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An Optimal Inventory Model : Income and Loan Availability Elasticity

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ABSTRACT:

The present paper considers the effect of income and loan elasticity on demand and determines the buyers' optional special order, quantity as well as the optimal time. When the supplier reduces his sale price, the buyer can offer a discount to push his sale, so as to increase the profit margin. The paper introduces income and loan availability elasticity effect in inventory management. The gain equation has been derived to find out the optimal special order quantity and profit associated with it under two different conditions i.e. when the remnant inventory is zero and when the remnant inventory is finite.

Keywords: Elasticity,Inventory model,Profit,Loan

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INTRODUCTION

The elasticity of demand is an economic terms which refers to the responsiveness of demand to the change in price. When the demand is not elastic, the drastic change in price may not affect the demand, for example a fall in price or rise in price does not change the demand of essential commodities. In case of a fall in price tends to reduce demand and rise in price tends to extend demand. But in case of the articles, which are sensitive to the change in price, the demand increases or decreases, such goods or product are price responsive. The present paper is devoted to introduce a new concept i.e. income and loan availability elasticity vis-à-vis inventory models. Income and loan availability elasticity may be defined as:

E_1 = Proportionate change in demand / Proportionate change in (income + loan availability).

The optional policies for no buyers discount and constant demand are derived in [1]. Optional ordering policies in response to permanent price increase when demand is assumed to be constant have been well documented by [2], [3], [4] and [5]. A model to determine the optional ordering policies for a finite horizon consisting of two distinct time intervals characterized by different inventory parameters is presented by [6]. Models which determine the optimal special order quantity when supplier reduces his price temporarily, assume that the reduced price is in effect buyer's replenishment time and the demand rate remains constant [1], [7] and [8]. In most cases a price discount results in an increase in demand. Ardalan [9] relaxes demand assumption made in most inventory system with price change. Ardalan[9] presents a procedure to include any relationship between price and demand to determine the combined to optimal price and optimal order quantity.

The present paper considers the effect of income and loan availability elasticity on demand and determines the buyer's optimal special order quantity and optimal time. When supplier reduces the price temporarily, the buyer may offer a discount to his customers to push his sale and increase his profit margin. This paper brings in to picture income and loan availability demand relationship, which deals with classification of products into essential and non-essential goods. For non-essential goods like Luxuries the income and loan availability demand relationship is considerable. We introduce the income and loan availability elasticity effect in to inventory analysis and obtain the gain equation to find optimal special order quantity under two conditions i.e. when there is finite remnant inventory and when it is zero. We also determine optimal time for placing order quantity when there is finite remnant inventory and extend it to find the optimal time for placing such an order. We further modify our model to find cost of saving of special purchase when supplier announces to increase his sale price by some amount from a specified date. Finally an optimal ordering policy is determined to get profit maximization.

MODEL ASSUMPTIONS

The present paper consider the effect of income and loan availability elasticity because of the fact that in the current liberalized market and consumerism, the financial institutions are very much liberal in financing consumer loans at very low percentage of interest, the low rate loan availability can be considered as an aid to buying capacity so we find a heavy demand on non essential goods such as luxury items Automobiles, Ornaments etc. As for example this above effect has an impact not only on the "Growth Rates of Industrial production of India but also on the globalization of Indian economy". The structure of interest (loan) rates and the growth rates of industrial production in India are cited in Table (1).

The models developed in this paper has taken care of the simultaneous effect of income change and; loan facilities on classical inventory models. This concept has very much importance to the stockiest to go for a better margin of profit and improvement in their sale.

If E_1 is the income and loan availability Elasticity, then from its definition.

E_1 = Proportionate change in demand/ Proportionate change in (Income + loan availability)

D = Buyer 's annual demand;

D_1 = Buyer's annual demand after discount to customer.

D_2 =Buyer's new annual demand after the effect of income and loan availability elasticity.

Thus, New annual demand = Old demand (Change in income + loan availability)

$D_2 = D_1 + (E_1 \times \text{proportionate change in (income + Loan availability) /100}) D_1$

$D_2 = D_1 [1 + E_1 \times \text{proportionate change in (income + Loan availability) /100}]$

$D_2 = D_1 (1+Y+L)$ where

$$\left.\begin{aligned} &L = (E_1 \times \text{proportionate change in loan availability /100}) \\ &\text{Where } Y = (E_1 \times \text{proportionate change in income/100}) \end{aligned}\right] \quad (1)$$

P = Suppliers regular price per unit; P_1 = Buyer's regular price per unit;

P_2 = Buyer's sale (reduced) price per unit; C = ordering cost per order,

H = PF = Holding cost per unit per year; F = Annual holding cost as a fraction of unit cost (carry cost parameters); E.O.Q. = the size of an order that minimizes the total inventory cost is known as economic order quantity.

