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Isomorphism in Kinematic Chains

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ABSTRACT :*The present work deals with the problem of detection of isomorphism which is frequently encountered in structural synthesis of kinematic chains. A new method based on theoretic approach, easy to compute and reliable is suggested in this paper. It is capable of detecting isomorphism in all types of planer kinematic chains.*

I. INTRODUCTION

Over the past several years much work has been reported in the literature on the structural analysis and synthesis of mechanisms [1-29]. Motives behind these studies range from the desire for an orderly classification system, to studies of mechanism degree of freedom to the hope of identifying the mechanisms. Methods for the recognition and identification of a given mechanism's kinematic structure can be divided into two main categories, graphical methods based on the visual inspection of various forms of simplified systematic diagrams and numerical methods many of which are based on the theory of graphs. However, even with the current numerical methods no efficient generalized method is known which will characterize the given mechanisms by inspection.

In this paper , a new method to detect isomorphism in mechanisms kinematic chains is presented by comparing the sum of absolute polynomial coefficient values [JJMP∑] and maximum absolute polynomial coefficient [JJMPmax] of Joint-Joint [JJM]matrix.

II. PROPOSEDTEST-BASIS

The kinematic chains are complex chains of combination of binary, ternary and other higher order links. These links are joined together by simple pin joints. It is the assembly of link/pair combination to form one or more closed circuits. While considering structural equivalence it is essential to consider type of links/joints and layout of the links in the assembly. An identification number is assigned to links. Thus a binary link has a value of two, ternary three, quarter nary four and so on. Link values are used to assigned values to [JJM] matrix and these are utilized to identify layout of the kinematicchains.

For detecting isomorphism in kinematic chains a [JJM] matrix is determined and characteristic polynomials [JJMP], composite structural invariants [JJMP∑] and [JJMPmax] of [JJM] matrix are compared.

III. THE JOINT-JOINT [JJM]MATRIX.

One possible symbolism for the topology or connectivity of kinematic chain is the link-link form of the incidence matrix, more properly referred to as the Joint-Joint [JJM] matrix. Once the joints of the chain have been numbered from 1 to n, the [JJM] matrix is defined as a square matrix of the order n, each row and each column representing the joint with the corresponding number. The elements of the matrix are then entered as either zero or type of the link, depending on the absence or presence of a direct kinematic connection between the joints corresponding to that row and column.

J_{ij} == Degree of link or type of link, if joint i is directly connected tojoint

=0, if joint i is not directly connected to joint j

------(1)

IV. STRUCTURAL INVARIANTS [JJMP∑] and[JJMPmax]

Polynomial coefficient values are the characteristic invariants for the chains and mechanisms. Many investigators have reported co-spectral graph (non-isomorphic graph having same eigen spectrum). But these eigen spectra (eigen values or polynomial coefficient values) have been determined from (0, 1) adjacency matrices. The proposed [JJM] matrix provides same set of polynomials of the co-spectral kinematic graph. To make this [JJM] matrix polynomial spectrum as a powerful single number characteristic index, new composite invariants are proposed. These indices are [JJMP∑] and [JJMPmax] of [JJM] matrix. The polynomial values of [JJM] matrix are obtained using MATLAB. It is hoped that these invariants are capable of characterizing all kinematic chains and mechanisms uniquely.

V. ISOMORPHISM

Two basic kinematic chains will be isomorphic if and only if, both the composite invariants **[**JJMP∑] and [JJMPmax] are identical respectively. If any set of structural invariants of one matrix of a chain does not match the corresponding set of structural invariant of the other chain, the two chains will be non-isomorphic. In other words, the two basic kinematic chains will be isomorphic if and only if the characteristic polynomial equations are identical other wise non-isomorphic.

Theorem: Two kinematic chains are isomorphic to each other, have identical characteristic polynomials for their associated [JJM]matrices.

VI. APPLICATIONS

EXAMPLE 1: The first example concerns three kinematic chains with 12 bars, 16 joints, one degree of freedom as shown in Fig.1, 2 and 3. The task is to examine whether these three chains are isomorphic. The [JJM] matrices for these chains are represented by J1, J2 and J3respectively.

