form it can be written as

$$\tau_x = \frac{\tau_x^* d}{\mu V_0} = \left(\frac{\partial u}{\partial y}\right)_{y=0}$$
$$= u_0'(0) + \epsilon u_1'(0).$$
$$= u_0'(0) + \epsilon u_{11}'(0) e^{i(\pi z - t)}.$$
$$= u_0'(0) + \epsilon H_1 \cos(\pi z - t + \phi_1). \quad (4.2)$$

The variations of amplitude and tangent of phase shift of shear stress due to primary flow for different values of ω, Re and Gr are shown in Tables.1 and 2. The amplitude increases with increase in both Re and Gr but it decreases with increase in ω. Tangent of phase shift increases with increase in ω whereas it oscillates with both Re and Gr.

Table.1: Amplitude and tangent of phase shift of the shear stress due to primary flow for $\Pr = 0.71, Gr = 5, \varepsilon = 0.25$

	H_1				$\tan\varphi_1$			
Re \ ω	4	5	6	7	4	5	6	7
2	21.47	18.41	15.47	12.96	.57	.96	1.46	2.25
3	23.48	22.60	21.08	19.33	.30	.56	.84	1.16
4	25.27	25.32	24.71	23.72	.20	.41	.61	.82
5	27.78	27.19	27.72	27.14	.21	.38	.54	.70

Table.2: Amplitude and tangent of phase shift of the shear stress due to primary flow for $\Pr = 0.71, Gr = 5, \varepsilon = 0.25$

	H_1				$\tan\varphi_1$			
Re \ Gr	5	6	7	8	5	6	7	8
2	18.41	22.09	25.78	29.46	.962255	.962256	.962256	.962255
3	22.60	27.12	31.64	36.16	.566386	.566586	.566586	.566586
4	25.32	30.39	35.46	40.52	.413472	.413472	.413473	.413472
5	27.19	33.59	39.19	44.79	.384586	.384586	.384586	.384587

The shear stress due to the secondary flow can be expressed as

$$\tau_z^* = \mu(\frac{\partial w^*}{\partial y^*})_{y^*=0} = \frac{\mu V_0}{d}(\frac{\partial w}{\partial y})_{y=0} \qquad (4.3)$$

In non-dimensional form the shear stress due to secondary flow at the plate $y = 0$ can be written as

$$\begin{aligned} \tau_z &= \frac{\tau_x^* d}{\mu V_0} = (\frac{\partial w}{\partial y})_{y=0} \\ &= w_0'(0) + \epsilon w_1'(0) \\ &= \epsilon w_{11}'(0) e^{i(\pi z - t)} \qquad (4.4) \\ &= \epsilon H_2 \cos(\pi z - t + \phi_2) \end{aligned}$$

Variations of the amplitude and tangent of phase shift of the shear stress due to secondary flow for different values of ω and Re is shown in Table.3. It is found that the amplitude increases with increase in both ω and Re. The tangent of phase shift decreases with increase in ω but increases with increase in Re.

Table.3 Amplitude and tangent of phase shift of the shear stress due to secondary flow for $\Pr = 0.71,\ \varepsilon = 0.25$

	H_2				$\tan\varphi_2$			
Re \ ω	2	3	4	5	2	3	4	5
2	4.86	4.88	4.90	4.92	20.57	13.74	10.34	8.31
3	5.58	5.59	5.61	5.63	25.00	16.70	12.55	10.08

4	6.36	6.37	6.38	6.39	30.66	20.47	15.38	12.33
5	7.19	7.20	7.21	7.22	37.82	25.23	28.95	15.18

The temperature profile θ is plotted for different values of Reynolds number and Prandtl number. It is found that the temperature θ decreases with increase in both Re or Pr .

The heat transfer coefficient from the plate to the fluid may be calculated as

$$q = -k(\frac{\partial T^*}{\partial y^*})_{y^*=0} = -\frac{k(T_w - T_\infty)}{d}(\frac{\partial \theta}{\partial y})_{y=0}. \quad (4.5)$$

In non-dimensional form it can be written as

$$Nu_1 = \frac{qd}{k(T_w - T_\infty)} = -(\frac{\partial \theta}{\partial y})_{y=0} = -\theta_0'(0) - \epsilon\theta_1'(0),$$
$$= -\theta_0'(0) - \epsilon\theta_{11}'(0)e^{i(\pi z - t)}, \quad (4.6)$$
$$= -\theta_0'(0) - \epsilon H_3 \cos(\pi z - t + \phi_3)$$

Table.4: Amplitude and tangent of phase shift of the rate of heat transfer at the plate $y = 0$

