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"Coefficient Bounds for a Subclass of Classes of Convex Functions and Starlike Functions in Limit Form"

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ABSTRACT: *We introduce a class of analytic functions and obtain sharp upper bounds of the functional* $|a_3 - \mu a_2^2|$ *for the analytic function* $f(z) = z + \sum_{n=2}^{\infty} a_n z^n, |z| < 1$ *belonging to this class with special character that it tends to the class of Starlike functions as* $\alpha \to \frac{\pi}{2}$.

KEYWORDS: *Univalent functions, Starlike functions, Close to convex functions and bounded functions, Analytic Functions, FeketeSzego Inequality, Convex Functions, Extremal function.*
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I. Introduction :

Let A denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1.1}$$

which are analytic in the unit disc $E = \{z: |z| < 1|\}$. Let S be the class of functions of the form (1.1), which are analytic univalent in E.

In 1916, Bieber Bach ([1], [2]) proved that $|a_2| \leq 2$ for the functions $f(z)S$. In 1923, Löwner proved that $|a_3| \leq 3$ for the functions $f(z)S$..

With the known estimates $|a_2| \leq 2$and $|a_3| \leq 3$, it was expected to try to find some relation between a_3 and ${a_2}^2$ for the class S,Fekete and Szegö [5] used Löwner's method to prove the following well known result for the class S.

Let $f(z)\ S$, then

$$|a_3 - \mu a_2^2| \leq \begin{cases} 3 - 4\mu, if\ \mu \leq 0; \\ 1, if\ 0 \leq \mu \leq 1; \\ 4\mu - 3, if \mu \geq 1 \end{cases} \tag{1.2}$$

The inequality (1.2) plays a very important role in determining estimates of higher coefficients for some sub classes S ([7], [8], [18]-[43]).

Let us define some subclasses of S.

We denote by S*, the class of univalent starlike functions

$$g(z) = z + \sum_{n=2}^{\infty} b_n z^n \in A$$

and satisfying the condition

$$Re\left(\frac{zg(z)}{g(z)}\right) > 0, z \in E. \tag{1.3}$$

We denote by K, the class of univalent convex functions

$$h(z) = z + \sum_{n=2}^{\infty} c_n z^n, z \in A$$

and satisfying the condition

$$Re\frac{((zh'(z))}{h'(z)} > 0, z \in E. \tag{1.4}$$

A function $f(z) \in A$ is said to be close to convex if there exists $g(z) \in S^*$ such that

$Re\left(\frac{zf'(z)}{g(z)}\right) > 0, z \in E.$ (1.5)

The class of close to convex functions is denoted by C and was introduced by Kaplan [7] and it was shown by him that all close to convex functions are univalent.

$$S^*(A,B) = \left\{ f(z) \in A; \frac{zf'(z)}{f(z)} \prec \frac{1+Az}{1+Bz}, -1 \le B < A \le 1, z \in E \right\} \quad (1.6)$$

$$K(A,B) = \left\{ f(z) \in A; \frac{(zf'(z))'}{f'(z)} \prec \frac{1+Az}{1+Bz}, -1 \le B < A \le 1, z \in E \right\} \quad (1.7)$$

It is obvious that $S^*(A,B)$ is a subclass of S^* and $K\,(A,B)$ is a subclass of K.
We introduce a new subclass as

$$\left\{ f(z) \in A; tan\alpha \left(\frac{zf'(z)}{f(z)}\right)^{\beta} + \left(\frac{(zf'(z))'}{f'(z)}\right)^{1-\beta} \prec (1+tan\alpha)\frac{1+w(z)}{1-w(z)}; z \in E \right\}$$

and we will denote this class as $KS^*(\alpha,\beta)$.
We will deal with two subclasses of $S^*(f,f',\alpha,\beta)$ defined as follows in our next paper:

$$KS^*(\alpha,\beta,A,B) = \left\{ f(z) \in A; tan\alpha \left(\frac{zf'(z)}{f(z)}\right)^{\beta} + \left(\frac{(zf'(z))'}{f'(z)}\right)^{1-\beta} \prec (1+tan\alpha)\,\frac{1+Az}{1+Bz}; z \in E \right\} \quad (1.8)$$

$$KS^*(A,B,\alpha,\beta,\gamma) = \left\{ f(z) \in A; tan\alpha \left(\frac{zf'(z)}{f(z)}\right)^{\beta} + \left(\frac{(zf'(z))'}{f'(z)}\right)^{1-\beta} \prec (1+tan\alpha)\left\{\frac{1+Az}{1+Bz}\right\}^{\gamma}; z \in E \right\} (1.9)$$

