

## Supercritical Evaporation of a Drop

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**ABSTRACT:** *The problem of the relaxation of a cold package of critical or supercritical fluid in a hotter environment of the same fluid is studied. An asymptotic theory valid in the limit of small values of the parameter  $\tau = \frac{\gamma_\infty}{\epsilon}$ , where  $\gamma_\infty$  is the ratio between the characteristic thermal diffusion time and the life time of the droplet and  $\epsilon$  the ratio between the fluid densities at the hot and cold regions is developed. Recession laws which are different from the classical  $d^2$  law can be derived from the zeroth order approximation solution in subcritical case [6] as well in critical and supercritical ones. Except for the critical case, additional assumptions on the thermodynamical properties of the gas phase restore the classical  $d^2$  law. A numerical resolution of the problem for a Van der Waals gas in supercritical conditions is performed to check the validity of the results of the asymptotic analysis. It is found that a transition region generally separate the two regions of the fluid in the supercritical conditions. The behavior of this region is numerically analyzed.*

**Keywords:** *Drop evaporation, Mass flow, Supercritical, Transition layer*

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### I. INTRODUCTION

Gas-liquid multiphase flows of purely fundamental or highly industrial interest depend on drops evaporation and condensation in laminar or turbulent environments at high or low temperatures. The drops evaporation and combustion cover a wide domain of applications from aircraft propulsion to the fight against fires and the protection of the environment. It is first acted to understand the process of evaporation and combustion of sprays as it is in this form that the drops are involved in different engines and industrial furnaces. But it involves the consideration and the description of the motion of a large number of drops in interaction with themselves and with the particles of the surrounding environment [1]. The mathematical modeling of such a situation is complex because of the nonlinear coupling of mass, momentum and energy transfers, so a major effort was undertaken to improve the understanding of evaporation and combustion processes of an isolated drop of fluid in order to obtain informations for the modeling of the entire flow.

The standard model in the study of drop evaporation and combustion was developed in the 1950s. Supercritical and subcritical cases (high pressure and low pressure) were separated from the start and gave rise to two distinct theories that are comparable, however, particularly with regard to the assumptions of constancy and uniformity of the thermodynamic coefficients and variables in the droplet, the absence of convection and the quasi-steadiness of the gas phase. The first results were about subcritical regime and are due to Spalding [2, 3] and Godsave [4] who established the quasi-steady theory and proved that the square of the diameter of a drop in combustion, assumed to be spherical, is a linear function of time. Spalding has also developed a similar asymptotic theory in the supercritical case. Despite the criticism they underwent especially after experimental results have invalidated some of the assumptions on which they are based, the results of these theories remain valid and are still widely used.

Numerous studies have been performed to test the validity of these assumptions and to examine their influence on the results [5]. The objective of this work is to extend to the critical and supercritical cases, the results obtained in our previous work in the subcritical case [6]. Pure fluids have remarkable properties when they are close to their critical point. Hypercompressible and very expandable while being very dense, they are the seat of a particular phenomenology that increases usually marginal effects and reduces others predominant when they are away from the critical point. Critical anomalies (divergence or cancellation of transport coefficients) make complicated the study of critical fluids.

A well-defined physical interface separates the liquid and gas phases when evaporation occurs in subcritical conditions while the concepts of latent heat and surface tension are meaningless in critical and supercritical conditions. Supercritical evaporation is a phenomenon of relaxation in a single phase. However, many experimental and numerical data suggest that at moderate supercritical pressures still exist two distinct fluid regions (reminiscent of the liquid and gaseous phases of the subcritical evaporation) separated by a thin region of transition comparable to the liquid-gas interface of the subcritical case but having a non-zero thickness [7]. In an isobaric process density gradients are very important. Thermal diffusion is effective only in a very thin layer for fluids near their critical point and huge density gradients occur leading to situations similar to that of

the subcritical evaporation: the initial density inhomogeneity due to the pocket can be assimilated to a droplet when the surrounding atmosphere is hotter [7, 8, 9].