The characteristic polynomial for chain 1 = 0.0000, 0.0000, -0.0000, -0.0000 0.0000 , 0.0000 , -0.0001, -0.0004 , 0.0016 , 0.0109 , -0.0056, -0.1239 , -0.1243, 0.4736, 0.7954 , -0.4396 , -1.0219

The characteristic polynomial for chain 2 = 0.0000 , 0.0000, -0.0000, -0.0000 , 0.0000 , 0.0000, -0.0001, -0.0004 , 0.0016, 0.0109 , -0.0056 , -0.1239, -0.1243 , 0.4736, 0.7954, -0.4396 , -1.0219,

The characteristic polynomialfor chain 3= 0.0000, 0.0000, -0.0000, -0.0000, 0.0000, 0.0000 ,-0.0001, -0.0004 , 0.0016 , 0.0108, -0.0059, -0.1238, -0.1160 , 0.5008, 0.8031 , -0.5288 , -1.1287

The values of composite invariant

For chain 1: **[**JJMP∑] = 2.9971e+010 , [JJMPmax] =1.0219e+010 For chain 2: **[**JJMP∑] = 2.9971e+010, [JJMPmax] =1.0219e+010 For chain 3: **[**JJMP∑] = 3.2201e+010, [JJMPmax]=1.1287e+010

Our method reports that chain 1 and 2 are isomorphic as the set of values of [JJMP∑] and [JJMPmax] are same for both the kinematic chain 1 and 2 respectively. Similarly, our method reports that kinemaic chains 1 and 3 are non-isomorphic as the values of structural invariants [JJMP∑] and [JJMPmax] are different for kinematic chain 1 and 3 respectively. Note that by using another method eigen vector [19] and artificial neural network [24], the same conclusion isobtained.

EXAMPLE 2 (Multidegree freedom chains): The second example concerns two kinemtic chains with 10 bars, 12 joints, three degree of freedom as shown in Fig.4 and Fig.5. The task is to examine whether these two chains are isomorphic. The [JJM] matrices for these chains are represented by J4 and J5respectively.

The characteristic polynomial for chain 4 = 0.0000, -0.0000, -0.0001, -0.0002, 0.0063, 0.0170, -0.1055, -0.3730, 0.4376, 2.1856, 0.0372, -3.5938, -1.6171

The characteristic polynomial for chain 5 = poly(J6) = 0.0000 , 0 , -0.0001 , -0.0002 , 0.0058 , 0.0161, -0.0965, -0.3525 , 0.4307 , 2.2696 , 0.1572 , -2.9393 , 0.7465.

The values of composite invariant

For chain 4: **[**JJMP∑]=8.3734e+006 , [JJMPmax] =3.5938e+006 For chain 5: **[**JJMP∑]=7.0147e+006 , [JJMPmax] =2.9393e+006

Our method reports that chain 4 and 5 are non-isomorphic as the set of values of [JJMP∑] and [JJMPmax] are different for both the kinematic chains. Note that by using other method summation polynomials [25], the same conclusion is obtained.

EXAMPLE 3: The third example concerns another example of two kinematic chains with 10 bars, 13 joints, single freedom as shown in Fig.6 and Fig.7. The task is to examine whether these two chains are isomorphic. The [JJM] matrices for these chains are represented by J6 and J7 respectively.

The characteristic polynomial for chain 6 = 0.0000 , 0 , -0.0000 , -0.0000, 0.0012 , 0.0026, -0.0324, -0.0947 , 0.2944 , 1.0537, -0.6359, -3.5648, -1.3812 , 0.4952.

The characteristic polynomial for chain 7 = 0.0000 , -0.0000 , -0.0000 , -0.0000 , 0.0012, 0.0026 , -0.0324, -0.0947, 0.2944, 1.0537 , -0.6359, -3.5648, -1.3812 , 0.4952.