Symbol $\prec$ stands for subordination, which we define as follows:
Principle of Subordination: Let $f(z)$ and $F(z)$ be two functions analytic in E. Then $f(z)$ is called subordinate to F(z) in E if there exists a function $w(z)$ analytic in E satisfying the conditions $w(0) = 0$ and $|w(z)| < 1$ such that $f(z) = F(w(z));\ zE$ and we write $f(z) \prec F(z)$.
By U, we denote the class of analytic bounded functions of the form
$w(z) = \sum_{n=1}^{\infty} d_n z^n, w(0) = 0, |w(z)| < 1.$ (1.10)
It is known that $|d_1| \le 1, |d_2| \le 1 - |d_1|^2$. (1.11)

II. PRELIMINARY LEMMAS:

For $0 < c < 1$, we write $w(z) = \left(\frac{c+z}{1+cz}\right)$ so that

$$\frac{1+w(z)}{1-w(z)} = 1 + 2cz + 2z^2 + ---. \quad (2.1)$$

III. MAIN RESULTS

THEOREM 3.1: Let $f(z) \in KS^*(\alpha,\beta)$, then

$$|a_3 - \mu a_2^2| \le \{ \frac{1+tan\alpha}{\{\beta tan\alpha + 2(1-\beta)\}^2}\left[\frac{\beta^2[tan^2\alpha - 1 - 9tan\alpha] + 12 - 9\beta - 8tan\alpha}{\{\beta tan\alpha + 3(1-\beta)\}} - 4\mu\right], if\mu \le \frac{8 - 5\beta^2 tan\alpha - \beta - 3\beta^2 - 4\beta tan\alpha}{4(1+tan\alpha)\{\beta tan\alpha + 3(1-\beta)\}}\ ; \quad (3.1)$$

$$\frac{1+tan\alpha}{3(1-\beta)+\beta tan\alpha}\ if\ \frac{8 - 5\beta^2 tan\alpha - \beta - 3\beta^2 - 4\beta tan\alpha}{4(1+tan\alpha)\{\beta tan\alpha + 3(1-\beta)\}} \le \mu \le \frac{2\beta^2 tan^2\alpha - 13\beta^2 tan\alpha + 3\beta^2 + 16 - 17\beta + 8tan\alpha}{4(1+tan\alpha)\{\beta tan\alpha + 3(1-\beta)\}}; \quad (3.2)$$

$$\frac{1+tan\alpha}{\{\beta tan\alpha + 2(1-\beta)\}^2}\left[4(1+tan\alpha)\mu - \frac{\beta^2(tan^2\alpha - 1 - 9tan\alpha) + 12 - 9\beta + 8tan\alpha}{\{\beta tan\alpha + 3(1-\beta)\}}\right], if\ \mu \ge \frac{2\beta^2 tan^2\alpha - 13\beta^2 tan\alpha + 3\beta^2 + 16 - 17\beta + 8tan\alpha}{4(1+tan\alpha)\{\beta tan\alpha + 3(1-\beta)\}} \quad (3.3)$$

The results are sharp.
Proof: By definition of $KS^*(\alpha,\beta,\gamma)$, we have

$$tan\alpha \left(\frac{zf'(z)}{f(z)}\right)^{\beta} + \left(\frac{(zf'(z))'}{f'(z)}\right)^{1-\beta} = (1+tan\alpha)\frac{1+w(z)}{1-w(z)}; w(z) \in U. \quad (3.4)$$

Expanding the series (3.4), we get

$$tan\alpha\left\{1 + \beta a_2 z + (2\beta a_3 + \frac{\beta(\beta-3)}{2}a_2^2)z^2 + ---\right\} + \{1 + 2(1-\beta)a_2 z + 2(1-\beta)(3a_3 - (\beta+2)a_2^2)z^2 + ---=(1+tan\alpha)1+2c1z+2c2+c12z2+---. \quad (3.5)$$

Identifying terms in (3.5), we get

$a_2 = \frac{2}{\beta tan\alpha + 2(1-\beta)} c_1$ (3.6)

$a_3 = \frac{1+tan\alpha}{\beta tan\alpha + 3(1-\beta)} c_2 + (1 + tan\alpha) \frac{[\beta^2(tan^2\alpha - 1 - 9tan\alpha) + 12 - 9\beta + 8tan\alpha]}{[2(1-\beta) + \beta tan\alpha]^2(\beta tan\alpha + 3(1-\beta)} {c_1}^2$.(3.7)