The assumption of quasi-steadiness of the gaseous environment surrounding the droplet is essential for the establishment of the  $d^2$  law. The validity of this hypothesis which seems to be limited to the case of low pressure where the density of the liquid is at least three times higher than the density of the gas [13] persists in the critical and supercritical evaporation at very high pressures when the densities of the cold and hot fluids are comparable [10, 11].

The rest of the paper is organized as follows. In Section 2, using the formalism developed in [6], we show that the  $d^2$  law remains valid under certain conditions in the supercritical evaporation while it could not be established for the critical evaporation. In section 3, we study numerically the behavior of a supercritical fluid pocket in a hotter environment of the same fluid. Particular emphasis was placed on the description and analysis of the transition zone.

## II. RECESSION LAW IN CRITICAL AND SUPERCRITICAL EVAPORATION

### 1. Position of the problem

A pocket of critical or supercritical fluid at a uniform temperature is suddenly introduced into a hotter infinite atmosphere of the same fluid initially at rest. The physical model adopted assumes the spherical symmetry of the pocket and the flow, the absence of viscosity and gravity and a uniform pressure in the flow domain. We denote by  $r'$  the distance to the center of the pocket and  $R'$  its radius. A law of state which will be specified in section 3 gives the pressure  $P'$  in the flow as a function of the density  $\rho'$  and temperature  $T'$ ;  $v'$  is the velocity of the flow. The radius, the density and temperature of the cold pocket at the initial time are respectively  $R'(0) = R'_{in}$ ,  $\rho'(0) = \rho'_{in}$  and  $T'(0) = T'_{in}$ . The conservation equations of the model are written in a reference frame tied to the border of the pocket and we introduce the variable  $x' = r' - R'$ .

We retain as in [6] two characteristic times in the description of the relaxation: the characteristic time of thermal diffusion  $t'_{dif}$  and the pocket life time  $t'_{res}$ . To take into account the gradient of density in the environment, we use two characteristic densities:  $\rho'_l$  in the pocket and  $\rho'_\infty$  in the surrounding gas. We introduce the dimensionless numbers  $\gamma_\infty = \frac{t'_{dif}}{t'_{res}}$ ,  $\epsilon = \frac{\rho'_\infty}{\rho'_l}$ ,  $Pe = \frac{Cp'_\infty R'_{in} \rho'_\infty}{\lambda'_\infty t'_{dif}}$  and the following dimensionless variables :  $t = \frac{t'}{t'_{res}}$ ,  $r_0 = \frac{R'}{R'_{in}}$ ,  $x = \frac{x'}{R'_{in}}$ ,  $v = \frac{v't'_{dif}}{R'_{in}}$ ,  $\dot{r}_0 = \frac{\dot{R}'t'_{res}}{R'_{in}}$ ,  $u = v - \gamma_\infty \dot{r}_0$ ,  $T = \frac{T'}{T'_{in}}$ ,  $\tilde{\rho} = \frac{\rho'}{\rho_\infty}$  et  $\bar{\rho} = \frac{\rho'}{\rho'_l}$ .

The dimensionless equations of the problem as in [6] take the form:

$$\begin{cases} \frac{\gamma_\infty}{\epsilon} \left( \frac{\partial \bar{\rho}}{\partial t} + \frac{2\bar{\rho}\dot{r}_0}{x+r_0} + \dot{r}_0 \frac{\partial \bar{\rho}}{\partial x} \right) + \frac{1}{(x+r_0)^2} \frac{\partial [(x+r_0)^2 \tilde{\rho} u]}{\partial x} = 0 \\ \frac{\gamma_\infty}{\epsilon} \bar{\rho} \left( \frac{\partial u}{\partial x} \dot{r}_0 + \gamma_\infty \ddot{r}_0 \right) + \tilde{\rho} \left( \gamma_\infty \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \right) = 0 \\ \frac{\gamma_\infty}{\epsilon} \left( \frac{\partial T}{\partial t} + \dot{r}_0 \frac{\partial T}{\partial x} \right) + u \frac{\partial T}{\partial x} = \frac{1}{Pe \tilde{\rho} C_p (x+r_0)^2} \frac{\partial}{\partial x} \left[ (x+r_0)^2 \lambda \frac{\partial T}{\partial x} \right] \end{cases} \quad (1)$$