The values of composite invariant

For chain 6: **[**JJMP∑] = 7.5562e+007, [JJMPmax] = 3.5648e+007 For chain 7: **[**JJMP∑]=7.5562e+007 , [JJMPmax] =3.5648e+007

Our method reports that chain 6 and 7 are isomorphic as the set of values of [JJMP∑] and [JJMPmax] are same for both the kinematic chains. Note that by using another method artificial neural network [24], the same conclusion is obtained.

EXAMPLE 4: The fourth example concerns two kinematic chains with 6 bars, 7 joints, one freedom as shown in Fig.8 and Fig.9. The task is to examine whether these two chains are isomorphic. The [JJM] matrices for these chains are represented by J8 and J9respectively.
The characteristic polynomial for chain 8 = 0.0010, -0.0000, -0.0700, -0.1080, 0.9010, 0.9720, -3.1680, 0.0000
The characteristic polynomial for chain 9 = 0.0010 , -0.0000 , -0.0670, -0.0990 , 1.0640 , 2.2470 , -2.1500, -3.3000
The values of composite invariant
For chain 8:**[**JJMP∑]= 5.2200e+003 , [JJMPmax] =3.1680e+003 For chain 9:**[**JJMP∑]= 8.9280e+003 , [JJMPmax] =3.3000e+003
Our method reports that chain 8 and 9 are non-isomorphic as the set of values of [JJMP∑] and [JJMPmax] are different for both the kinematic chains. Note that by using another method s distance matrix [2], and summation polynomials [25], the same conclusion is obtained.

VII. CONCLUSION

In this paper, a simple, efficient and reliable method to identify isomorphism is proposed. By this method, the isomorphism of mechanisms kinematic chains can easily be identified. It incorporates all features of the chain and as such violation of the isomorphism test is rather difficult. In this method, the characteristic polynomials, composite structural invariants [JJMP∑] and [JJMPmax] of [JJM] matrix of the kinematic chain. The advantage is that they are very easy to compute using MATLAB software. It is not essential to determine both the composite invariants to compare two chains, only in case the [JJMP∑] is same then it is needed to determine [JJMPmax]for both kinematic chains. The [JJM] matrices can be written with very little effort, even by mere inspection of the chain. The proposed test is quite general in nature and can be used to detect isomorphism of not only planar kinematic chains of one degree of freedom, but also kinematic chains of multi degree of freedom. The characteristic polynomials and composite structural invariants are very informative and from them valuable information regarding topology of kinematic chains can be predicted. According to this method, we can find that the [JJM] matrix is a map of mechanism kinematic chains and characteristic polynomials and other characteristic invariants may reflect some nature and inner property of the mechanism. The inner relation between characteristic value and mechanism kinematic chain need further study.

