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Evolution of the concept of work done and Kinetic Energy

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Abstract
Science is discovered while technology is fabricated to serve the need of the hour by applying the existing knowledge. Technology along with the serving to the society also helps the science to create new knowledge and validate the same. The bridge between the science and technology is to create future fertile mind to advance both science and technology. This is possible only when how we spread the existing well-established knowledge and their practices to the new generation of intellect which provides: the thought line of creation of the existing knowledge, their practices and critically examining the limitations of the existing knowledge, quantitatively and qualitatively both. In this communication, we have made an attempt to see the limitations of the force-method to solve a problem and a thought-line which evolved the concept of work-done and kinetic energy of a particle, system of particles or a rigid body.
Keywords: *Force-method; Work-done, Kinetic energy*

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I. Introduction

From its conception, the human mind has an intrinsic characteristic: thinking, imagining, creating knowledge and applying it to make life easy and powerful. The oldest question in human history of mankind was that: light is made up of what? And how this nature works around us? The quest of understanding of human mind addressed both the problems successfully. With the passage of time, finally both the problems were solved as: light is made up of electric and magnetic fields and concept of electromagnetic wave was established by Maxwell and the mechanical working of nature was qualitatively stated by Galileo and quantitively formulated by Newton in form of Newton's laws of motion. This is known as force-method to solve a problem in which forces may be of contact type or action at a distance. By the force-method we can find the instanteous acceleration, velocity, and position of a particle if force is well known and we can characterise the state of motion or rest of a particle, system of particles or a rigid body. The question is: what is the limitation of the force-method or can it solve any problem?

II. Limitations of the Force-Method

Consider a particle of mass $\mathbf{m}$, experiencing a net force $\vec{\mathbf{F}}$at any time$\mathbf{t}$, then what is its acceleration $\vec{\mathbf{a}}(\mathbf{t})$, velocity$\vec{\mathbf{v}}(\mathbf{t})$ and position vector $\vec{\mathbf{r}}(\mathbf{t})$ at any time $\mathbf{t}$? By Newton's 2nd law of motion, we have:$\vec{\mathbf{F}} = \mathbf{m}\vec{\mathbf{a}}(\mathbf{t})$.
Therefore, $\vec{\mathbf{a}}(\mathbf{t}) = \vec{\mathbf{F}}/\mathbf{m}$. Now, for velocity we have,

$$\vec{\mathbf{a}}(\mathbf{t}) = \left(\frac{\overrightarrow{\mathbf{dv}}}{\mathbf{dt}}\right)$$

or $$\overrightarrow{\mathbf{dv}} = \vec{\mathbf{a}}(\mathbf{t})\mathbf{dt}$$

or $$\overrightarrow{\mathbf{dv}} = \left(\frac{\vec{\mathbf{F}}}{\mathbf{m}}\right)\mathbf{dt}$$

or $\vec{\mathbf{v}}(\mathbf{t}) = \vec{\mathbf{v}}(\mathbf{0}) + \left(\frac{\mathbf{1}}{\mathbf{m}}\right)\int_{\mathbf{0}}^{\mathbf{t}} \vec{\mathbf{F}}\,\mathbf{dt}$(1)

where $\vec{\mathbf{v}}(\mathbf{t})$ and $\vec{\mathbf{v}}(\mathbf{0})$ are final and initial velocities of the particle, out of them $\vec{\mathbf{v}}(\mathbf{0})$ is known in advance.
If, we are able to integrate the part $\int_{\mathbf{0}}^{\mathbf{t}} \vec{\mathbf{F}}\,\mathbf{dt}$ of Equ. (1) we can solve the problem of finding $\vec{\mathbf{v}}(\mathbf{t})$ of the particle at any time $\mathbf{t}$. The problem lies here in $\vec{\mathbf{F}}$. In most of the natural interacting or developing forces as: Elastic force, Gravitational force, Coulomb force, Viscous force, etc., the force $\vec{\mathbf{F}}$is the function of space i.e. $\vec{\mathbf{F}} = \mathbf{f}\,(\mathbf{r})\hat{\boldsymbol{r}}$while the integration is with respect to time $\mathbf{t}$, hence we cannot proceed beyond Equ. (1) to solve the problem, and this is the limitation of force-method.
The attempt to overcome this limitation, evolved the concept of work-done by a force and kinetic energy gained by the particle, system of particles or a rigid body on which work is done.

