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Interior operator over primary interval-valued intuitionistic fuzzy M group

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Abstract: *In this study, Interior Operator over primary interval-valued intuitionistic fuzzy M group as well as Primary interval-valued intuitionistic anti fuzzy M group were defined. Some results based on Primary interval-valued intuitionistic fuzzy M group and anti fuzzy M group are also established.*

Keywords: *Interval-valued intuitionistic fuzzy set, Primary interval-valued intuitionistic fuzzy M group, Primary interval-valued intuitionistic anti fuzzy M group, Interior operator.*

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I. Introduction

The concept of fuzzy sets was initiated by L.A.Zadeh [11] then it has become a vigorous area of research in engineering, medical science, graph theory. Rosenfeld [10] gave the idea of fuzzy subgroup. Bipolar valued fuzzy sets was introduced by K.M.Lee [6] are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0,1] to [-1,0]. In a bipolar valued fuzzy set, the membership degree 0 means that the elements are irrelevant to the corresponding property, the membership degree (0,1] indicates that elements somewhat satisfy the property and the membership degree[-1,0) indicates that elements somewhat satisfy the implicit counter property. The author W.R.Zhang [12] commenced the concept of bipolar fuzzy sets as a generalization of fuzzy sets in 1994. K.Chakrabarthy, R.Biwas and S.Nanda [4] investigated note on union and intersection of intuitionistic fuzzy sets. G.Prasannavengeteswari, K.Gunasekaran and .S.Nandakumar [8] introduced the definition of Primary Bipolar Intuitionistic M Fuzzy Group and anti M Fuzzy Group. A.Balasubramanian, K.L.Muruganantha Prasad, K.Arjunan [3] introduced the definition of Bipolar Interval Valued Fuzzy Subgroups of a Group. G.Prasannavengeteswari, K.Gunasekaran and .S.Nandakumar [9] introduced the definition of primary interval-valued intuitionistic fuzzy M group. In this study Interior Operator over Primary interval-valued Intuitionistic Fuzzy M Group and anti Fuzzy M Group and some properties of the same are proved.

II. Preliminaries

Definition:1

Let G be a non-empty set, let A be an interval-valued intuitionistic fuzzy set (IVIFS) in G and be an object of the form $A = \{\langle x, [\mu_A^-(x), \mu_A^+(x)], [\upsilon_A^-(x), \upsilon_A^+(x)]\rangle \mid x \in G\}$, where $\mu_A^+: G \to [0, 1]$, $\mu_A^-: G \to [0, 1]$ and $\upsilon_A^+: G \to [0, 1]$, $\upsilon_A^-: G \to [0, 1]$ and $(\forall x \in G)$ $(\mu_A^-(x) \leq \mu_A^+(x), \upsilon_A^-(x) \leq \upsilon_A^+(x), \mu_A^+(x) + \upsilon_A^+(x) \leq 1)$ are called the degree of positive membership, the degree of negative membership, the degree of positive non-membership, and the degree of negative non-membership, respectively.

Definition:2

Let G be an M group and let A be an intuitionistic fuzzy subgroup of G, then A is called a primary intuitionistic fuzzy M group of G. if for all $x, y \in$ G and $m \in$ M, then either $\mu_A^+(mxy) \leq \mu_A^+(x^p)$ and $\upsilon_A^+(mxy) \geq \upsilon_A^+(x^p)$, for some $p \in Z_+$ or else $\mu_A^+(mxy) \leq \mu_A^+(y^q)$ and $\upsilon_A^+(mxy) \geq \upsilon_A^+(y^q)$, for some $q \in Z_+$ and either $\mu_A^-(mxy) \geq \mu_A^-(x^p)$ and $\upsilon_A^-(mxy) \leq \upsilon_A^-(x^p)$, for some $p \in Z_+$ or else $\mu_A^-(mxy) \geq \mu_A^-(y^q)$ and $\upsilon_A^-(mxy) \leq \upsilon_A^-(y^q)$, for some $q \in Z_+$.