$$J1=\begin{pmatrix}
0&3&3&2&0&0&0&0&0&0&0&0&0&0&0&0\\
3&0&3&0&0&0&0&2&0&0&0&0&0&0&0&0\\
3&3&0&0&0&0&0&0&0&0&0&0&0&0&0&2\\
2&0&0&0&3&3&0&0&0&0&0&0&0&0&0&0\\
0&0&0&3&0&3&0&0&0&0&0&0&0&3&3&0\\
0&0&0&3&3&0&3&0&0&3&0&0&0&0&0&0\\
0&0&0&0&0&3&0&3&3&3&0&0&0&0&0&0\\
0&2&0&0&0&0&3&0&3&0&0&0&0&0&0&0\\
0&0&0&0&0&0&3&3&0&0&0&3&3&0&0&0\\
0&0&0&0&0&3&3&0&0&0&3&3&0&0&0&0\\
0&0&0&0&0&0&0&0&0&3&0&3&0&2&0&0\\
0&0&0&0&0&0&0&0&3&3&3&0&3&0&0&0\\
0&0&0&0&0&0&0&0&3&0&0&3&0&0&3&3\\
0&0&0&0&3&0&0&0&0&0&2&0&0&0&3&0\\
0&0&0&0&3&0&0&0&0&0&0&0&3&3&0&3\\
0&0&2&0&0&0&0&0&0&0&0&0&3&0&3&0
\end{pmatrix}
\qquad
J2=\begin{pmatrix}
0&3&3&2&0&0&0&0&0&0&0&0&0&0&0&0\\
3&0&3&0&0&0&0&2&0&0&0&0&0&0&0&0\\
3&3&0&0&0&0&0&0&0&0&0&0&0&0&0&2\\
2&0&0&0&3&3&0&0&0&0&0&0&0&0&0&0\\
0&0&0&3&0&3&0&0&0&0&0&0&0&3&3&0\\
0&0&0&3&3&0&3&0&0&3&0&0&0&0&0&0\\
0&0&0&0&0&3&0&3&3&3&0&0&0&0&0&0\\
0&2&0&0&0&0&3&0&3&0&0&0&0&0&0&0\\
0&0&0&0&0&0&3&3&0&0&0&3&3&0&0&0\\
0&0&0&0&0&3&3&0&0&0&3&3&0&0&0&0\\
0&0&0&0&0&0&0&0&0&3&0&3&0&2&0&0\\
0&0&0&0&0&0&0&0&3&3&3&0&3&0&0&0\\
0&0&0&0&0&0&0&0&3&0&0&3&0&0&3&3\\
0&0&0&0&3&0&0&0&0&0&2&0&0&0&3&0\\
0&0&0&0&3&0&0&0&0&0&0&0&3&3&0&3\\
0&0&2&0&0&0&0&0&0&0&0&0&3&0&3&0
\end{pmatrix}$$

$$J3=\begin{pmatrix}
0&3&3&2&0&0&0&0&0&0&0&0&0&0&0&0\\
3&0&3&0&0&0&0&3&0&0&0&0&0&3&0&0\\
3&3&0&0&0&0&0&0&0&0&0&0&0&0&0&2\\
2&0&0&0&3&3&0&0&0&0&0&0&0&0&0&0\\
0&0&0&3&0&3&0&0&0&0&0&0&0&0&2&0\\
0&0&0&3&3&0&3&0&0&3&0&0&0&0&0&0\\
0&0&0&0&0&3&0&3&3&3&0&0&0&0&0&0\\
0&3&0&0&0&0&3&0&3&0&0&0&0&3&0&0\\
0&0&0&0&0&0&3&3&0&0&0&3&3&0&0&0\\
0&0&0&0&0&3&3&0&0&0&3&3&0&0&0&0\\
0&0&0&0&0&0&0&0&0&3&0&3&0&2&0&0\\
0&0&0&0&0&0&0&0&3&3&3&0&3&0&0&0\\
0&0&0&0&0&0&0&0&3&0&0&3&0&0&3&3\\
0&3&0&0&0&0&0&3&0&0&2&0&0&0&0&0\\
0&0&0&0&2&0&0&0&0&0&0&0&3&0&0&3\\
0&0&2&0&0&0&0&0&0&0&0&0&3&0&3&0
\end{pmatrix}$$

$$J4=\begin{pmatrix}
0&2&0&0&0&0&0&3&0&3&0&0\\
2&0&2&0&0&0&0&0&0&0&0&0\\
0&2&0&2&0&0&0&0&0&0&0&0\\
0&0&2&0&3&0&0&0&0&0&3&0\\
0&0&0&3&0&3&0&0&0&0&3&3\\
0&0&0&0&3&0&3&0&0&0&0&3\\
0&0&0&0&0&3&0&3&3&0&0&0\\
3&0&0&0&0&0&3&0&3&3&0&0\\
0&0&0&0&0&0&3&3&0&0&0&2\\
3&0&0&0&0&0&0&3&0&0&2&0\\
0&0&0&3&3&0&0&0&0&2&0&0\\
0&0&0&0&3&3&0&0&2&0&0&0
\end{pmatrix}$$