	H_3				$\tan\varphi_3$			
ω / Re	2	3	5	7	2	3	5	7
2	3.99	3.59	2.94	2.77	.14	.16	.06	.13
3	5.01	4.73	4.07	3.50	.11	.16	.18	.12
4	6.04	5.84	5.30	4.61	.10	.14	.20	.22
5	7.12	6.97	6.54	5.98	.08	.12	.19	.24

and the heat transfer coefficient at the plate $y = 1$ is given by

$$Nu_2 = \frac{qd}{k(T_w - T_0)} = -(\frac{\partial \theta}{\partial y})_{y=1} = -\theta_0'(1) - \epsilon\theta_1'(1),$$
$$= -\theta_0'(1) - \epsilon\theta_{11}'(1)e^{i(\pi z - t)}, \quad (4.7)$$
$$= -\theta_0'(1) - \epsilon H_4 \cos(\pi z - t + \phi_4)$$

Table.5: Amplitude and tangent of phase shift of the rate of heat transfer at the plate $y = 1$.

	H_4				$\tan\varphi_4$			
ω / Re	2	3	5	7	2	3	5	7
2	2.07	1.90	1.61	1.52	.14	.16	.07	.09
3	2.53	2.41	2.10	1.84	.11	.15	.18	.12
4	3.03	2.93	2.68	2.38	.09	.14	.20	.22
5	3.56	3.49	3.28	3.01	.08	.12	.19	.24

The variations of amplitude and tangent of phase shift of rate of heat transfer at the plate $y = 0$ and $y = 1$ for several values of Re and ω is shown in Tables.4 and 5. It is found that the amplitude decreases with increase in ω but increases with increase in Re at both the plates. The tangent of phase shift oscillates with Re and ω at both the plates.

V. Conclusion

The present paper deals the unsteady flow and heat transfer through the vertical channel subject to the time dependent periodic suction when the left plate fluctuates with time. It is found that the primary velocity increases with increase in Grashoff number. It is also found that increases near the left plate and decreases away from the plate with increase in Reynolds number. The secondary velocity increases near the left plate and decreases away from the plate with increase in Reynolds number. The amplitude of the shear stress due to primary flow increases with increase in both Renolds number and grashoff number but decreases with increase in frequency parameter. The amplitude of the shear stress due to secondary flow increases with increase in both frequency parameter or Renolds number. The temperature profile decreases with increase in both Renolds number or Prandtl number. It is found that the amplitude of the Nusselt number decreases with increase in frequency parameter but increases with increase in Renolds number at both the plates.

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Nomenclature

$A_i, i = 1,..3$	constants;
A, B, C, D,	constants;
C_1, C_2,	constants;
$D_i, i = 1,\cdots 14$	constants;
d,	channel width;
$E_i, i = 1,\cdots 10$,	constants;
g,	gravitational acceleration ;
H_1, H_2	amplitude of the shear stresses ;
H_3, H_4	Amplitude of the Nusselt numbers ;
K,	constant;
K_1, K_2	constants;
Nu_1, Nu_2,	Nusselt number at the left and right plates;
p^*,	pressure;
p,	dimensionless pressure;
Pr,	Prandtl number;
q,	local heat transfer at the plate;

$r_i, i = 1, \cdots 4$,	constants;
Re,	Reynolds number;
T^*,	temperature of the fluid;
T_w,	plate temperature ($y^* = 0$);
u^*, v^*, w^*,	velocity components in x, y, z axes
$u, v, w,$	Dimensionless Velocity components in x, y, z axes respectively;
V_0,	constant suction velocity;
x^*, y^*, z^*,	Cartesian coordinates system;
$x, y, z,$	dimensionless Cartesian coordinate system;
μ	viscosity;
β,	coefficient of thermal expansion;
θ,	non-dimensional temperature;
ν,	kinematic viscosity;
ε,	amplitude of the suction velocity;
ρ,	density of the fluid;
ω,	frequency parameter;
$\tan\varphi_i, i = 1, \cdots 4$	tangents of phase shifts