From (3.6) and (3.7), we obtain

$a_3 - \mu a_2^2 = \frac{1+tan\alpha}{\beta tan\alpha + 3(1-\beta)} c_2 + \frac{\beta^2(tan^2\alpha - 1 - 9tan\alpha) + 12 - 9\beta + 8tan\alpha}{\beta tan\alpha + 3(1-\beta)} - 4\mu c_1^2 \frac{(1+tan\alpha)^2}{\{\beta tan\alpha + 2(1-\beta)\}^2}$.(3.8)

Taking absolute value and using Triangular inequality, (3.8) can be rewritten as

$|a_3 - \mu a_2^2| \leq \frac{(1+tan\alpha)}{\beta tan\alpha + 3(1-\beta)} |c_2| + \frac{1+tan\alpha}{\{\beta + 2(1-\beta) tan\, tan\, \alpha\}^2} \left| \frac{\beta^2(tan^2\alpha - 1 - 9tan\alpha) + 12 + 8tan\alpha - 9\beta}{\beta tan\alpha + 3(1-\beta)} - 4\mu(1 + tan\alpha) \right| |c_1^2|$.
(3.9)

Using (1.9) in (3.9), Simple calculations yield

$|a_3 - \mu a_2^2| \leq \frac{(1+tan\alpha)}{\beta tan\alpha + 3(1-\beta)} + \frac{1+tan\alpha}{\{\beta tan\alpha + 2(1-\beta)\}^2} \left[\left| \frac{\beta^2(tan^2\alpha - 1 - 9tan\alpha) + 12 + 8tan\alpha - 9\beta}{\{\beta tan\alpha + 3(1-\beta)\}} - 4(1 + tan\alpha)\mu \right| - \right.$
(1+tanα)tanαβ+2(1−β)2βtanα+31−β|c1|2. (3.10)

Case I: $\mu \leq \frac{\beta^2(tan^2\alpha - 1 - 9tan\alpha + 12 + 8tan\alpha - 9\beta}{4(tan\alpha + 1)\{\beta tan\alpha + 3(1-\beta)\}}$.

In this case, (3.10) can be rewritten as

$|a_3 - \mu a_2^2 \leq \frac{1+tan\alpha}{\beta tan\alpha + 3(1-\beta)} + \frac{1+tan\alpha}{\{\beta tan\alpha + 2(1-\beta)\}^2} \left[\frac{-3\beta^2 - 5\beta^2 tan\alpha - \beta + 8 - 4\beta tan\alpha}{\beta tan\alpha + 3(1-\beta)} - 4\mu(1 + tan\alpha) \right] |c_1|^2$. (3.11)

Subcase I (a):

$$\mu \leq \frac{8 - 5\beta^2 tan\alpha - \beta - 3\beta^2 - 4\beta tan\alpha}{4(1 + tan\alpha)\{\beta tan\alpha + 3(1-\beta)\}}.$$

Using (1.9), (3.11) becomes

$$|a_3 - \mu a_2^2| \leq \frac{1 + tan\alpha}{\{\beta tan\alpha + 2(1-\beta)\}^2} \left[\frac{\beta^2(tan^2\alpha - 1 - 9tan\alpha) + 12 - 9\beta - 8tan\alpha}{\{\beta tan\alpha + 3(1-\beta)\}} - 4(1 + tan\alpha)\mu \right] \quad (3.12)$$

Subcase I (b):

$$\mu \geq \frac{8 - 5\beta^2 tan\alpha - \beta - 3\beta^2 - 4\beta tan\alpha}{4(1 + tan\alpha)\{\beta tan\alpha + 3(1-\beta)\}}.$$

We obtain from (3.11)

$|a_3 - \mu a_2^2| \leq \frac{1+tan\alpha}{\beta tan\alpha + 3(1-\beta)}$. (3.13)

Case II:

$$\mu \geq \frac{\beta^2(tan^2\alpha - 1 - 9tan\alpha) + 12 - 9\beta + 8tan\alpha}{4(1 + tan\alpha)(\beta tan\alpha + 3(1-\beta)}$$