With the initial conditions :

$$T(0, x) = \begin{cases} T_{in}(x), & x \leq 0 \\ T_\infty(x), & x > 0 \end{cases} \quad (2)$$

$$\rho(0, x) = \begin{cases} \rho_{in}(x), & x \leq 0 \\ \rho_\infty(x), & x > 0 \end{cases} \quad (3)$$

$$u(0, x) = \begin{cases} u_{in}(x), & x \leq 0 \\ u_\infty(x), & x > 0 \end{cases} \quad (4)$$

The boundary conditions are :

$$T(t, x) = T_l(t, x), \quad x \leq 0 \quad (5)$$

$$\lim_{x \rightarrow +\infty} T(t, x) = T_\infty(t) \quad (6)$$

$$\rho(t, x) = \rho_l(t, x), \quad x \leq 0 \quad (7)$$

$$\lim_{x \rightarrow +\infty} \rho(t, x) = \rho_\infty(t) \tag{8}$$

$$u(t, x) = u(t, x), \quad x \leq 0 \tag{9}$$

$$\lim_{x \rightarrow +\infty} u(t, x) = u_\infty(t) \tag{10}$$

And the jump relations are:

$$\rho_l u_l = \rho_g u_g \tag{11}$$

$$\lambda_l \left(\frac{\partial T}{\partial x}\right)_l + P_e \rho_l u_l L_0 = \lambda_g \left(\frac{\partial T}{\partial x}\right)_g, \quad L_0 = \frac{h'_l - h'_g}{Cp'_\infty T'_\infty} \tag{12}$$

$h'$  is the total enthalpy corresponding to the relative motion.

The formalism developed in [6] applies and we obtain in first approximation, the quasi steady equations of the model:

$$\left\{ \begin{aligned} & \frac{1}{(x + r_0)^2} \frac{\partial [(x + r_0)^2 \rho_0 u_0]}{\partial x} = 0 \\ & \rho_0 \left( \gamma_\infty \frac{\partial u_0}{\partial t} + u_0 \frac{\partial u_0}{\partial x} \right) = 0 \\ & u_0 \frac{\partial T_0}{\partial x} = \frac{1}{P_e \rho_0 Cp_0 (x + r_0)^2} \frac{\partial}{\partial x} \left[ (x + r_0)^2 \lambda_0 \frac{\partial T_0}{\partial x} \right], \quad \rho_0 = \tilde{\rho}_0 \end{aligned} \right. \tag{13}$$

**2. The recession laws**

The model equations have been established without the need to clarify the law of state. The developed formalism is valid in both subcritical and critical or supercritical regimes taking into account the changes that the critical phenomena introduce in the thermodynamic coefficients.

In critical and supercritical regimes, the thermal conductivity can be written [7, 10] :

$$\lambda'_0 = \lambda_0^{gas} + \lambda_0^C + \lambda_0^E \tag{14}$$

With :

$$\lambda^{gas} = \lambda'_\infty \frac{T'}{T_\infty} \text{ or } \lambda'_\infty \sqrt{\frac{T'}{T_\infty}}, \quad \lambda^C = \lambda'_{in} \left| \frac{T' - T'_c}{T'_c} \right|^{-1/3}, \quad \lambda^E = \lambda'_1 \frac{\rho'}{\rho'_c}$$

The first two terms depend only on the temperature while the latter depends only on the density. When the pressure is constant,  $\rho'$  depends only on the temperature. The heat capacity at constant pressure  $C'_p$  depends also on the temperature. Experiments and numerical results [7, 8, 9] suggest the existence of two distinct regions in the flow at critical and moderate supercritical conditions and the existence of a sphere of radius depending on the initial and boundary conditions out of which the macroscopic variables of the flow do not depend on time.