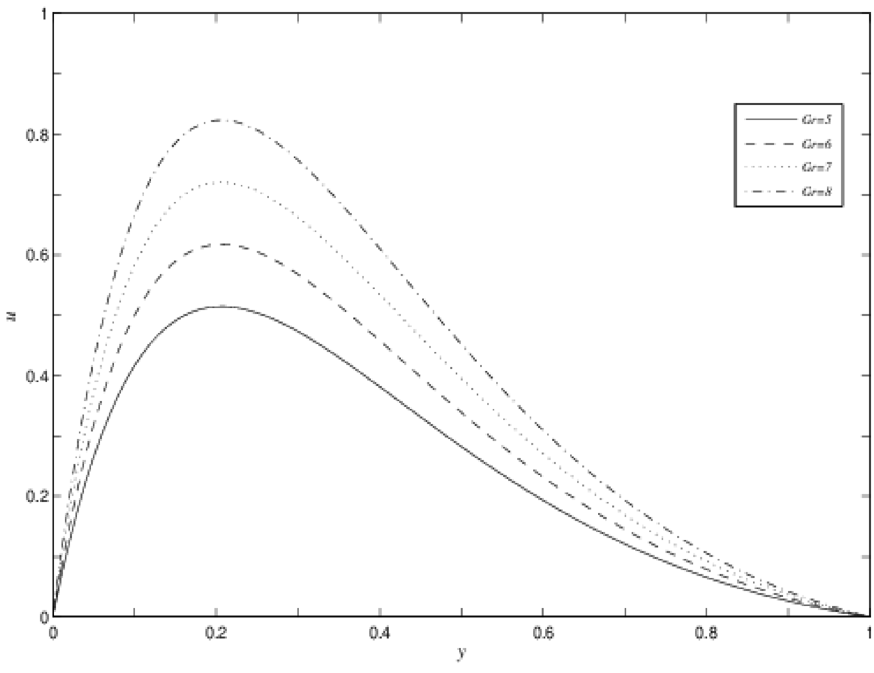

Fig.2: Primary velocity u for $\mathrm{Re} = 5,\ \mathrm{Pr} = 0.71, \omega = 5, t = 0.2.$