Preceding as in case I, we get

$|a_3 - \mu a_2^2| \leq \frac{1+tan\alpha}{3(1-\beta) + \beta tan\alpha} + \frac{1+tan\alpha}{\{tan\alpha\beta + 2(1-\beta)\}^2} \left[4(1 + tan\alpha)\mu - \frac{2\beta^2 tan^2\alpha - 13\beta^2 tan\alpha + 3\beta^2 + 16 - 17\beta + 8tan\alpha}{\{\beta tan\alpha + 3(1-\beta)\}} - 4\mu(1 + \right.$
tanα)|c1|2. (3.14)

Subcase II (a):

$$\mu \leq \frac{2\beta^2 tan^2\alpha - 13\beta^2 tan\alpha + 3\beta^2 + 16 - 17\beta + 8tan\alpha}{4(1 + tan\alpha)\{\beta tan\alpha + 3(1-\beta)\}}$$

(3.14) takes the form

$$|a_3 - \mu a_2^2| \leq \frac{1 + tan\alpha}{\beta tan\alpha + 3(1-\beta)} \quad (3.15)$$

Combining subcase I (b) and subcase II (a), we obtain

$|a_3 - \mu a_2^2| \leq \frac{1+tan\alpha}{\beta tan\alpha + 3(1-\beta)} if \frac{8 - 5\beta^2 tan\alpha - \beta - 3\beta^2 - 4\beta tan\alpha}{4(1+tan\alpha)\{\beta tan\alpha + 3(1-\beta)\}} \leq \mu \leq \frac{2\beta^2 tan^2\alpha - 13\beta^2 tan\alpha + 3\beta^2 + 16 - 17\beta + 8tan\alpha}{4(1+tan\alpha)[\beta tan\alpha + 3(1-\beta)]}$(3.16)

Subcase II (b): $\mu \geq \frac{2\beta^2 tan^2\alpha - 13\beta^2 tan\alpha + 3\beta^2 + 16 - 17\beta + 8tan\alpha}{4(1+tan\alpha)[\beta tan\alpha + 3(1-\beta)]}$

Preceding as in subcase I (a), we get

$|a_3 - \mu a_2^2| \leq \frac{1+tan\alpha}{\{\beta tan\alpha + 2(1-\beta)\}^2} \left[4(1 + tan\alpha)\mu - \frac{\beta^2(tan^2\alpha - 1 - 9tan\alpha) + 12 - 9\beta + 8tan\alpha}{\{\beta tan\alpha + 3(1-\beta)\}} \right]$. (3.17)

Combining (3.12), (3.16) and (3.17), the theorem is proved.

Extremal function for (3.1) and (3.3) is defined by

$$f_1(z) = (1 + az)^b$$

Where $a = \frac{2\gamma\{\beta + 3(1-\beta) tan\, tan\, \alpha\}}{\{4(1-\beta)(\beta+2) tan\, tan\, \alpha - \beta(\beta-3)\}\{\beta + 2(1-\beta) tan\, tan\, \alpha\}\}^3 - 2\gamma}$

And $b = \frac{\{4(1-\beta)(\beta+2) tan\, tan\, \alpha - \beta(\beta-3)\}\{\beta + 2(1-\beta) tan\, tan\, \alpha\}\}^3 - 2\gamma}{\{\beta + 3(1-\beta) tan\, tan\, \alpha\}\{\beta + 2(1-\beta) tan\, tan\, \alpha\}\}}$

Extremal function for (3.2) is defined by $f_2(z) = z(1 + cz^2)^d$,
Where $c = \frac{\tan\tan\alpha}{\beta+3(1-\beta)\tan\tan\alpha}$ and $d = \frac{\gamma}{\tan\tan\alpha}$

Corollary 3.2: Putting $\gamma = 1, \beta = 0$ and applying limit as $\alpha \to \frac{\pi}{2}$ in the theorem, we get

$$|a_3 - \mu a_2^2| \leq \begin{cases} 1-\mu, if \mu \leq 1; \\ \frac{1}{3} if 1 \leq \mu \leq \frac{4}{3}; \\ \mu - 1, if \mu \geq \frac{4}{3} \end{cases}$$

These estimates were derived by Keogh and Merkes [9] and are results for the class of univalent convex functions.

Corollary 3.3: Putting $\alpha = 0, \beta = 1, \gamma = 0$ in the theorem, we get

$$|a_3 - \mu a_2^2| \leq \begin{cases} 3 - 4\mu, if\ \mu \leq \frac{1}{2}; \\ 1, if\ \frac{1}{2} \leq \mu \leq 1; \\ 4\mu - 3, if \mu \geq 1 \end{cases}$$

These estimates were derived by Keogh and Merkes [9] and are results for the class of univalent starlike functions.

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