This ensures the non vanishing of the latent heat  $L_0$  and the quasi steady evolution of the flow to the vanishing of the cold pocket after an unsteady transitional phase.

When  $\lambda_0$  and  $Cp_0$  depend only on the temperature, the third relation of the equations (13) can be developed as:

$$\frac{\partial^2 T}{\partial x^2} + a \frac{\partial T}{\partial x} + b \left(\frac{\partial T}{\partial x}\right)^2 = 0, \quad a = \frac{2}{x + r_0} - \frac{P_0 Cp_0 f(t)}{\lambda_0 (x + r_0)^2}, \quad b = \frac{1}{\lambda_0} \frac{\partial \lambda_0}{\partial T} \tag{15}$$

We infer from this :

$$\frac{\partial T}{\partial x} = \frac{\exp\left[-\int_0^x a(t, v) dv\right]}{k(t) + J(t, x)}, \quad J(t, x) = \int_0^x b(t, w) \exp\left[-\int_0^x a(t, v) dv\right] dw \tag{16}$$

This gives by using the jump relation on the energy conservation

$$k(t)r_0 = -\frac{\epsilon \lambda_0^0}{\gamma_\infty L_0}, \quad \lambda_0^0(t) = \lim_{x \rightarrow 0^+} \lambda_0 \tag{17}$$

By integrating equation (16) we obtain

$$\frac{k(t)}{\lambda_0^0(t)} = \frac{J(t, x)}{\lambda_0(T) - \lambda_0^0(t)} \tag{18}$$

We infer from this by making  $x$  tend towards  $+\infty$

$$k(t) = \frac{\lambda_0^0 J(t)}{\lambda_0^\infty - \lambda_0^0}, \quad \lambda_0^\infty = \lim_{x \rightarrow +\infty} \lambda_0 [T(t, x)], \quad J(t) = \lim_{x \rightarrow +\infty} J(t, x) \tag{19}$$

In the particular case where  $b$  and  $\frac{Cp_0}{\lambda_0}$  do not depend explicitly on  $x$ , we obtain :

$$J(t) = \frac{r_0^2}{\alpha \lambda_0^\infty} \left( \frac{\partial \lambda_0}{\partial T} \right)_\infty \left[ \exp \left( \frac{\alpha}{r_0} \right) - 1 \right], \quad \alpha = \frac{P_e f(t) C p_0^\infty}{\lambda_0^\infty}, \quad \left( \frac{\partial \lambda_0}{\partial T} \right)_\infty = \lim_{x \rightarrow +\infty} \frac{\partial \lambda_0}{\partial T} \quad (20)$$

Returning to dimensional variables, we have:  $\frac{\alpha}{r_0} = \frac{u'_0 r'_0}{K'}$ ,  $K'$  being the thermal diffusivity. Hence we have:

- Near the critical point,  $K'$  tends towards zero, so  $\frac{\alpha}{r_0}$  tends towards infinity. The recession law of the pocket radius is given by the implicit relation :

$$\int_1^{r_0} \frac{1}{\alpha} \left[ \exp \left( \frac{\alpha}{s} \right) - 1 \right] s^2 ds = \frac{\epsilon}{P_e \gamma_\infty} \int_0^t \frac{\lambda_0^\infty (\lambda_0^\infty - \lambda_0^0)}{L_0 \left( \frac{\partial \lambda_0}{\partial T} \right)_\infty} ds \quad (21)$$

which obviously is not a polynomial law.

