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Generalized Walsh and Hadamard Transforms

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Abstract
Proposed here are some new transforms, which can be considered as the generalized version of the transforms developed by Walsh and Hadamard. Unlike Walsh and the Hadamard transforms, where the transform kernels are symmetric matrices of 1 and –1, here any two distinct real numbers, may be used as the elements of the transform kernels. The kernel may or may not be symmetric. The proposed generalized Hadamard transform is symmetric, whereas the generalized Walsh transform is asymmetric. We also propose a symmetric transform kernel known as pseudo Walsh transform whose sequency is similar to that of the generalized Walsh kernel. The existing Walsh and the Hadamard transforms can be obtained from them as special cases. The generalized transforms may find use in many fields of signal processing, image processing and statistical data analysis.
Keywords: *Asymmetric matrix, Generalized Hadamard transform, Generalized Walsh transform, Transform kernel, Orthogonality, Pseudo Walsh transform, Sequency, Symmetric matrix.*

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I. INTRODUCTION

Transforms play important roles in engineering and applied science fields. Working on the transformed data is, sometimes much easier and more useful information may be obtained from them. Fourier Transform, cosine transform, sine transform, etc are the common transforms based on sinusoids because of the cyclic nature of the sinusoids. Walsh transform and Walsh Hadamard transform in which the kernel is made of 1 and –1 are much simpler transforms as compared to sinusoid-based transforms. As a result, these transforms can be implemented without any cost of multiplication operation. Hence Walsh and Hadamard transforms are much faster than FFT. Because of simplicity and speed, people took interests in these transforms and used in different applications. Kak [9] applied Walsh transform for measuring randomness of a finite sequence. Similarly, Yuen [16] used it for testing random number sequences. Shih and Han [15] showed that it can be used for solving first order partial differential equations. Corrington [5] used it for solving differential and integral equations. Edwards [6] used it in designing logic gates. Also these transforms are used for astronomical signal/image processing, coding and filtering operations. For introductory theory and applications of Walsh transform, readers are referred to Beer [2]. In Statistics, Hadamard matrices are used in designing optimal weighting experiments as well as experimental design (analysis of variance) models such as factorial designs, Youden designs and Latin Squares. For comprehensive review of Hadamard matrices and their use in Statistics can be found in [7].

In this paper, we develop the generalized orthogonal transform kernels corresponding to Walsh and Hadamard transforms. Also, we introduce a new kind of transform known as pseudo Walsh transform. We will show that Walsh transform and Hadamard transform are special cases of these generalized transforms. The generalized Walsh transform and generalized Hadamard transform are interrelated i.e., one can be obtained from the other as in the original Walsh and Hadamard transforms. In the generalized form Walsh transform and the Walsh Hadamard transform are different so far as the kernel composition is concerned as they are ordered differently. The generalized Walsh kernel is asymmetric, whereas the generalized Hadamard kernel is symmetric. Pseudo Walsh kernel is symmetric but it cannot be generated in the same way as Walsh kernel is generated.

For notaion, capital letters are used to represent matrices or vectors and small letters as elements of a matrix or vector. A vector will be denoted by a bold capital letter. We organize the papers into four main sections. Section 2 gives the derivation of the generalized transforms. We first discuss the existing Walsh and Hadamard transforms in the series form as it is written so in most of the literatures and the same will be formulated in the matrix form. Section 3 gives derivation of the generalized transforms of the Walsh and Hadamard transform as well as the formulation of the new transform and a brief analysis of their computational complexity. Section 4 gives discussion and Section 5 gives brief conclusions.

II. EXISTING WALSH AND HADAMARD TRANSFORMS

In Walsh transform and the Hadamard transform, an analogue term of frequency known as sequency is used. By sequency we generally mean the average number of zero crossing per unit time. With referred to a transform kernel of a specific size, it refers to the number of change in sign in a particular sequence or the row of the kernel. The size of a transform kernel is generally a power of 2. In other words, the signal to be transformed must be a length of power 2. In the original transform kernel all the elements are of 1 or –1. The minimum kernel in both Walsh and Hadamard transform is a 2×2 symmetric matrix.