Q = Lot size or order quantity in units (a special order quantity);

Tc = Total annual cost

Q_r = Regular optimal order quantity = $\sqrt{(2CD/PF)}$ (2)

Further if, 'd' is the amount which supplier offers on his sale price and Qd is regular optimal order quantity using the reduced price.

If income and loan availability elasticity effect is there, then

$$Qd = [2D_2\, C/(p-d)\, F]^{1/2} = [2D_1\,(1+Y+L)\, C/(P-d)\, F]^{1/2} \quad (3)$$

If however the income and loan availability elasticity effect is not there then

Y = 0;

$$Qd = [2D_1 C/(P-d)\, F]^{1/2} \quad (4)$$

Q_0 = Special optimal order quantity when remnant inventory is zero.

Q_q = Special optimal order quantity when remnant inventory is not zero.

G_r = Gain associated with regular policy during Ts;

Gs = Gain associated with special order during T_S.

T_S = The time interval between the time the buyer receives the special order of size "Q" and his next replenishment time.

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Optimal Special Order Quantity when Remnant Inventory is zero and effect of income and loan Availability Elasticity is significant.

Suppose D_2 is the buyer's new annual demand after change in income and loan availability elasticity of the customer. At replenishment time buyer places an order of Q units at reduced price (offered by supplier). So the buyer too offers a discount to his customers on all the Q units. T_S is the time interval after which the suppliers reduced price is not available and buyer reverts to his usual ordering policy.

Now the gain associated within the time interval T_S

$$G_s = [P_2 - (P\text{-}d)]\, Q - [Q^2\, (P - d)\, F]/[2D_1\, (1+Y+L)] - C \tag{5}$$

If suppliers reduced price and buyers discount were permanent then classical inventory formula of E.O.Q could be used to determine optimal order quantity. As a result these are temporary business manoeuvirs and so this cannot be used, but these can be taken as rudimentary tool to obtain optimal policy for special order Qd.

If 'd' be the discount offered on Qd units then purchase cost of these Qd units would be $(P - d)\, Qd$.

The number of units bought at usual regular price during

$$T_s = [D/D_1\, (1+Y+L)](Q - Qd) \tag{6}$$

Therefore, the total buying cost is

$$(P - d)\, Qd+[PD/D_1\, (1+Y+L)](Q\text{-}Qd) \tag{7}$$

The holding cost of the first order is

$$Qd^2\, (P\text{-}d)\, F/2D_1\, (1+Y+L) \tag{8}$$

The holding cost during the rest of Ts is

$$[Qr(Q\text{-}Qd)PF]/[2D_1(1+Y+L)] \tag{9}$$

The total holding cost during Ts is

$$[Qd^2\, (P\text{-}d)\, F]/[2D_1(1+Y+L)]+[Qr(Q\text{-}Qd)PF]/[2D_1(1+Y+L)] \tag{10}$$

The number of orders placed during Ts is

$$(1+D\, (Q\text{-}Qd)/D_1\, (1+Y+L)\, Qr] \tag{11}$$

The ordering cost associated with this order will be given by

$$C[1+D(Q\text{-}Qd)/D_1(1+Y+L)Qr] \tag{12}$$

The gain associated with the usual (regular) ordering policy during Ts is given by:

$$Gr=[P_2-(P-d)]Qd+(P_1-P)D(Q-Qd)/D_1(1+Y+L)-(P-d)FQd^2/2D_1(1+Y+L)$$
$$-Qr(Q-Qd)PF/2D_1(1+Y+L)-C[1+D(Q-Qd)/D_1(1+Y+L)Qr] \quad (13)$$

In order to maximize gain the difference between Gr and Gs should be maximized.

$$G = [P_2 -(P-d)]\ Q-Q^2\ (P-d)\ F/2D_1(1+Y+L)-C$$
$$-[P_2\ (P-d)]Qd-(P_1-P)D(Q-Qd)/D_1(1+Y+L)+(P-d)F\ Qd^2/2D_1(1+Y+L)$$
$$+Qr(Q-Qd)PF/2D_1(1+Y+L)+C[1+D(Q-Qd)/D_1(1+Y+L)Qr] \quad (14)$$

Differentiating G with respect to Q and equating dG/dQ = 0, we get

$$Q_0 = [D_1\ (1+Y+L)\ (P_2-P+d)-(P_1-P)\ D]/[(P-d)\ F]+[Qr\ P/(P-d)]$$

Putting Q_0 in Equation (14) to get the optimal value of $G = C\ (Q_0- Qd)\ ^2/Qd^2$

Optimal Special Order Quantity when Remnant Inventory is not zero and Effect of Income and Loan Availability Elasticity are significant.