$$J5=\begin{pmatrix}
0&2&0&0&0&0&0&3&3&0&0&0\\
2&0&2&0&0&0&0&0&0&0&0&0\\
0&2&0&3&0&0&0&0&0&0&0&3\\
0&0&3&0&2&0&0&0&0&0&0&3\\
0&0&0&2&0&3&0&0&0&0&3&0\\
0&0&0&0&3&0&2&0&0&0&3&0\\
0&0&0&0&0&2&0&2&0&0&0&0\\
3&0&0&0&0&0&2&0&3&0&0&0\\
3&0&0&0&0&0&0&3&0&2&0&0\\
0&0&0&0&0&0&0&0&2&0&3&3\\
0&0&0&0&3&3&0&0&0&3&0&3\\
0&0&3&3&0&0&0&0&0&3&3&0
\end{pmatrix}$$

$$J6=\begin{pmatrix}
0&4&0&0&0&0&2&0&4&0&0&0&0\\
4&0&2&0&0&0&0&0&0&4&0&0&0\\
0&2&0&3&0&0&0&0&0&0&0&0&3\\
0&0&3&0&3&0&0&0&0&0&0&3&3\\
0&0&0&3&0&2&0&0&0&0&0&3&0\\
0&0&0&0&2&0&3&3&0&0&0&0&0\\
2&0&0&0&0&3&0&3&0&0&0&0&0\\
0&0&0&0&0&3&3&0&2&0&0&0&0\\
4&0&0&0&0&0&0&2&0&4&0&0&0\\
0&4&0&0&0&0&0&0&4&0&3&0&3\\
0&0&0&0&0&0&0&0&0&3&0&2&3\\
0&0&0&3&3&0&0&0&0&0&2&0&0\\
0&0&3&3&0&0&0&0&0&3&3&0&0
\end{pmatrix}$$

$$J7=\begin{pmatrix}
0&4&0&0&0&0&2&0&4&0&0&0&0\\
4&0&3&0&0&0&0&0&0&4&0&0&3\\
0&3&0&2&0&0&0&0&0&0&0&0&3\\
0&0&2&0&3&0&0&0&0&0&0&3&0\\
0&0&0&3&0&2&0&0&0&0&0&3&0\\
0&0&0&0&2&0&3&3&0&0&0&0&0\\
2&0&0&0&0&3&0&3&0&0&0&0&0\\
0&0&0&0&0&3&3&0&2&0&0&0&0\\
4&0&0&0&0&0&0&2&0&4&0&0&0\\
0&4&0&0&0&0&0&0&4&0&2&0&0\\
0&0&0&0&0&0&0&0&0&2&0&3&3\\
0&0&0&3&3&0&0&0&0&0&3&0&3\\
0&3&3&0&0&0&0&0&0&0&3&3&0
\end{pmatrix}$$

$$J8=\begin{pmatrix} 0&2&0&0&0&3&3\\ 2&0&2&0&0&0&0\\ 0&2&0&3&0&0&3\\ 0&0&3&0&2&0&3\\ 0&0&0&2&0&2&0\\ 3&0&0&0&2&0&3\\ 3&0&3&3&0&3&0 \end{pmatrix} \qquad J9=\begin{pmatrix} 0&2&0&0&3&0&3\\ 2&0&3&0&0&3&0\\ 0&3&0&2&0&2&0\\ 0&0&2&0&2&0&0\\ 3&0&0&2&0&0&3\\ 0&3&3&0&0&0&2\\ 3&0&0&0&3&2&0 \end{pmatrix}$$