2.1. Walsh Transform

In Walsh's case, we denote the minimum transform kernel by W_1^1. The upper suffix 1 is used to denote it a special or original Walsh kernel and the lower suffix denotes the half of the kernel size.

$$W_1^1 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}.$$

The way of obtaining other higher size kernels is different in both cases. We will first give the way of obtaining Walsh's other higher size kernels from the above minimum kernel. The next higher Walsh kernel is,

i.e., $$W_2^1 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

Proceeding in this way, we can write W_{2n}^1 kernel in terms of the next lower kernel W_n^1 as

$$W_{2n}^1 = \begin{bmatrix} W_n^1(1) & W_n^1(1) \\ W_n^1(1) & -W_n^1(1) \\ W_n^1(2) & -W_n^1(2) \\ W_n^1(2) & W_n^1(2) \\ W_n^1(3) & W_n^1(3) \\ W_n^1(3) & -W_n^1(3) \\ W_n^1(4) & -W_n^1(4) \\ W_n^1(4) & W_n^1(4) \\ \vdots & \vdots \\ W_n^1(n) & -W_n^1(n) \\ W_n^1(n) & W_n^1(n) \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Here $W_n^1(r)$ denotes the rth row of the W_n^1 kernel.
Alternative way of finding Walsh transform kernel, can be found in [4,11,12].

Properties:

- The Walsh transformation kernel is symmetric, and orthogonal.
- Each row index or column index gives the sequency of the corresponding row or the column in the kernel.
- The determinant of the minimum kernel is –2.
- The determinant of any other higher kernel of size $2^n \times 2^n$ is $(2^n)^{2^{n-1}}$ for $n > 1$.

Let $S = [s_1, s_2, \cdots, s_N]$ be any discrete signal. Let $Wal\ (n, i)$ denotes the nth order sequency of the transform kernel i.e., the nth row of the kernel of size $N \times N$ and $T(n)$, the nth transform coefficient. Then we can write

$$T(k) = \sum_{i=1}^{N} \mathbf{S}(i).Wal\ (k, i)\ \text{........(2)}$$

Where $k = 1, 2, 3, \cdots, N$ and $N = 2^n$.

The original signal components of S can be obtained from the transform components T using the following inverse relation.

$$\mathbf{S}(k) = \frac{1}{N}\sum_{i=1}^{N} \mathbf{S}(i).Wal\ (k, i)\ \text{..............(3)}$$

Where $k = 1, 2, 3, \cdots, N$.

The term $1/N$ is the normalizing factor.

In matrix form, let W denotes the Walsh kernel of size $N \times N$ and $\mathbf{S}$ denotes the vector of length N, then the transform coefficient vector $\mathbf{T}$ can be written as

$$\mathbf{T} = \mathbf{S} * W\ \cdots\cdots\cdots (4)$$

And the corresponding inverse transform is

$$\mathbf{S} = \frac{1}{2^n}\mathbf{T} * W\ \cdots\cdots\cdots (5)$$

Using equation (4) and (5), we can perform the forward transform operation on a signal vector $\mathbf{S}$ with the transform kernel W.

2.2. Hadamard Transform

The minimum kernel in Hadamard transform is the same as the minimum Walsh kernel W_1^1. This is the only kernel where Walsh kernel is exactly the same as the Hadamard kernel. The way of obtaining other higher size kernels from this minimum kernel is totally different from the way of obtaining Walsh kernel using equation (1).

The minimum Hadamard kernel denoted by H_1^1 is

$$H_1^1 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

The next higher size Hadamard kernel H_2^1 is

$$H_2^1 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$, which is totally different from the W_2^1 of the Walsh kernel.

The other higher size Hadamard kernels are obtained from the following relation.

$$H_{2n}^1 = \begin{bmatrix} H_n^1 & H_n^1 \\ H_n^1 & -H_n^1 \end{bmatrix}\ \text{..............(6)}$$

Hadamard kernel can be derived from the Walsh kernel using gray code representation of the sequency. For alternative way of finding Hadamard kernel refer [10].

Properties

- Hadamard kernel is also symmetric and orthogonal.
- Sequency of a row or a column of the kernel is given by the decimal representation of the Gray code of the bit reversed binary values of the corresponding row or the column index.
- The determinant of the minimum kernel is –2.
- The determinant of any other higher kernel of size $2^n \times 2^n$ is $(2^n)^{2^{n-1}}$ for $n > 1$.