- In supercritical regime,  $K'$  is non zero and as  $\frac{\alpha}{r_0}$  is small near the critical point, we can make the approximation  $\exp \left( \frac{\alpha}{r_0} \right) - 1 \cong \frac{\alpha}{r_0}$  and obtain

$$r_0^2 = 1 - \frac{2\epsilon}{P_e \gamma_\infty} \int_0^t \frac{\lambda_0^\infty (\lambda_0^\infty - \lambda_0^0)}{L_0 \left( \frac{\partial \lambda_0}{\partial T} \right)_\infty} ds \quad (22)$$

In the case where the integrand  $\frac{\lambda_0^\infty (\lambda_0^\infty - \lambda_0^0)}{L_0 \left( \frac{\partial \lambda_0}{\partial T} \right)_\infty}$  is constant, we obtain the classical  $d^2$  law.

The formalism we have used doesn't allow establishing the classical recession law of  $d^2$  near the critical point. Nevertheless, it confirms the validity of the  $d^2$  law in supercritical evaporation.

### III. NUMERICAL STUDY OF THE SUPERCRITICAL DROPLET EVAPORATION

The asymptotic analysis made in Section 2 allowed us to deduce the recession laws of the pocket radius of cold fluid in hotter environment of the same fluid. We now numerically solve the system of conservation equations to study the macroscopic variables of the flow and to test the validity of the assumptions used in the previous section. When we introduce a drop or a pocket of cold fluid in a hotter isobaric atmosphere at supercritical pressure of the same fluid, there may be several types of qualitative behavior depending on the initial conditions. If the drop is subcritical and the gaseous environment is supercritical initially, the drop passes through critical state during the evaporation process. If the droplet and its environment are supercritical, there is a relaxation phenomenon which never passes through the critical state. In our study, we consider only the second case to avoid taking into account critical divergency and complex laws of recession.

#### 1. Position of the problem

We consider a spherical pocket of carbon dioxide ( $CO_2$ ) of initial radius  $r'(0) = R'_{in}$ , initial density  $\rho'(0) = \rho'_{in}$  and initial uniform pressure  $P'(0) = P'_{in}$  suddenly introduced into a combustion chamber filled with the same hotter supercritical fluid but at the same constant pressure  $P'_{in}$  and at uniform density  $\rho'_\infty$ . The dimensions of the pocket and the chamber are chosen such that their ratio  $\frac{R'_{in}}{R'_{ch}}$  is  $10^{-2}$ ,  $R'_{ch}$  is the radius of the chamber, this gives a ratio  $10^{-6}$  for the initial volumes of spheres. It is therefore reasonable to assume that the effects of the disturbance caused by the presence of the pocket reach the walls of the room infinitesimally. We consider that the process is isobaric throughout its duration at pressure  $P'_{in}$ . The temperature  $T'_\infty$  of the wall of the room is kept constant. The ratio of initial densities  $\frac{\rho'_\infty}{\rho'_{in}}$  of the fluid far from the pocket and in the pocket is 0,6. We have to solve the system of the equations of mass, momentum and energy conservation, the equation of Van der Waals is the law of the state chosen. The system is written in spherical coordinates and we have:

$$\left\{ \begin{array}{l} \frac{\partial \rho'}{\partial t'} + \frac{1}{r'^2} \frac{\partial (r'^2 \rho' v')}{\partial r'} = 0 \\ \rho' \left( \frac{\partial v'}{\partial t'} + v' \frac{v'}{r'} \right) = - \frac{\partial P'}{\partial r'} \\ \rho' C'_v \left( \frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial r'} \right) = \frac{1}{r'^2} \frac{\partial}{\partial r'} \left( r'^2 \lambda' \frac{\partial T'}{\partial r'} \right) - (P' + a' \rho'^2) \left( \frac{2v'}{r'} + \frac{\partial v'}{\partial r'} \right) \\ P' = \frac{\rho' R' T'}{1 - b' \rho'} - a' \rho'^2 \end{array} \right. \quad (23)$$

The initial conditions on the density and temperature are given by:

$$\rho'(r', 0) = \begin{cases} \rho'_{in} & r' \leq R'_{in} \\ \rho'_{\infty} & r' > R'_{in} \end{cases} \quad (24)$$

$$T'(r', 0) = \begin{cases} T'_{in} & r' \leq R'_{in} \\ T'_{\infty} & r' > R'_{in} \end{cases} \quad (25)$$