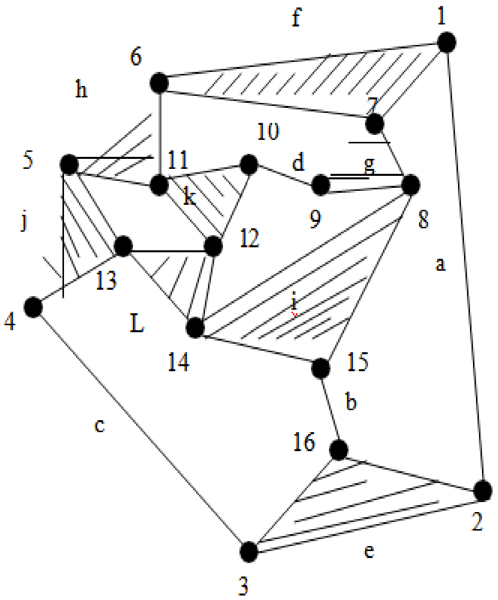

Fig.1: Twelve bar chain,singlefreedom

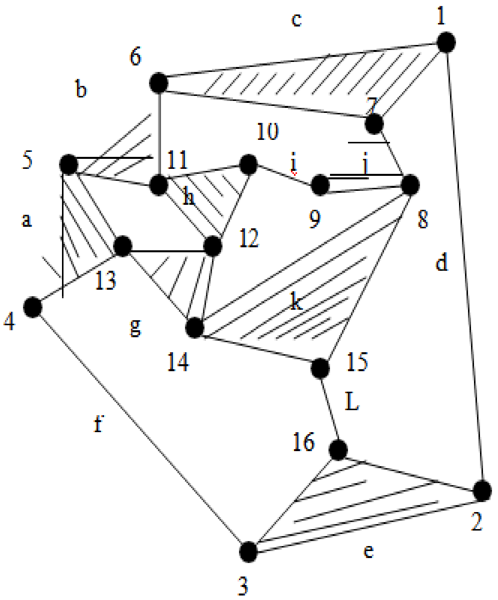

Fig.2: Twelve bar chain, singlefreedom

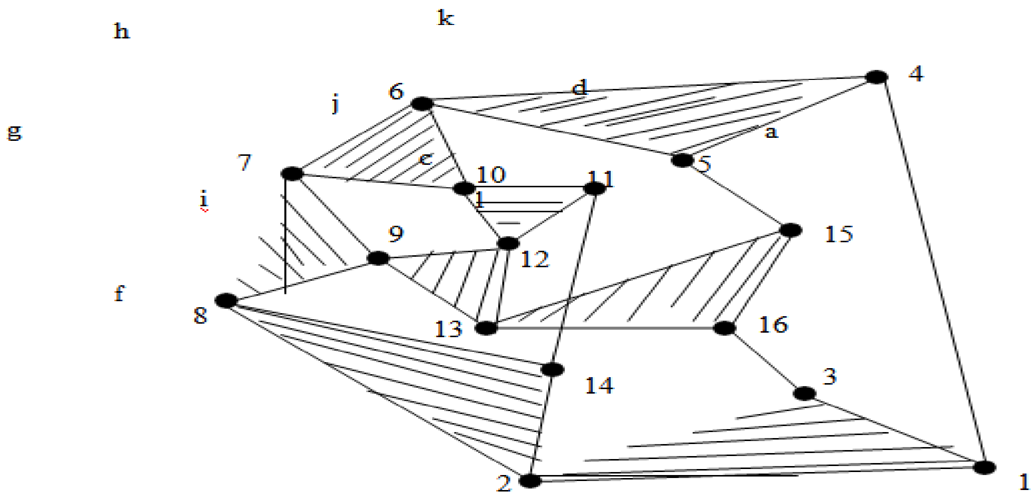

Fig.3: Twelve bar chain, single freedom

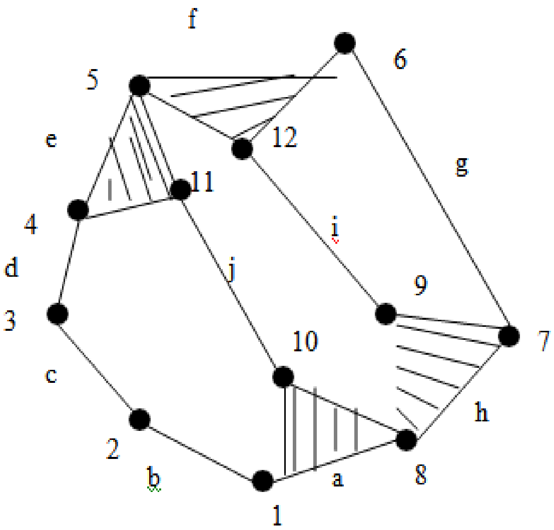

Fig.4: Ten bar chain,threefreedom

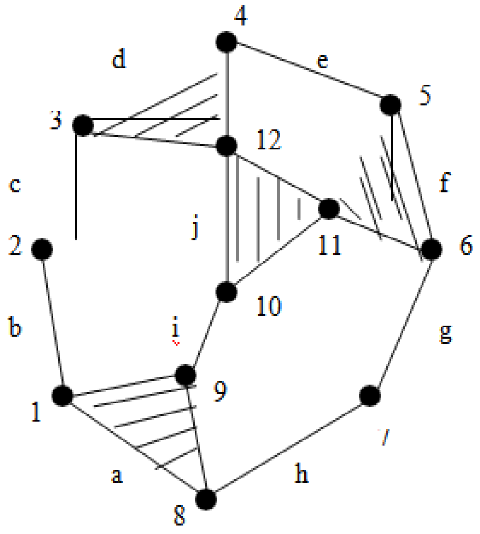

Fig.5: Ten bar chain, threefreedom