The Hadamard transform is also known as Walsh-Hadamard transform. For series notation, Wah will denote the Walsh Hadamard transform kernel.

Let $Wah\ (n, i)$ denotes the nth row of the Walsh Hadamard kernel of size $N \times N$. Then a discrete signal $\mathbf{S}$ of length 2^n can be transformed as

$$\mathbf{T}(k) = \sum_{i=1}^{N} \mathbf{S}(i).Wah\ (k, i) \text{ ....(7)}$$

where $k = 1, 2, 3, \cdots, N$

The inverse transform of equation (7), gives the original signal $\mathbf{S}$ as

$$\mathbf{S}(k) = \frac{1}{N} \sum_{i=1}^{N} \mathbf{T}(i).Wah\ (k, i) \cdots\cdots\cdots (8)$$

In matrix notation, let H denotes the Walsh Hadamard kernel of size $2^n \times 2^n$, and $\mathbf{S}$ is a vector of length 2^n, then we can write the transformed coefficient vector $\mathbf{T}$ as

$$\mathbf{T} = \mathbf{S} * H \cdots\cdots\cdots (9)$$

And the corresponding inverse transform as

$$\mathbf{S} = \frac{1}{2^n} \mathbf{T} * H \cdots\cdots\cdots (10)$$

Here the factor $1/2^n$ is the normalizing factor.

The Walsh transform kernel and the Hadamard transform kernel are both symmetric, they are composed of same basis vectors but in different orders. The Walsh transform kernel can be obtained from the Hadamard kernel by arranging the Hadamard kernel in ascending order of sequency of the basis vectors i.e., the rows of the transform kernel. Similarly, we can obtain Hadamard kernel from the Walsh kernel.

III. GENERALIZED WALSH AND WALSH TRANSFORM

In section 2, we have seen that kernels of Wash and Hadamard transforms has only 1 or –1 as its elements. Here, we will develop a new transform kernel whose elements are any two real numbers, not both zeros. We will show that such transform kernels do exist and Walsh and Hadamard kernel are special cases of them. These transform kernels can be considered as generalized Walsh and Hadamard transform kernel. Only matrix notation will be used in the generalized transform formulation.

3.1. Generalized Walsh Transform (GWT)

Let w_1, w_2 be any two real numbers, not both zeros, then the cyclic 2×2 matrix, is the minimum transform kernel of the generalized Walsh kernel.

$$W_1 = \begin{bmatrix} w_1 & w_2 \\ w_2 & -w_1 \end{bmatrix}$$

Then, the next higher transform kernel W_2 is as in the case of Walsh kernel as

$$W_2 = \begin{bmatrix} w_1 & w_2 & w_1 & w_2 \\ w_1 & w_2 & -w_1 & -w_2 \\ w_2 & -w_1 & -w_2 & w_1 \\ w_2 & -w_1 & w_2 & -w_1 \end{bmatrix}$$

Proceeding in this way, we can obtain W_{2n} kernel from the next lower kernel W_n using equation (1). Here, the transformation kernel is not symmetric but the transform kernels generated in this way are orthogonal. Hence they can be used for linear transformation of a signal or a data sequence.

Properties:

- The generalized Walsh transform kernel is asymmetric but orthogonal.
- The row or column index of the kernel gives the sequency of the corresponding row or the column of the kernel.
- The determinant of the minimum kernel is $-(w_1^2 + w_2^2)$.

- The determinant of any other higher kernel of size $2^n \times 2^n$ is $[2^{n-1}(w_1^2 + w_2^2)]^{2^{n-1}}$ for $n > 1$

This can be easily proved from the properties of finding determinant of Kronecker product of two matrices.

Thus, we see that the properties of the Walsh transform are also preserved in the generalized Walsh transform.

Let W denotes the generalized transform kernel of size $2^n \times 2^n$, and S be the signal vector of length 2^n, then we can write the transform coefficient vector T as

$$\mathbf{T} = \mathbf{S}.W \quad \ldots\ldots\ldots(11)$$

And the corresponding inverse transform $\mathbf{S}$ is given by

$$\mathbf{S} = \frac{1}{2^{n-1}(w_1^2 + w_2^2)} \mathbf{T}.W' \quad \ldots\ldots\ldots\ldots(12)$$

Where the term $1/2^{n-1}(w_1^2 + w_2^2)$ is the normalizing factor.