Example:

$$\mu_A^+(x) = \begin{cases} 0.7 & if\ x = 1 \\ 0.6 & if x = -1 \\ 0.4 & if\ x = i, -i \end{cases} \qquad \upsilon_A^+(x) = \begin{cases} 0.2 & if\ x = 1 \\ 0.3 & if x = -1 \\ 0.5 & if\ x = i, -i \end{cases}$$

$$\mu_A^-(x) = \begin{cases} 0.6 & if\ x = 1 \\ 0.5 & if x = -1 \\ 0.3 & if\ x = i, -i \end{cases} \qquad \upsilon_A^-(x) = \begin{cases} 0.1 & if\ x = 1 \\ 0.2 & if x = -1 \\ 0.5 & if\ x = i, -i \end{cases}$$

Definition:3

Let G be an M group and let A be an intuitionistic anti fuzzy subgroup of G, then A is called a primary intuitionistic anti fuzzy M group of G. if for all $x, y \in$ G and $m \in$ M, then either $\mu_A^+(mxy) \geq \mu_A^+(x^p)$ and $\upsilon_A^+(mxy) \leq \upsilon_A^+(x^p)$, for some $p \in Z_+$or else $\mu_A^+(mxy) \geq \mu_A^+(y^q)$ and $\upsilon_A^+(mxy) \leq \upsilon_A^+(y^q)$, for some $q \in Z_+$ and either $\mu_A^-(mxy) \leq \mu_A^-(x^p)$ and $\upsilon_A^-(mxy) \geq \upsilon_A^-(x^p)$, for some $p \in Z_+$ or else $\mu_A^-(mxy) \leq \mu_A^-(y^q)$ and $\upsilon_A^-(mxy) \geq \upsilon_A^-(y^q)$, for some $q \in Z_+$

Example:

$$\mu_A^+(x) = \begin{cases} 0.4 & if\ x = 1 \\ 0.6 & if x = -1 \\ 0.7 & if\ x = i, -i \end{cases} \qquad \upsilon_A^+(x) = \begin{cases} 0.5 & if\ x = 1 \\ 0.3 & if x = -1 \\ 0.2 & if\ x = i, -i \end{cases}$$

$$\mu_A^-(x) = \begin{cases} 0.3 & if\ x = 1 \\ 0.5 & if x = -1 \\ 0.4 & if\ x = i, -i \end{cases} \qquad \upsilon_A^-(x) = \begin{cases} 0.4 & if\ x = 1 \\ 0.2 & if x = -1 \\ 0.1 & if\ x = i, -i \end{cases}$$

Definition:4

Let *G* be an *M* group and *A* be an interval-valued intuitionistic fuzzy subgroup of *G*, then *A* is called a primary interval-valued intuitionistic fuzzy *M* group of *G*. If for all $x, y \in G$ and $m \in M$, then either $\mu_A^+(mxy) = \inf M_A(mxy) \leq \inf M_A(x^p) = \mu_A^+(x^p)$ and $\upsilon_A^+(mxy) = \sup N_A(mxy) \geq \sup N_A(x^p) = \upsilon_A^+(x^p)$, for some $p \in Z_+$ or else $\mu_A^+(mxy) = \inf M_A(mxy) \leq \inf M_A(y^q) = \mu_A^+(y^q)$ and $\upsilon_A^+(mxy) = \sup N_A(mxy) \geq \sup N_A(y^q) = \upsilon_A^+(y^q)$, for some $q \in Z_+$ and either $\mu_A^-(mxy) = \sup M_A(mxy) \geq \sup M_A(x^p) = \mu_A^-(x^p)$ and $\upsilon_A^-(mxy) = \inf N_A(mxy) \leq \inf N_A(x^p) = \upsilon_A^-(x^p)$, for some $p \in Z_+$ or else $\mu_A^-(mxy) = \sup M_A(mxy) \geq \sup M_A(y^q) = \mu_A^-(y^q)$and $\upsilon_A^-(mxy) = \inf N_A(mxy) \leq \inf N_A(y^q) = \upsilon_A^-(y^q)$, for some $q \in Z_+$