The thermal conductivity is calculated as

$$\lambda'(T') = \lambda'_0 \left( \sqrt{\frac{T'}{T'_c}} + 0,078 \frac{\rho'}{\rho'_c} + 0,014 \left| \frac{T'}{T'_c} - 1 \right|^{-1/3} \right) \quad (26)$$

As in [11], the specific heat at constant volume  $c'_v$  is assumed constant and equal to the ideal gas specific heat. Similarly, the parameters  $a'$  and  $b'$  are calculated using experimental values of the critical temperature and pressure for  $CO_2$ . We put the macroscopic variables into dimensionless form by setting  $\rho = \frac{\rho'}{\rho'_c}$ ,  $T = \frac{T'}{T'_c}$ ,  $P = \frac{P'}{\rho'_c R T'_c}$ ,  $\rho'_c$  and  $T'_c$  are the critical values of the density and temperature of  $CO_2$ . We introduce the characteristic velocity  $U'$  of the flow and the Mach number  $Ma = \frac{U'}{C'_0}$ , where  $C'_0$  is the sound velocity calculated in the ideal gas at temperature  $T'_c$ . Then, the dimensionless time and velocity are given by  $t = \frac{U' t'}{R'_{in}}$  and  $v = \frac{v'}{c'_0}$ . It is assumed that the flow is a low Mach number flow. The formalism used is the same as in [11, 14]. An asymptotic expansion in terms of the parameter  $Ma$  allows having in first approximation the following system [14, 15]:

$$\left\{ \begin{array}{l} \frac{\partial \rho_0}{\partial t} + \frac{1}{r^2} \frac{\partial (r^2 \rho_0 v_0)}{\partial r} = 0 \\ \frac{\partial \rho_0}{\partial r} = 0 \\ \rho_0 \left( \frac{\partial v_0}{\partial t} + v_0 \frac{\partial v_0}{\partial r} \right) = -\frac{1}{\gamma} \frac{\partial P_1}{\partial r} \\ \rho_0 \left( \frac{\partial T_0}{\partial t} + v_0 \frac{\partial T_0}{\partial r} \right) = \frac{t'_a}{t'_{dif}} \frac{\gamma}{Ma} \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \lambda \frac{\partial T_0}{\partial r} \right) - (\gamma - 1) \left( P_0 + \frac{9}{8} \rho_0^2 \right) \left( \frac{2v_0}{r} + \frac{\partial v_0}{\partial r} \right) \\ P_0 = \frac{P_0 T_0}{1 - \frac{\rho_0}{3}} - \frac{9}{8} \rho_0^2 \end{array} \right. \quad (27)$$

This system is then solved by a finite volume code using the SIMPLER algorithm [12, 14].

## 2. The numerical results

The numerical computations have been performed for  $\epsilon = \frac{\rho'_{\infty}}{\rho'_{in}} = 0,6$  and  $\frac{T'_{\infty} - T'_{in}}{T'_{in}} = 10^{-5}$ . The initial configuration is slightly supercritical and we assume that the initial density of the drop is the critical density of  $CO_2$ . Hence we have  $\rho_{in} = 1$ .

### 2.1 Results for the temperature and the density

Figure 1 and figure 2 represent respectively the evolution of the temperature ( $Temp = \theta = \frac{T'}{T'_{\infty}}$ ) and the density ( $\rho = \frac{\rho'}{\rho'_{in}}$ ) of the flow at different times versus  $r$ , the ratio of the distance to the center of the droplet and the initial radius of the droplet ( $\frac{r'}{R'_{in}}$ ).