The generalized Walsh transform kernel is not symmetric and hence, the transpose of the kernel is used in the equation of inverse transform. Equation (11) gives the forward transform coefficients and the equation (12), gives the inverse transform coefficients of the generalized Walsh transform where the kernel is made of any two real numbers. Thus, we see that any two numbers other than 1's can be used as an orthogonal transform kernel. Depending on the values of w_1 and w_2 we can consider the following cases.

Case-1: When one of w_1 and w_2 is zero.

The transform kernel is still orthogonal and non-singular and hence is inversible. But the kernel remains asymmetric. Hence, the forward transform and inverse transform equations (11) and (12) are still valid. When this happens, we can reduce the computation time considerably.

Case-2: When both w_1 and w_2 are equal but not equal to 1.

Under such condition, the transform kernel becomes symmetric and orthogonal and is equal to Walsh kernel multiplied by a constant factor. The forward transform kernel is itself the inverse kernel and hence no need of transposition in the inverse transform equation.

Special Case: When $w_1 = w_2 = 1$.

When $w_1 = w_2 = 1$, then equation (11) and (12) becomes the Walsh forward and inverse transform equations (2) and (3) or (4) and (5). In other words, Walsh transform can be considered as a special case of the newly proposed generalized Walsh transform.

3.2. Generalized Hadamard Transform (GHT)

Similar to the generalized Walsh transform, we can generalize Hadamard transform. In generalized Hadamard transform, the elements of the kernel are written in terms of $\pm h_1$ or $\pm h_2$, any two real numbers not all zeros, instead of ± 1. The minimum generalized Hadamard kernel is

$$H_1 = \begin{bmatrix} h_1 & h_2 \\ h_2 & -h_1 \end{bmatrix}.$$

The next higher kernel H_2 is similarly found out as in the case of Walsh Hadamard kernel as

$$H_2 = \begin{bmatrix} h_1 & h_2 & h_1 & h_2 \\ h_2 & -h_1 & h_2 & -h_1 \\ h_1 & h_2 & -h_1 & -h_2 \\ h_2 & -h_1 & -h_2 & h_1 \end{bmatrix}.$$

We see that unlike generalized Walsh transform kernel W_2, H_2 is symmetric. Similarly, all other higher size generalized Hadamard transform kernels are symmetric. The generalized transform kernel H_{2n} can be obtained from its next lower kernel H_n using the relation (6) of the Hadamard kernel.

Properties:

- The GHT kernel is symmetric and orthogonal.
- Sequency of a row or a column of the kernel is given by the decimal representation of the Gray code of the bit reversed binary values of the corresponding row or the column index.
- The determinant of the minimum kernel is $-(h_1^2 + h_2^2)$.
- The determinant of any other higher kernel of size $2^n \times 2^n$ is $[2^{n-1}(h_1^2 + h_2^2)]^{2^{n-1}}$ for $n > 1$. This can be easily proved from the properties of finding determinant of Kronecker product of two matrices.

Let H denotes the generalized transform kernel of size $2^n \times 2^n$, and $\mathbf{S}$ be the signal vector of length 2^n , then the transform coefficient vector $\mathbf{T}$ can be written as

$\mathbf{T} = \mathbf{S} * H$ ……..(13)

And the corresponding inverse transform as

$$\mathbf{S} = \frac{1}{2^{n-1}(h_1^2 + h_2^2)} \mathbf{T} * H \quad \ldots\ldots\ldots..(14)$$

Where the factor $1/2^{n-1}(h_1^2 + h_2^2)$ is the normalizing factor.

Here, the transform kernel is symmetric and hence no need of transposition of the kernel in the inverse equation.

The GWT kernel can be obtained from the GHT kernel in the same way as we obtain the Walsh transform kernel from the Hadamard transform kernel and vice versa. In other words, GWT kernel is the GHT kernel arranged in ascending order of sequency. Depending on the values of h_1 and h_2 , we can consider the following cases.

Case-1: When one of h_1 and h_2 is zero.

The transform kernel is still orthogonal and non-singular and hence invertible. Also, the transform kernel is symmetric. Under such condition, the transform kernel can be thought of a constant multiple of a kernel made of 1’s and 0’s and hence computation becomes much simpler.