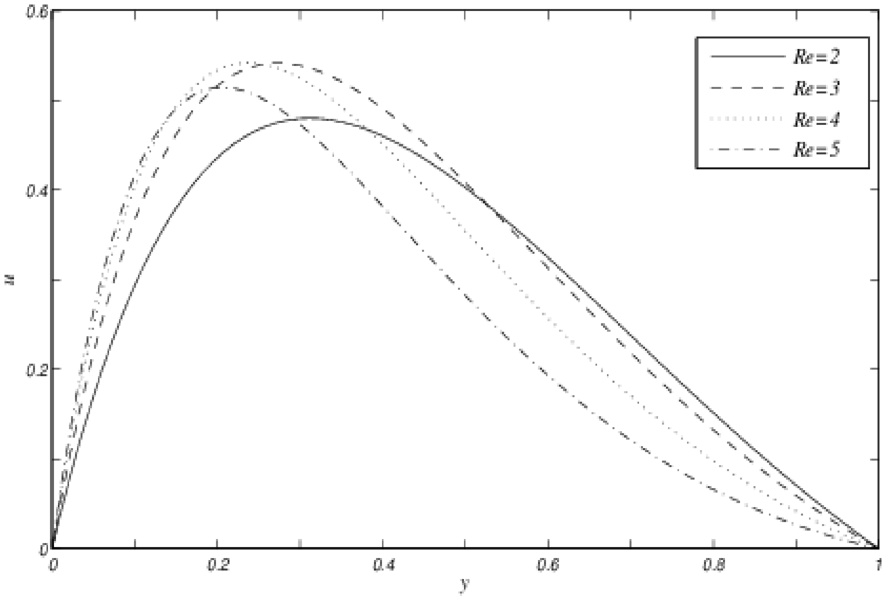

Fig.3: Primary velocity u for $Gr = 5, \omega = 5, \Pr = 0.71, t = 0.2.$

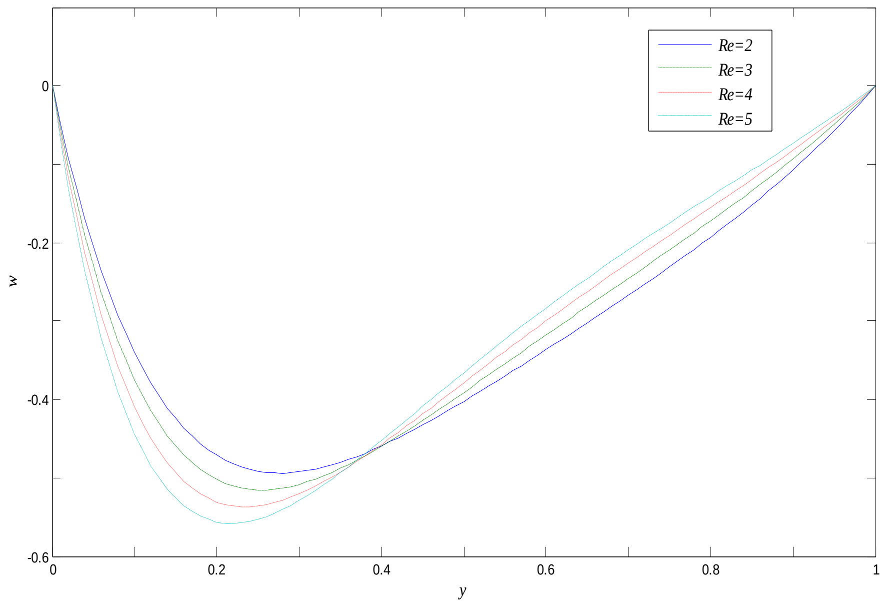

Fig.4: Secondary velocity w for $\omega = 5, \Pr = 0.71, t = 0.2.$

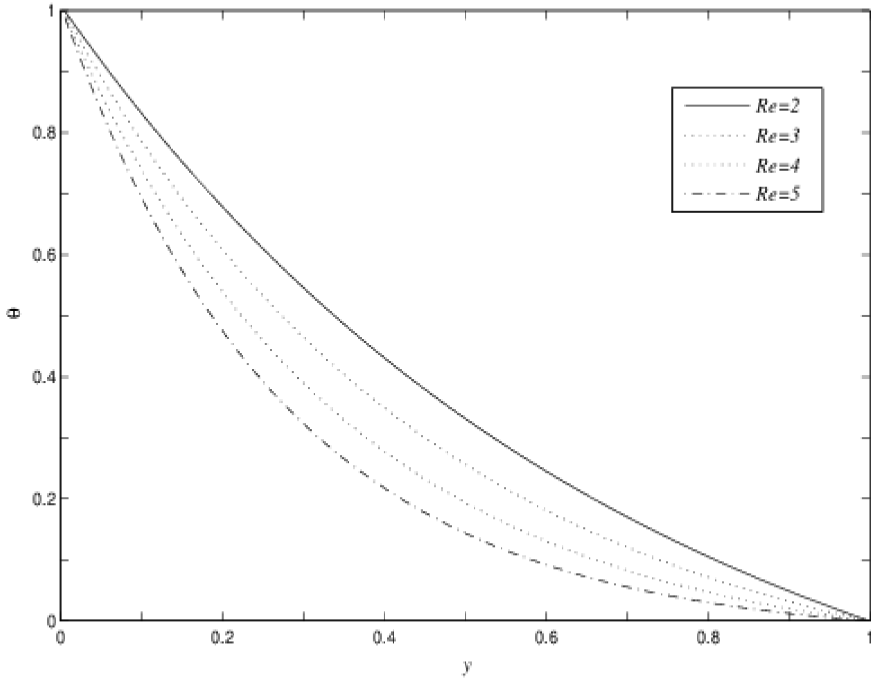

Fig.5: Temperature profile θ for $\omega = 5, \mathrm{Pr} = 0.71, t = 0.2.$

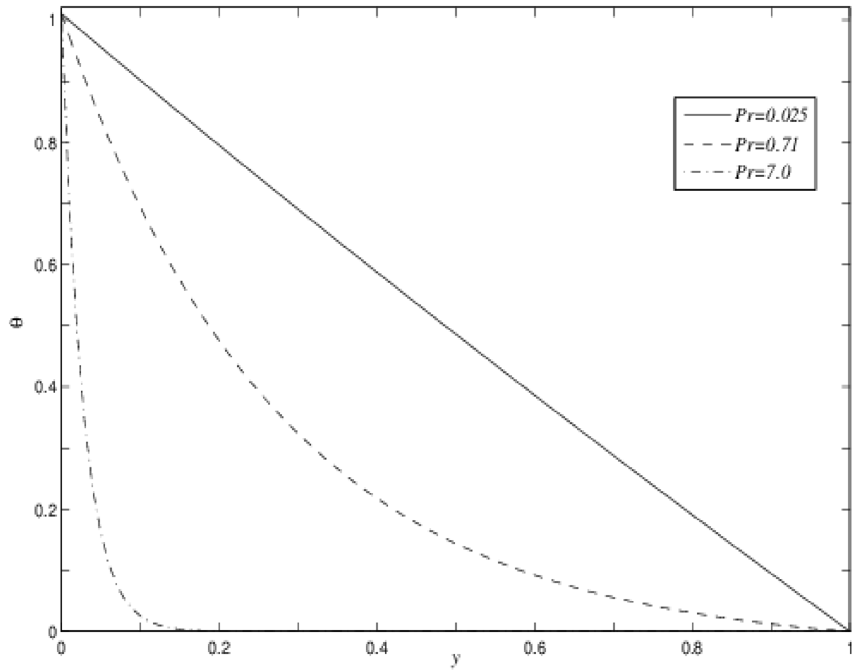

Fig.6: Temperature profile θ for $\mathrm{Re} = 5, \omega = 5,\ t = 0.2.$