If effect of income and loan availability elasticity is considerable and there is sufficient remnant inventory, then buyer has two options either to buy at the reduced price of supplier and subsequently offer discount to his customer or just avoid the business ploy and follow his usual optimal order policy.

If ‘q’ be the level of remnant inventory then to determine special order quantity the difference between Gr and Gs should be maximized. If per capita income rised and loan availability rises increase in purchasing power of customer, the gain associated with special order quantity (Gs) and gain associated with usual ordering policy (Gr) is expressed as -

$$Gs=Q(P_2-P+d)-q^2PF/2D-qQ(P-d)F/D-[(p-d)FQ^2]/[2D_1(1+Y+L)]-C \quad (15)$$

$$Gr=[DQ(P_1-P)/D_1(1+Y+L)-(q^2PF/2D)]$$
$$-(QrQPF/2D_1(1+Y+L)]-[DQC/D_1(1+Y+L)Qr] \quad (16)$$

The increase in gain due to special order is G = Gs – Gr.

$$G=Q(P_2-P+d)-q^2PF/2D-qQ(p-d)F/D-(p-d)FQ^2/2D_1(1+Y+L)-C$$
$$-DQ(P_1-P)/D_1(1+Y+L)+q^2PF/2D+QrQPF/2D_1(1+Y+L)+DQC/D_1(1+Y+L)Qr \quad (17)$$

To maximize profit differentiating G with respect to Q and equating dG/dQ = 0

$$\Rightarrow Qq=[(P_2-P+d)D_1(1+Y+L)-D(P_1-P)]/[(P-d)F] +QrP/(P-d)-qD_1(1+Y+L)/D \quad (18)$$

$$\Rightarrow Qq=Qo-[qD_1(1+Y+L)/D] \quad (19)$$

To determine the optimal time for placing special order Qq, a theorem is deduced: -

Theorem – 1. Higher the income and loan availability elasticity smaller the level of remnant inventory and it determines the optimal time for placing optimal special order.

Proof : When remnant inventory is ‘q’ the special order quantity is ‘Qq’

When remnant inventory is ‘q_{-1}’ the optimal order is given by $Qq+D_1(1+Y+L)/D$.

If E_i is income and loan availability elasticity then new demand due to it is D_2 given by $D_2 = D_1(1+Y+L)$.

Using the gain equation 17, putting Q=Qq when remnant inventory is q units and gain G as Gq and $Q = Q_{q-1}$ when remnant inventory is (q_{-1}) units and gain G as $G_{q-1.}$

Where $Q_{q-1} = Qq+[D_1(1+Y+L)/D]$

Equation 17 $\Rightarrow$

$$Gq= Qq(P_2-P+d) -qQq(P-d)F/D - (P-d)FQq^2/2D_1 (1+Y+L) -C-DQq(P_1-P) /D_1(1+Y+L)+QrQqPF/2D_1(1+Y+L)+DQqC/D_1(1+Y+L)Qr \quad (20)$$

For remnant inventory (Q_{q-1})

Equation 17 $\Rightarrow$

$$G_{q-1} = Q_{q-1} (P_2-P+d)-(q_{-1})Q_{q-1}(P-d)F/D- (p-d)F(Q_{q-1})^2/2D_1(1+Y+L) +QrQ_{q-1} PF/2D_1(1+Y+L)+DQ_{q-1}C/D_1(1+Y+L)Qr - DQ_q(P_1-P)/D_1(1+Y+L) -C \quad (21)$$

Here we need to Prove $G_{q-1} > G_q$ i.e. $G_{q-1} - G_q$ = positive.

$$G_{q-1} - G_q = D_1(1+Y+L) (P_2-P+d)/D-D_1 (1+Y+L)q (P-d)F/D^2 +D_1(1+Y+L)(P-d)F/2D^2-P_1+P+QrPF/2D+C/Qr \quad (22)$$

Using equation (18) in equation (22) we get

$$[G_{q-1}-G_q][D/(P-d)F]=Qq+D_1(1+Y+L)/2D \quad (23)$$

Which is a positive value, this proves the theorem. (Proved)

Theorem –2

Higher the income and loan availability elasticity larger the level of remnant inventory and higher is the cost of saving when price rise is imminent.