Case-2: When both h_1 and h_2 are equal but not equal to 1.

Under such condition, the transform kernel equal to Hadamard kernel multiplied by a constant factor. The forward transform kernel is itself the inverse kernel and hence no need of transposition in the inverse transform equation.

Special Case: When $h_1 = h_2 = 1$.

When $h_1 = h_2 = 1$, the transform kernel becomes Hadamard kernel and equations (13) and (14) represent the Hadamard transform of equation (7) and (8).

3.3. Pseudo Walsh Transform (PWT)

We know the original Walsh transform kernel with ± 1 as elements is a symmetric matrix. But the GWT kernel is asymmetric. It can be seen that the GWT does not preserve the symmetry property of the Walsh transform kernel. We are interested to develop a generalized transform kernel which is symmetric but at the same time have the same sequency as that of Walsh transform kernel. In addition, the transform kernel will be an orthogonal symmetric matrix with p_1, p_2 any two real numbers, both not zeros. We will call such a symmetric kernel having the same sequency as that of Walsh transform kernel as pseudo Walsh transform.

The minimum kernel of pseudo Walsh transform kernel is

$$\mathbf{P}_1 = \begin{bmatrix} p_1 & p_2 \\ p_2 & -p_1 \end{bmatrix},$$

The next higher kernel $\mathbf{P}_2$ having the same sequency as second order Walsh kernel is

$$\mathbf{P}_2 = \begin{bmatrix} p_1 & p_2 & p_1 & p_2 \\ p_2 & p_1 & -p_2 & -p_1 \\ p_1 & -p_2 & -p_1 & p_2 \\ p_2 & -p_1 & p_2 & -p_1 \end{bmatrix}$$

We see $\mathbf{P}_2$ is a symmetric matrix, having the same sequency as the generalized Walsh transform kernel W_2. Its kernel cannot be generated as in the way a GWT or Walsh transform is generated from its minimum kernel $\mathbf{P}_1$. But so far as the sequency and the symmetry is concerned it is similar to Walsh transform, hence it is entitled as pseudo generalized Walsh transform. Its kernel is a cyclic matrix of p_1 and p_2 having the same sequency as Walsh transform. Pseudo Walsh kernel is symmetric and orthogonal similar to GHT kernel, but the generalized Walsh transform or Hadamard transform can't be obtained from it by simple ordering of rows or columns of the kernel.

Properties:

- The PWT kernel is symmetric and orthogonal.
- The row or column index of the kernel gives the sequency of the corresponding row or the column of the kernel.
- The determinant of the minimum kernel is $-(p_1^2 + p_2^2)$.
- The determinant of any other higher kernel of size $2^n \times 2^n$ is $[2^{n-1}(p_1^2 + p_2^2)]^{2^{n-1}}$ for $n > 1$

The PWT kernel may be obtained by writing a symmetric matrix of p_1 and p_2 of the specified size in the increasing order of sequency of the row or column index.

Let $\mathbf{P}$ be the PWT kernel of size $2^n \times 2^n$ and S be the signal vector of length 2^n, then the transform coefficient vector $\mathbf{T}$ can be obtained from equation (13) by replacing H by P, i.e.,

$$\mathbf{T} = \mathbf{S} * P \cdots\cdots\cdots (15)$$

The inverse transform is given by

$$\mathbf{S} = \frac{1}{2^{n-1}(p_1^2 + p_2^2)} \mathbf{T} * P \cdots\cdots\cdots (16)$$

Where the factor $1/2^{n-1}(p_1^2 + p_2^2)$ is the normalizing factor.

Similar to the cases of GHT and GWT, we can consider the following cases depending on the values of p_1 and p_2.

Case-1: When one of p_1 and p_2 is zero.

Under such condition, it is similar to the case-1 of GHT.

Case-2: When both p_1 and p_2 are equal but not equal to 1.

Under such condition, it is similar to the case-2 of GHT.

Special case: When $p_1 = p_2 = 1$.

Under this condition, it becomes the Walsh transform kernel.

3.4. Computational Complexity

We know that all the three generalized kernels introduced above are made up of two different elements. Their computational complexity will be the same and hence we can analyze the computational aspect of any one of them. The generalized Hadamard kernel is discussed here.