Definition:5

Let *G* be an *M* group and *A* be an interval-valued intuitionistic fuzzy subgroup of *G*, then *A* is called a primary interval-valued intuitionistic anti fuzzy *M* group of *G*. If for all $x, y \in G$ and $m \in M$, then either $\mu_A^+(mxy) = \inf M_A(mxy) \geq \inf M_A(x^p) = \mu_A^+(x^p)$ and $\upsilon_A^+(mxy) = \sup N_A(mxy) \leq \sup N_A(x^p) = \upsilon_A^+(x^p)$, for some$p \in Z_+$ or else $\mu_A^+(mxy) = \inf M_A(mxy) \geq \inf M_A(y^q) = \mu_A^+(y^q)and\upsilon_A^+(mxy) = \sup N_A(mxy) \leq \sup N_A(y^q) = \upsilon_A^+(y^q)$, for some $q \in Z_+$ and either $\mu_A^-(mxy) = \sup M_A(mxy) \leq \sup M_A(x^p) = \mu_A^-(x^p)$ and $\upsilon_A^-(mxy) = \inf N_A(mxy) \geq \inf N_A(x^p) = \upsilon_A^-(x^p)$, for some $p \in Z_+$ or else $\mu_A^-(mxy) = \sup M_A(mxy) \leq \sup M_A(y^q) = \mu_A^-(y^q)and\upsilon_A^-(mxy) = \inf N_A(mxy) \geq \inf N_A(y^q) = \upsilon_A^-(y^q)$, for some $q \in Z_+$.

Definition:6

Let A be an interval valued intuitionistic fuzzy set of E then the interior operator I is defined by,

$I(\mathrm{A}^+) = \{x, \min \mu_A^+(y), \max \upsilon_A^+(y) \,/ x \in E, y \in E\}$ and

$I(\mathrm{A}^-) = \{x, \max \mu_A^-(y), \min \upsilon_A^-(y) \,/ x \in E, y \in E\}$

3. Some Operations on primary interval-valued intuitionistic fuzzy M group and primary interval-valued intuitionistic anti fuzzy M group

Theorem:1

If A is a primary interval-valued intuitionistic fuzzy M group of G then $I(A)$ is primary interval-valued intuitionistic fuzzy M group of G.

Proof:

Consider $x, y \in A$ and $m \in M$

Consider $\mu_{\mathrm{I(A)}}^+(mxy) = \min \mu_A^+(mab)$

$= \min(\inf M_A(mab))$

$\leq \min(\inf M_A(a^p))$
$= \min \mu_A^+(a^p)$
$= \mu_{I(A)}^+(x^p)$

Therefore $\mu_{I(A)}^+(mxy) \leq \mu_{I(A)}^+(x^p)$, for some $p \in Z_+$

Consider $\upsilon_{I(A)}^+(mxy)) = \max(\upsilon_A^+(mab))$
$= \max(\sup N_A(mab))$
$\geq \max(\sup N_A(a^p))$
$= \max \upsilon_A^+(a^p)$
$= \upsilon_{I(A)}^+(x^p)$

Therefore $\upsilon_{I(A)}^+(mxy) \geq \upsilon_{I(A)}^+(x^p)$, for some $p \in Z_+$

Consider $\mu_{I(A)}^-(mxy) = \max(\mu_A^-(mab))$
$= \max(\sup M_A(mab))$
$\geq \max(\sup M_A(a^p))$
$= \max \mu_A^-(a^p)$
$= \mu_{I(A)}^-(x^p)$

Therefore $\mu_{I(A)}^-(mxy) \geq \mu_{I(A)}^-(x^p)$, for some $p \in Z_+$

Consider $\upsilon_{I(A)}^-(mxy)) = \min(\upsilon_A^-(mxy))$
$= \min(\inf N_A(mab))$
$\leq \min(\inf N_A(a^p))$
$= \min(\upsilon_A^-(x^p))$
$= \upsilon_{I(A)}^-(x^p)$

Therefore $\upsilon_{I(A)}^-(mxy) \leq \upsilon_{I(A)}^-(x^p)$, for some $p \in Z_+$

Therefore $I(A)$ is a primary interval-valued intuitionistic fuzzy M group of G.