III. Evolution of the concept of Work-done and Kinetic energy

In Equ. (1) the problem is that $\vec{\mathbf{F}}$is the function of space except the time dependent applied forces and the integration is with respect to time. Let us integrate $\vec{\mathbf{F}}$with respect to space as an attempt to address the problem,

$$\vec{\mathbf{F}} \bullet \overrightarrow{\mathbf{dr}} = \mathbf{m}\vec{\mathbf{a}} \bullet \overrightarrow{\mathbf{dr}}$$

or $\vec{\mathbf{F}} \bullet \overrightarrow{\mathbf{dr}} = \mathbf{m}\vec{\mathbf{a}} \bullet \vec{\mathbf{v}}\mathbf{dt}$

or $\vec{\mathbf{F}} \bullet \overrightarrow{\mathbf{dr}} = \mathbf{m}[\vec{\mathbf{a}}\mathbf{dt}] \bullet \vec{\mathbf{v}}$

or $\vec{\mathbf{F}} \bullet \overrightarrow{\mathbf{dr}} = \mathbf{m}[\overrightarrow{\mathbf{dv}} \bullet \vec{\mathbf{v}}]$(2)

since$\mathbf{d}(\vec{\mathbf{v}} \bullet \vec{\mathbf{v}}) = \overrightarrow{\mathbf{dv}} \bullet \vec{\mathbf{v}} + \vec{\mathbf{v}} \bullet \overrightarrow{\mathbf{dv}}$

or $\mathbf{d}(\mathbf{v^2}) = \mathbf{2}\overrightarrow{\mathbf{dv}} \bullet \vec{\mathbf{v}}$

or $\overrightarrow{\mathbf{dv}} \bullet \vec{\mathbf{v}} = \mathbf{d}(\frac{\mathbf{v^2}}{\mathbf{2}})$

Therefore, the Equ. (2) will be reduced in the form,

$$\vec{\mathbf{F}} \bullet \overrightarrow{\mathbf{dr}} = \mathbf{d}(\frac{\mathbf{mv^2}}{\mathbf{2}})$$

i.e. $\vec{\mathbf{F}}(\overrightarrow{\mathbf{r}}) \bullet \overrightarrow{\mathbf{dr}} = \mathbf{d}(\frac{\mathbf{mv^2}}{\mathbf{2}})$ (3)

Integrating this Equ. (3) from initial to final, we get

$\int_{\vec{\mathbf{r}}_i}^{\vec{\mathbf{r}}} \vec{\mathbf{F}}(\overrightarrow{\mathbf{r}}) \bullet \overrightarrow{\mathbf{dr}} = \int_{\mathbf{v}_i}^{\mathbf{v}} \mathbf{d}(\frac{\mathbf{mv^2}}{\mathbf{2}})$(4)

Now, we can integrate the L.H.S. because $\vec{\mathbf{F}}$is the function of space known.
Therefore,

$\mathbf{v^2} = \mathbf{v_i}^2 + (\frac{\mathbf{2}}{\mathbf{m}}) \int_{\vec{\mathbf{r}}_i}^{\vec{\mathbf{r}}} \vec{\mathbf{F}}(\overrightarrow{\mathbf{r}}) \bullet \overrightarrow{\mathbf{dr}}$(5)

and now, we can find out the final speed**v**at the final position vector $\vec{\mathbf{r}}$but without knowing what are the variations in between initial and final positions.

The integration $\int_{\vec{\mathbf{r}}_i}^{\vec{\mathbf{r}}} \vec{\mathbf{F}}(\overrightarrow{\mathbf{r}}) \bullet \overrightarrow{\mathbf{dr}}$ is defined as the work-done by the force $\vec{\mathbf{F}}(\overrightarrow{\mathbf{r}})$in displacing the particle from $\vec{\mathbf{r}}_i$to $\vec{\mathbf{r}}$and $\frac{\mathbf{mv^2}}{\mathbf{2}}$is defined asthe kinetic energy of the particle at the position at which its speed is**v**. The work-done by the force $\vec{\mathbf{F}}(\overrightarrow{\mathbf{r}})$in displacing the particle from $\vec{\mathbf{r}}_i$to $\vec{\mathbf{r}}$is measured in terms of change in kinetic energy of the article as $\frac{\mathbf{mv^2}}{\mathbf{2}} - \frac{\mathbf{mv_i}^2}{\mathbf{2}}$ where due to work-done the speed is changed from $\vec{\mathbf{v}}_i$to $\vec{\mathbf{v}}$, increase or decrease as the case may be.

IV. Conclusions:

i. The dependence of most of the natural and developed forces in space restricts to know $\vec{\mathbf{v}}\mathbf{(t)}$ as a function of time.
ii. In attempt to integrate $\vec{\mathbf{F}}(\overrightarrow{\mathbf{r}})$ with respect to space, invites the concept of path integral of a vector function.
iii. The concept of work-done and kinetic energy was developed in scalar form as **Work-Energy Theorem**and it became an easy and powerful tool to solve the problems.

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