Proof: If the supplier changes his sale policy and he stops the discount given to customer and announces a new increased sale price.

Let P be the amount by which supplier tends to increase his sale price at some date ti. The units purchased before ti will cost 'p' but purchase after ti will cost (P+p).

If D be the annual demand and Ei be the income and loan availability elasticity and 'q' be level of remnant inventory.

New demand due to income and loan availability elasticity = Old demand +change in demand

$$D^* = D(1+Y+L) \tag{24}$$

The optimal order quantity before price increase

$$Qr = (2CD^*/PF)^{1/2} = [2CD(1+Y+L)/PF]^{1/2} \tag{25}$$

The optimal order quantity after the price rise will be:

$$Qr^* = [2CD^*/(P+p)F]^{1/2} = [2CD(1+Y+L)/(P+p)F]^{1/2} \tag{26}$$

To obtain optimal special order size, it is necessary to maximize the cost difference during td – tr with and without special time order.

Total cost = Purchased cost+ Holding cost + Ordering cost

$$Tcr = PQs + [QsPFq/D(1+Y+L)] + Qs^2PF/2D(1+Y+L)] + [q^2PF/2D(1+Y+L)] + C \tag{27}$$

If no special order is placed prior to ti the total cost of the system during tr to td when several purchases of Qr* are made at new price (P+p)

$$Tcs = (P+p)Qs + (Qr^*/2)(P+p)F(Qs/D^*) + (q/2)PF(q/D^*) + (Qs/Qr^*)C$$

Using equation (24) and Equation (26)

$$Tcs = (P+p)Qs + (\tfrac{1}{2})[2CD(1+Y+L)/(P+p)F]^{1/2}(P+p) \\ F[Qs/D(1+Y+L)] + q^2PF/2D(1+Y+L) + (Qs/Qr^*)C \tag{28}$$

To find the optimal order size the difference between Tcr and Tcs should be minimized G = Tcs-Tcr

$$\Rightarrow G = Qs[p + C[F(P+p)/2D(1+Y+L)]^{1/2} \\ - PFq/D(1+Y+L) - [PFQs^2/2D(1+Y+L)] - C \tag{29}$$

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Differentiating with respect to Qs and putting dG/dQs = 0, we get optimal special order quantity Qso.

$$Qso=Qr^*/2+p/2PF[2D(1+Y+L)+Qr^*F]-q \tag{30}$$

This is optimal special order size, putting equation 30 in equation 29 we get the optimal cost of saving associated with Qso

$$G\Rightarrow Qso[p+[CF+(P+p)/2D(1+Y+L)]^{1/2}-PFq/D(1+Y+L)]-PFQso^2/2D(1+Y+L)]-C$$

Theorem 3:

If tf <tr the optimal ordering policy is

- **(i) To order Qq at ‘tf’ if Qq > Qd otherwise**
- **(ii) Order Qr at ‘tr’**

Proof: The value of the remnant inventory Qr for which Gr and Gs are equal can be determined by equating equation 15 and 16.

$$Q(P_2-p+d)-q^2PF/2D-qQ(P-d)F/D-[(P-d)FQ^2]/[2D_{1(}1+Y+L)]-C$$

$$=[DQ(P_1-P)/D_1(1+Y+L)]-[q^2PF/2D]-[QrQPF/2D_1(1+Y+L)]-[DQC/D_1(1+Y+L)Qr]$$

Simplifying and putting Q=Qq and q=qc using equation (18) we get

$$Qq/2=Qd^2/2Qq=Qq^2=Qd^2 \tag{31}$$

Since Gs> Gr for any Qq> Qd as long as Equation 31 holds good and optimal order size Qq at ‘tf’ would maximize the gain. Otherwise the usual (regular) ordering policy will be optimal.

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Conclusions

When a supplier reduces his sale price temporarily, then buyer might follow a policy which increases his demand. However due to income and loan availability elasticity effect, the change in demand may be much greater than expected. In fact income and elasticity brings product classification, which is an important economic consideration. The mathematical treatment of influence of income elasticity of a buyer's policy is discussed. We determine the case of finding optimal order quantity and profit when supplier reduced his sale price in both situation i.e. with/without remnant inventory. The special cost of saving is determined when rise in price is imminent and finally ordering policy is discussed.

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