Theorem:2

If A is a primary interval-valued intuitionistic fuzzy M group of G then $I(I(A)) = I(A)$ is a primary interval-valued intuitionistic fuzzy M group of G.

Proof:

Consider $x, y \in A$ and $m \in M$

Consider $\mu_{I(I(A))}^+(mxy) = \min \mu_{I(A)}^+(mab)$
$= \min(\min \mu_A^+(mxy))$
$= \min(\min(\inf M_A(mxy)))$
$\leq \min(\min(\inf M_A(x^p)))$
$= \min(\min \mu_A^+(x^p))$
$= \min \mu_A^+(a^p)$
$= \mu_{I(A)}^+(x^p)$

Therefore $\mu_{I(I(A))}^+(mxy) \leq \mu_{I(A)}^+(x^p)$, for some $p \in Z_+$

Consider $\upsilon_{I(I(A))}^+(mxy)) = \max(\upsilon_{I(A)}^+(mab))$
$= \max(\max \upsilon_A^+(mxy))$
$= \max(\max(\sup N_A(mxy)))$
$\geq \max(\max(\sup N_A(x^p)))$
$= \max(\max \upsilon_A^+(x^p))$
$= \max \upsilon_A^+(a^p)$
$= \upsilon_{I(A)}^+(x^p)$

Therefore $\upsilon_{I(I(A))}^+(mxy) \geq \upsilon_{I(A)}^+(x^p)$, for some $p \in Z_+$

Consider $\mu_{I(I(A))}^-(mxy) = \max \mu_{I(A)}^-(mab)$
$= \max(\max \mu_A^-(mxy))$
$= \max(\max(\sup M_A(mxy)))$
$\geq \max(\max(\sup M_A(x^p)))$
$= \max(\max \mu_A^-(x^p))$
$= \max \mu_A^-(a^p)$
$= \mu_{I(A)}^+(x^p)$

Therefore $\mu^{-}_{I(I(A))}(mxy) \geq \mu^{-}_{\mathrm{I(A)}}(x^p)$, for some $p \in Z_+$

Consider $\upsilon^{-}_{I(I(A))}(mxy)) = \min(\upsilon^{-}_{I(A)}(mab))$

$= \min(min\, \upsilon^{-}_{A}(mxy))$

$= \min(\min(\inf N_A(mxy)))$

$\leq \min(\min(\inf N_A(x^p)))$

$= \min(\min \upsilon^{-}_{A}(x^p))$

$= \min \upsilon^{-}_{A}(a^p)$

$= \upsilon^{-}_{\mathrm{I(A)}}(x^p)$

Therefore $\upsilon^{-}_{I(I(A))}(mxy) \leq \upsilon^{-}_{\mathrm{I(A)}}(x^p)$, for some $p \in Z_+$

Therefore $I(I(A)) = I(A)$ is a primary interval-valued intuitionistic fuzzy M group of G.

Theorem:3

If A and B are primary interval-valued intuitionistic fuzzy M group of G, then $I(A \cap B = I(A) \cap I$(B) is a primary interval-valued intuitionistic fuzzy M group of G.