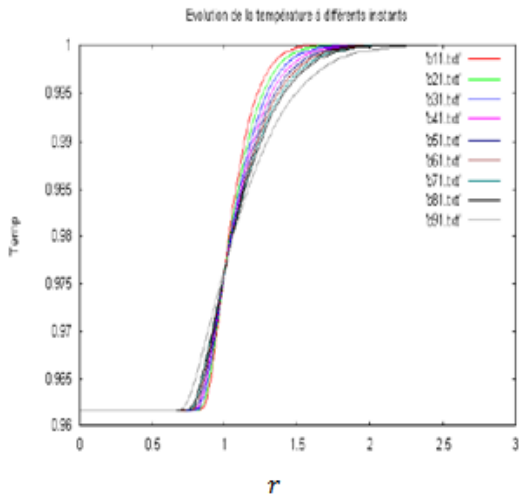


Figure 1: Temperature evolution at different times

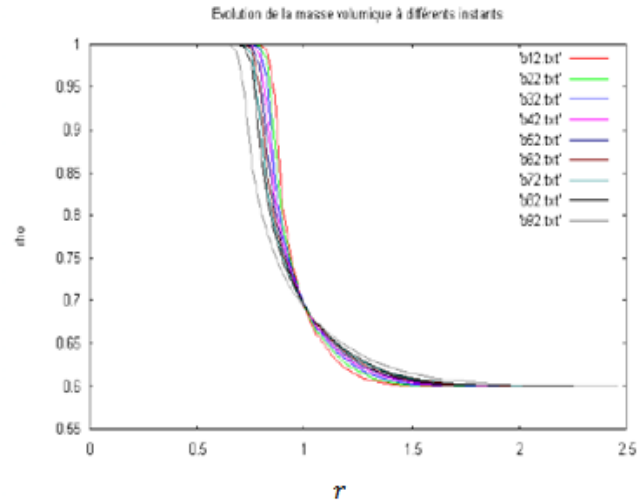


Figure 2: density evolution at different times

The initial conditions induce the partition of the flow domain into three separate zones: two steady zones which are the inner of the drop and the surrounding gas separated by a very thin transition layer in the beginning of the process. This transition layer whose thickness depends on the time, gradually expands to invade the entire flow field.

In contrast with the quasi-steady approximation and surely due to the constancy of the macroscopic variables in the drop and in the gaseous environment far from the drop and out of the transition zone, the temperature and density profiles are monotonic. They are increasing for the temperature and decreasing for the density in the transition layer.

## 2.2 Results for the mass flow

The figures 3.a and 3.b show the evolution of the mass flow at different times as function of  $r$ . The initial velocity was taken zero throughout the flow so the initial mass flow is zero both in the drop and in the hotter gas environment. Subsequent changes are therefore due to the discontinuities in the initial and boundary conditions of the temperature and the density. The profiles of the mass flow show generally at the beginning of the process two shocks which are represented by a maximum and a minimum of the mass flow. These shocks occur in a very small neighborhood on both sides of the drop border. The first shock is due to thermal discontinuities in the vicinity of the border of the pocket and leads to a maximum of the mass flow. The value of this maximum is not a monotonic function of time. It increases from the beginning of the process until a time  $t_{max}$  at which it begins to decrease.