We know, the minimum GHT kernel is $H_1 = \begin{bmatrix} h_1 & h_2 \\ h_2 & -h_1 \end{bmatrix}$

For a signal $\mathbf{S}_1 = [s_1 \quad s_2]$, the transform component vector $\mathbf{T}$ is given by

$\mathbf{T}(1) = s_1.h_1 + s_2.h_2$

$\mathbf{T}(2) = s_1.h_2 - s_2.h_1$

We require 4 multiplication operations for a signal of length 2.

For a signal of length 4, say $\mathbf{S}_2 = [s_1 \quad s_2 \quad s_3 \quad s_4]$, the corresponding Hadamard kernel is

$$H_2 = \begin{bmatrix} h_1 & h_2 & h_1 & h_2 \\ h_2 & -h_1 & h_2 & -h_1 \\ h_1 & h_2 & -h_1 & -h_2 \\ h_2 & -h_1 & -h_2 & h_1 \end{bmatrix}$$

and the transform vector $\mathbf{T}$ is given by

$\mathbf{T}(1) = s_1.h_2 + s_2.h_2 + s_3.h_1 + s_4.h_2$, this can be written as

$\mathbf{T}(1) = h_1(s_1 + s_3) + h_2(s_2 + s_4)$

$\mathbf{T}(2) = s_1.h_2 - s_2.h_1 + s_3.h_2 - s_4.h_1$

$\mathbf{T}(2) = h_2(s_1 + s_3) - h_1(s_2 + s_4)$

Similarly, we can write

$\mathbf{T}(3) = h_1(s_1 - s_3) + h_2(s_2 - s_4)$

$\mathbf{T}(4) = h_2(s_1 - s_3) - h_1(s_2 - s_4)$

Total number of multiplication operations here is 8, i.e., 2 times the length of the signal. The number of multiplication operations for a signal of length 16 will be 32, for that of 32 will be 64, and so on. In other words, we can have an algorithm for computing the GHT in 2 N multiplication operations where N is the length of the signal. Thus, the generalized Hadamard transform is linear time algorithm. In computational complexity terminology, GHT has O(n) complexity. Similar is the case for inverse transform as the inverse kernel is also the same. As the kernels of generalized Walsh and pseudo Walsh transforms are also made up of two different elements, they also have the linear time complexity.

IV. DISCUSSION

We have discussed the existing Walsh and Hadamard transform kernels and the ways of obtaining the higher order kernels. We see that Walsh and Hadamard transforms are not the only possible orthogonal real transforms. The orthogonality of these kernels is not because of their kernel elements being 1 or –1 nor the kernels being the symmetric, as many people used to think so. Any two real numbers following a certain sequency order can be used to generate orthogonal kernels. These orthogonal kernels may or may not be symmetric. Fourier transform kernel, Walsh and Hadamard transform kernel etc, are symmetric. But whether an orthogonal kernel is symmetric or not has no impact on the possibility of getting inverse of the transform. In our proposed generalized transform, the generalized Walsh transform kernel is asymmetric, but it is invertible; its transpose being its inverse kernel. We also introduced a new transform other than GWT and GHT known as Pseudo Walsh transform. PWT has also interesting properties because it is symmetric and its sequency is similar to that of Walsh transform. The generalized transforms here are much simpler than sinusoid based transforms, they may be used in many other applications in image and signal processing such as filtering, coding, edge detection etc and in statistical data analysis. Also, we know that these transforms take O(n) complexity for their computation, which is much less than that of FFT, hence it can be more attractive than FFT so far as the speed is concerned.

V. CONCLUSION

The Walsh and Hadamard transforms used only 1 and -1 as elements in their transform kernels which are symmetric. The elements of the transform kernels can be chosen any two real numbers in the proposed generalized Walsh and Hadamard transforms. This flexibility of choosing any two real numbers will enable us to find suitable kernels for different signal and image processing applications. Also, the transform kernel may or may not be symmetric. The generalized transforms are much simpler and easier to understand and implement as compared with sinusoids based transforms. Their basis can be chosen easily, and hence this gives the flexibility of choosing a most suitable basis for any particular application, instead of using same kernel for different applications. Also, they are faster than sinusoid based transforms including FFT. Moreover, we may get the same Walsh and Hadamard transforms from these generalized transforms.

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