Proof:

Consider $x, y \in A \cap B$ then $x, y \in A$ and $x, y \in B$ and $m \in M$

Consider $\mu^{+}_{\mathrm{I}(A\cap B)}(mxy) = \min(\mu^{+}_{A\cap B}(mab))$

$= \min(\min(\mu^{+}_{A}(mab), \mu^{+}_{B}(mab)))$

$= \min(\min(\inf M_A(mab), \inf M_B(mab)))$

$\leq \min(\min(\inf M_A(a^p), \inf M_B(a^p)))$

$= \min(\min(\mu^{+}_{A}(a^p), \mu^{+}_{B}(a^p)))$

$= \min(\min \mu^{+}_{A}(a^p), \min \mu^{+}_{B}(a^p))$

$= \min\left(\mu^{+}_{\mathrm{I(A)}}(x^p), \mu^{+}_{\mathrm{I(B)}}(x^p)\right)$

$= \mu^{+}_{\mathrm{I}(A)\cap\mathrm{I(B)}}(x^p)$

Therefore $\mu^{+}_{\mathrm{I}(A\cap B)}(mxy) \leq \mu^{+}_{\mathrm{I}(A)\cap\mathrm{I(B)}}(x^p)$, for some $p \in Z_+$

Consider $\upsilon^{+}_{\mathrm{I}(A\cap B)}(mxy) = \max(\upsilon^{+}_{A\cap B}(mab))$

$= \max(\max(\upsilon^{+}_{A}(mab), \upsilon^{+}_{B}(mab)))$

$= \max(\max(\sup N_A(mab), \sup N_B(mab)))$

$\geq \max(\max(\sup N_A(a^p), \sup N_B(a^p)))$

$= \max\left(\max(\upsilon^{+}_{A}(a^p), \upsilon^{+}_{B}(a^p)\right)$

$= \max(\max \upsilon^{+}_{A}(a^p), \max \upsilon^{+}_{B}(a^p))$

$= \max\left(\upsilon^{+}_{\mathrm{I(A)}}(x^p), \upsilon^{+}_{\mathrm{I(B)}}(x^p)\right)$

$= \upsilon^{+}_{\mathrm{I}(A)\cap\mathrm{I(B)}}(x^p)$

Therefore $\upsilon^{+}_{\mathrm{I}(A\cap B)}(mxy) \geq \upsilon^{+}_{\mathrm{I}(A)\cap\mathrm{I(B)}}(x^p)$, for some $p \in Z_+$

Consider $\mu^{-}_{\mathrm{I}(A\cap B)}(mxy) = \max(\mu^{-}_{A\cap B}(mab))$

$= \max(\max(\mu^{-}_{A}(mab), \mu^{-}_{B}(mab)))$

$= \max(\max(\sup M_A(mab), \sup M_B(mab)))$

$\geq \max(\max(\sup M_A(a^p), \sup M_B(a^p)))$

$= \max(\max(\mu^{-}_{A}(a^p), \mu^{-}_{B}(a^p)))$

$= \max(\max \mu^{-}_{A}(a^p), \max \mu^{-}_{B}(a^p))$

$= \max\left(\mu^{-}_{\mathrm{I(A)}}(x^p), \mu^{-}_{\mathrm{I(B)}}(x^p)\right)$

$= \mu^{-}_{\mathrm{I}(A)\cap\mathrm{I(B)}}(x^p)$

Therefore $\mu^{-}_{\mathrm{I}(A\cap B)}(mxy) \geq \mu^{-}_{\mathrm{I}(A)\cap\mathrm{I(B)}}(x^p)$, for some $p \in Z_+$

Consider $\upsilon^{-}_{\mathrm{I}(A\cap B)}(mxy) = \min(\upsilon^{-}_{A\cap B}(mab))$

$= \min(\min(\upsilon^{-}_{A}(mab), \upsilon^{-}_{B}(mab)))$

$= \min(\min(\inf N_A(mab), \inf N_B(mab)))$

$\leq \min(\min(\inf N_A(a^p), \inf N_B(a^p)))$

$= \min(\min(\upsilon^{-}_{A}(a^p), \upsilon^{-}_{B}(a^p)))$

$= \min(\min \upsilon^{-}_{A}(a^p), \min \upsilon^{-}_{B}(a^p))$

$= \min\left(\upsilon^{-}_{\mathrm{I(A)}}(x^p), \upsilon^{-}_{\mathrm{I(B)}}(x^p)\right)$