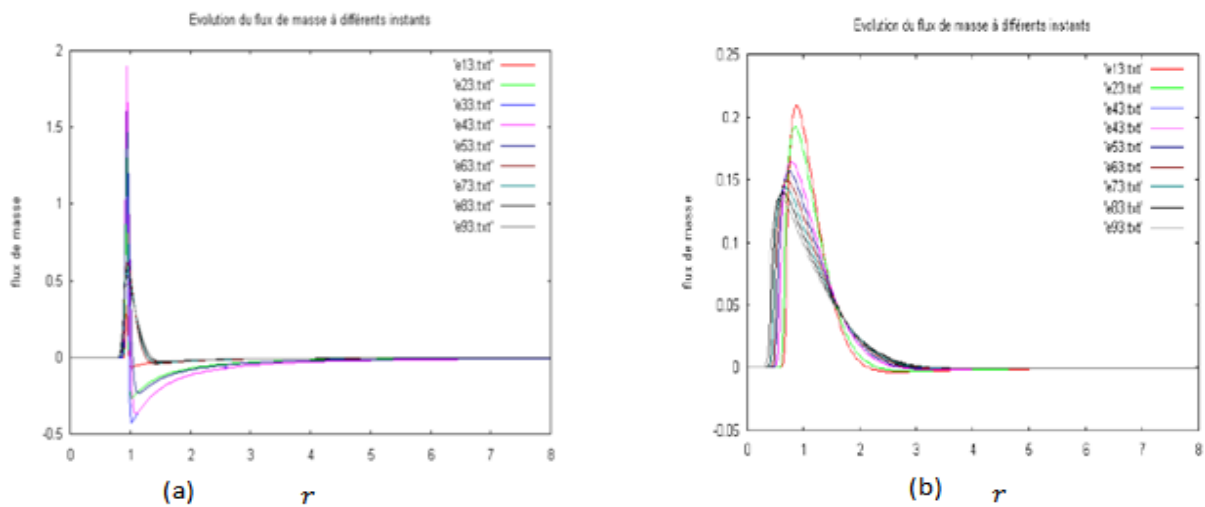


Figure 3: Mass flow evolution at different times

The second shock is due to mass discontinuities in the vicinity of the border and leads to a minimum of the mass flow. Its value is also a non-monotonic function of time and behaves as the maximum. The only difference is that it is canceled before the maximum which disappears with the drop. In addition, we clearly notice on the

curves that the values of the maximum and the minimum do not reach their maxima at the same time. The existence of these extrema of the mass flow whose nature (maximum and minimum) is explained by the fact that the heat and mass flows discontinuities propagate in opposite directions confirms the results of the quasi steady analysis reported in [6] as regards the mass flow discontinuities in the vicinity of the border of the subcritical drop. The difference is the fact that in subcritical conditions the shock due to the mass discontinuities is completely masked by the shock due to thermal gradients.

This numerical simulation allows us to study in the case of  $CO_2$  the relative behavior of these extrema. The magnitude of the minimum of the mass flow decreases and its position moves towards the gaseous phase during the process, while the base of the maximum moves towards the center of the drop, but remains in the vicinity of its border. Hence, the thickness of the transition layer which is very low at earlier stage is gradually enlarged. A remarkable fact is that as long as the minimum of the mass flow exist, the position of its maximum does not change, even if its value increases or decreases with the time (Fig. 3.a).

Thus, in this phase of the drop evolution, if we take the position of the maximum of the mass flow as the border of the drop, the volume of the drop is constant. Otherwise if we set the border of the drop at the position of the minimum of the mass flow the volume of the drop increases. It is obvious that these extrema delimit a transition layer which prevents the evaporation of the drop (Fig. 3.b). The minimum of the mass flow always disappears first. Then the maximum of the mass flow begins to move toward the center of the drop. The disappearance of the mass flow maximum coincides with the vanishing of the drop (Fig. 3.b).

#### IV. CONCLUSION

We study the process of relaxation of a cold package of a near-critical fluid in a hotter supercritical environment of the same fluid. Using a formalism developed in [6] we establish the classical  $d^2$  law for supercritical evaporation. A numerical resolution of the Navier-Stokes equations is then performed to analyze the transition layer between the cold package and the hotter gaseous environment. We find in the slightly supercritical case the existence of two extrema of the mass flow which delimit the transition layer. The relaxation of the cold package is effective only after the vanishing of the minimum of the mass flow. It is obvious that the presence of the minimum of the mass flow which delays the process of the relaxation of the cold package depends on the initial and boundary conditions. For instance, for fixed values of  $\mu = \frac{T'_\infty - T'_{in}}{T'_{in}}$  one can avoid the appearance of the minimum of the mass flow by weakening the value of  $\frac{\rho'_\infty}{\rho'_{in}}$ . However as the values of  $T'_{in}$  and  $\rho'_{in}$  are linked by the law of state one of the logical results of this study is to find equations of state which permit the use of these weaker values of  $\epsilon$ .

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