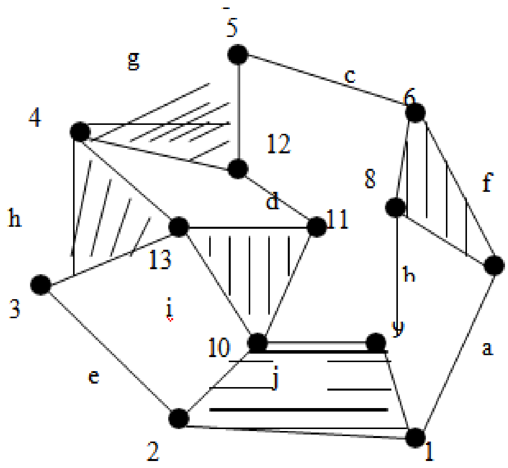

Fig.6: Ten bar chain, single freedom

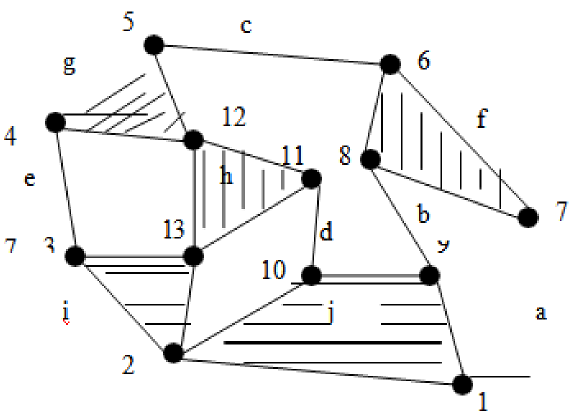

Fig.7: Ten bar chain, single freedom

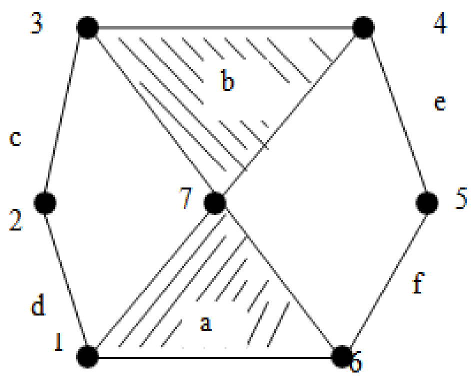

Fig.8: Six bar chain,singlefreedom

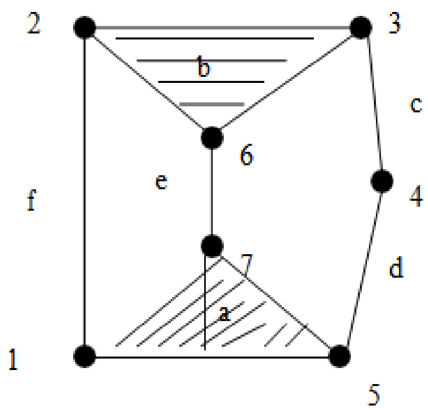

Fig.9: Six bar chain, single freedom

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