$= \upsilon^{-}_{\mathrm{I}(A)\cap\mathrm{I(B)}}(x^p)$

Therefore $\upsilon^{-}_{\mathrm{I}(A\cap B)}(mxy) \leq \upsilon^{-}_{\mathrm{I}(A)\cap\mathrm{I(B)}}(x^p)$, for some $p \in Z_+$

Therefore $I(A \cap B) = I(A) \cap I$(B) is a primary interval-valued intuitionistic fuzzy M group of G.

Theorem:4

If A is a primary interval-valued intuitionistic fuzzy M group of G, then $\square(I(\mathrm{A})) = I(\square(\mathrm{A}))$ is also a primary interval-valued intuitionistic fuzzy M group of G.

Proof:

Consider $x, y \in A$ and $m \in M$

Consider $\mu^{+}_{\square(I(\mathrm{A}))}(mxy) = \mu^{+}_{\mathrm{I(A)}}(mxy)$

$= \min(\mu^{+}_{\mathrm{A}}(mab))$

$= \min(\inf M_A(mab))$

$\leq \min(\inf M_A(a^p))$

$= \min(\mu^{+}_{\mathrm{A}}(a^p))$

$= \min \mu^{+}_{\square\mathrm{A}}(a^p)$

$= \mu^{+}_{I(\square(\mathrm{A}))}(x^p)$

Therefore $\mu^{+}_{\square(I(\mathrm{A}))}(mxy) \leq \mu^{+}_{I(\square(\mathrm{A}))}(x^p)$, for some $p \in Z_+$

Consider $\upsilon^{+}_{\square(I(\mathrm{A}))}(mxy) = 1 - \mu^{+}_{\mathrm{I(A)}}(mxy)$

$= 1 - \min(\mu^{+}_{\mathrm{A}}(mab))$

$= 1 - \min(\inf M_A(mab))$

$\geq 1 - \min(\inf M_A(a^p))$

$= 1 - \min(\mu^{+}_{\mathrm{A}}(a^p))$

$= 1 - \min(\mu^{+}_{\square\mathrm{A}}(x^p))$

$= 1 - \mu^{+}_{I(\square(\mathrm{A}))}(x^p)$

$= \upsilon^{+}_{I(\square(\mathrm{A}))}(x^p)$

Therefore $\upsilon^{+}_{\square(I(\mathrm{A}))}(mxy) \geq \upsilon^{+}_{I(\square(\mathrm{A}))}(x^p)$, for some $p \in Z_+$

Consider $\mu^{-}_{\square(I(\mathrm{A}))}(mxy) = \mu^{-}_{\mathrm{I(A)}}(mxy)$

$= \max(\mu^{-}_{\mathrm{A}}(mab))$

$= \max(\sup M_A(mab))$

$\geq \max(\sup M_A(a^p))$

$= \max(\mu^{-}_{\mathrm{A}}(a^p))$

$= \max(\mu^{-}_{\square\mathrm{A}}(a^p))$

$= \mu^{-}_{I(\square(\mathrm{A}))}(x^p)$

Therefore $\mu^{-}_{\square(I(\mathrm{A}))}(mxy) \geq \mu^{-}_{I(\square(\mathrm{A}))}(x^p)$, for some $p \in Z_+$

Consider $\upsilon^{-}_{\square(I(\mathrm{A}))}(mxy) = 1 - \mu^{-}_{\mathrm{I(A)}}(mxy)$

$= 1 - \max(\mu^{-}_{\mathrm{A}}(mab))$

$= 1 - \max(\sup M_A(mab))$

$\leq 1 - \max(\sup M_A(a^p))$

$= 1 - \max(\mu^{-}_{\mathrm{A}}(a^p))$

$= 1 - \max(\mu^{-}_{\square\mathrm{A}}(a^p))$

$= 1 - \mu^{-}_{I(\square(\mathrm{A}))}(x^p)$

$= \upsilon^{-}_{I(\square(\mathrm{A}))}(x^p)$

Therefore $\upsilon^{-}_{\square(I(\mathrm{A}))}(mxy) \leq \upsilon^{-}_{I(\square(\mathrm{A}))}(x^p)$, for some $p \in Z_+$

Therefore $\square(I(\mathrm{A})) = I(\square(\mathrm{A}))$ is a primary interval-valued intuitionistic fuzzy M group of G.

Theorem:5

If A is a primary interval-valued intuitionistic fuzzy M group of G, then $\lozenge\,(I(\mathrm{A})) = I(\lozenge\,(\mathrm{A}))$ is also a primary interval-valued intuitionistic fuzzy M group of G.

Proof:

Consider $x, y \in A$ and $m \in M$

Consider $\mu^{+}_{\lozenge(I(\mathrm{A}))}(mxy) = 1 - \upsilon^{+}_{\mathrm{I(A)}}(mxy)$

$= 1 - \max(\upsilon^{+}_{\mathrm{A}}(mab))$

$= 1 - \max(\sup N_A(mab))$

$\leq 1 - \max(\sup N_A(a^p))$

$= 1 - \max(\upsilon^{+}_{\mathrm{A}}(a^p))$

$= 1 - \max(\upsilon^{+}_{\lozenge\mathrm{A}}(a^p))$

$= 1 - \upsilon^{+}_{I(\lozenge(\mathrm{A}))}(a^p)$

$= \mu^{+}_{I(\Diamond(A))}(x^p)$

Therefore $\mu^{+}_{\Diamond(I(A))}(mxy) \leq \mu^{+}_{I(\Diamond(A))}(x^p)$, for some $p \in Z_+$

Consider $\upsilon^{+}_{\Diamond(I(A))}(mxy) = \upsilon^{+}_{I(A)}(mxy)$

$= \max(\upsilon^{+}_{A}(mab))$

$= \max(\sup N_A(mab))$

$\geq \max(\sup N_A(a^p))$

$= \max(\upsilon^{+}_{A}(a^p))$

$= \max(\upsilon^{+}_{\Diamond A}(a^p))$

$= \upsilon^{+}_{I(\Diamond(A))}(x^p)$

Therefore $\upsilon^{+}_{\Diamond(I(A))}(mxy) \geq \upsilon^{+}_{I(\Diamond(A))}(x^p)$, for some $p \in Z_+$

Consider $\mu^{-}_{\Diamond(I(A))}(mxy) = 1 - \upsilon^{-}_{I(A)}(mxy)$

$= 1 - \min(\upsilon^{-}_{A}(mab))$

$= 1 - \min(\inf N_A(mab))$

$\geq 1 - \min(\inf N_A(a^p))$

$= 1 - \min(\upsilon^{-}_{A}(a^p))$

$= 1 - \min(\upsilon^{-}_{\Diamond A}(a^p))$

$= 1 - \upsilon^{-}_{I(\Diamond(A))}(a^p)$

$= \mu^{-}_{I(\Diamond(A))}(x^p)$

Therefore $\mu^{-}_{\Diamond(I(A))}(mxy) \geq \mu^{-}_{I(\Diamond(A))}(x^p)$, for some $p \in Z_+$

Consider $\upsilon^{-}_{\Diamond(I(A))}(mxy) = \upsilon^{-}_{I(A)}(mxy)$

$= \min(\upsilon^{-}_{A}(mab))$

$= \min(\inf N_A(mab))$

$\leq \min(\inf N_A(a^p))$

$= \min(\upsilon^{-}_{A}(a^p))$

$= \min(\upsilon^{-}_{\Diamond A}(a^p))$

$= \upsilon^{-}_{I(\Diamond(A))}(x^p)$

Therefore $\upsilon^{-}_{\Diamond(I(A))}(mxy) \leq \upsilon^{-}_{I(\Diamond(A))}(x^p)$, for some $p \in Z_+$

Therefore $\Diamond(I(A)) = I(\Diamond(A))$ is a primary interval-valued intuitionistic fuzzy M group of G.

Theorem:6

If A is a primary interval-valued intuitionistic anti fuzzy M group of G then $I(A)$ is primary interval-valued intuitionistic anti fuzzy M group of G.

Theorem:7

If A is a primary interval-valued intuitionistic anti fuzzy M group of G then $I(I(A)) = I(A)$ is primary interval-valued intuitionistic anti fuzzy M group of G.

Theorem:8

If A and B are primary interval-valued intuitionistic anti fuzzy M group of G, then $I(A \cap B) = I(A) \cap I(B)$ is a primary interval-valued intuitionistic anti fuzzy M group of G.

Theorem:9

If A is a primary interval-valued intuitionistic anti fuzzy M group of G, then $\Box(I(A)) = I(\Box(A))$ is also a primary interval-valued intuitionistic anti fuzzy M group of G.

Theorem:10

If A is a primary interval-valued intuitionistic anti fuzzy M group of G, then $\Diamond(I(A)) = I(\Diamond(A))$ is also a primary interval-valued intuitionistic anti fuzzy M group of G.

References

[1] Atanassov, K. T. (1999). *Intuitionistic Fuzzy Sets:Theory and Application*, Springer Physica-Verlag.

[2] Atanassov, K. T. (2020). *Interval-Valued Intuitionistic Fuzzy Sets*, Springer Cham.

[3] Balasubramanian, A., Murugenantha Prasad, K. L., &Arjunan, K. (2015). Bipolar Interval Valued Fuzzy Subgroups of a Group, *Bulletin of Mathematics and Statistics Research*,3(3), 234–239.

[4] Chakrabarthy, K., Biswas, R., & Nanda, S. (1997). A note on union and intersection of intuitionistic fuzzy sets. *Notes on Intuitionistic Fuzzy Sets*, 3(4), 34–39.

[5] Gunasekaran, K., &Gunaseelan, D. (2017). Some Operations on Bipolar Intuitionistic *M*-Fuzzy Group and Anti *M*-Fuzzy Group, *International Journals of Advanced Research in Science, Engineering and Technology*, 4(3), 3511–3518.

[6] Lee, K. M. (2000). Bipolar valued fuzzy sets and their operations. *Proceeding of International Conference on Intelligent Technologies*, Bangkok, Thailand, 307–312.

[7] Palanivelrajan, M., &Nandakumar, S. (2012). Intuitionistic Fuzzy Primary and Semiprimary Ideal. *Indian Journal of Applied Research*, 59(1), 159–160.
[8] Prasannavengeteswari, G., Gunasekaran, K., &Nandakumar, S. (2021). Primary Bipolar Intuitionistic *M* Fuzzy Group. *Journal of Shanghai Jiaotong University*, 17(4), 82–92.
[9] Prasannavengeteswari, G., Gunasekaran, K., &Nandakumar, S. (2022). primary interval-valued intuitionistic fuzzy M group. Notes on Intuitionistic Fuzzy Sets, 28(2),120-131.
[10] Rosenfeld, A. (1971). Fuzzy Groups, *Journal of Mathematical Analysis and Its Application*, 35, 512–517.
[11] Zadeh, L. A. (1965). Fuzzy sets.*Information and Control*, 8, 338–353.
[12] Zhang, W. R. (1998). Bipolar fuzzy sets. *Proceeding of FUZZ-IEEE*, 835–840.
[13] Zimmermann, H. J. (1985). *Fuzzy Set Theory and Its Applications*, Kluwer-NijhoffPublishing Co.