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Oscillation of Second Order Biased and Unbiased Delay Differential Equations

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Abstract
In this dissertation to discuss the oscillations of solutions to a class of second order half linear unbiased differential equations with delayed arguments. New oscillation criteria are established and they essentially improve the well – known results reported in the literature, including those for biased differential equations. To find the solution of oscillation to second order unbiased delay differential equation using classical Riccati transformation technique.

Keywords: *s_0 – initial point, unbiased – not difference to other,H_1,H_2 -measurements*

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I. Introduction

For the sake of smallness and directness, we let

$$Q(s) = (1 - p(\theta(s))^{\alpha} q(s)$$

$$Q(S) = \int_{s}^{\infty} Q(t)dt,$$

$$R(t) = \int_{s_1}^{s} u^{-\frac{1}{\alpha}}(t)dt,$$

$$\dot{R}(s) = R(s) + \frac{1}{\alpha}\int_{s_1}^{s} R(t)R^{\alpha}\dot{\theta}(t))Q(t)dt,$$

$$R(s) = \exp\left(-\alpha\int_{\theta(s)}^{s} \frac{dt}{R(t)u^{\frac{1}{\alpha}}(t)}\right)$$

for $s \geq s_1$, where $s_1 \geq s_0$ is large sufficient.

To prove oscillation criteria, we require the following supporting consequences.

Lemma 3.1

Let form (2.1.2) hold and assume that x(s) is a non negative solution of (2.1.1) on $[s_0, \infty)$. Then there exists a $s_1 \geq t0$ such that, for $s \geq s_1$,

$$y(s) > 0, \qquad y'(s) > 0,$$

$$(u(s)\, y'(s)^{\alpha})' \leq 0. \qquad (3.1.1)$$

Lemma 3.2

Let $g(u)= Au–Bu^{(\alpha+1)/\alpha}$, where A and B >0 are constants, α is a proportion of odd natural numbers. Then g attains its maximum value on ℝ at $u^*=(\alpha A/((\alpha+1)B))^{\alpha}$ and

$$\max_{u\in\mathbb{R}} g = g(u*) = \frac{\alpha^{\alpha}}{(\alpha+1)^{\alpha+1}}\left(\frac{A^{\alpha+1}}{B^{\alpha}}\right) \qquad (3.1.2)$$

3.2 Main results

At the present, we state and prove the our first oscillation consequence which extend obtain for the linear delay differential equation (2.1.3) to the half-linear unbiased delay differential equation (2.1.1).

Theorem 3.1

Let the condition (2.1.2) be satisfied. If

$$\lim_{s\to\infty} \sup \int_{\theta(s)}^{s} Q(t)R^{\alpha}(\theta(t))dt > 1$$,(3.2.1) θ is non decreasing

(Or)

$$\lim_{s\to\infty} \inf \int_{\theta(s)}^{s} Q(t)R^{\alpha}(\theta(t))dt > \frac{1}{e}, \qquad (3.2.2)$$

then(2.1.1) is oscillatory.

Proof

Assume that (2.1.1) has an non oscillatory solution x(s) on $[s_0, \infty)$. with no loss of generalization, we may assume that, there exists an $s_1 \geq s_0$ such that x(s)> 0, x(τ (s)) > 0, and x(θ(s)) > 0 for $s \geq s_1$. By the definition of y(s), we obtain, for $s \geq s_1$,

$$x(s) \geq y(s) - p(s)x\,\tau(s) \geq y(s) - p(s)y\tau(s) \geq 1 - p(t)\,z(t),$$

which simultaneously to (2.1.1) implies that

$$(u(s)(y'(s)^{\alpha})' \leq -Q(t)z^{\alpha}(\theta(t)) \qquad (3.2.3)$$

On the additional hand, it follows from the monotonicity of $u^{\frac{1}{\alpha}}(s)y'(s)$ that,

$$y(s) = y(s_1) + \int_{s_1}^{s} \frac{1}{u^{\frac{1}{\alpha}}(s)} u^{\frac{1}{\alpha}}(s)y'(s)ds \geq R(s)u^{\frac{1}{\alpha}}(s)y'(s)ds \qquad (3.2.4)$$

A simple calculation shows that

$$\left(y(s) - R(s)u^{\frac{1}{\alpha}}(s)y'(s)\right)' = -R(s)(u^{\frac{1}{\alpha}}(s)y'(s))' \qquad (3.2.5)$$

Apply the chain rule, it is simple to see that

$$R(s)\left(u(s)\left(y'(s)\right)^{\alpha}\right)' = \alpha R(s)\left(u^{\frac{1}{\alpha}}(s)y'(s)\right)^{\alpha-1}\left(u^{\frac{1}{\alpha}}(s)y'(s)\right)'.$$

By good value of (3.2.3), the later equal opportunity yields

$$-R(s)\left(u^{\frac{1}{\alpha}}(s)y'(s)\right)' \geq \frac{1}{\alpha}R(s)\left(u^{\frac{1}{\alpha}}(s)y'(s)\right)^{1-\alpha} Q(s)y^{\alpha}(\theta(s)). \qquad (3.2.6)$$

Combining (3.2.5) and (3.2.6), we obtain

$$\left(y(s) - R(s)u^{\frac{1}{\propto}}(s)y'(s)\right)' \geq \frac{1}{\propto} R(s)\left(u^{\frac{1}{\propto}}(s)y'(s)\right)^{\frac{1}{\propto}} Q(s)y^{\propto}(\theta(s)). \qquad (3.2.7)$$

Integrating (3.2.7) from s_1 to s, we have

$$y(s) \geq R(s)u^{\frac{1}{\propto}}(s)y'(s) + \frac{1}{\propto}\int_{s_1}^{s}\left(u^{\frac{1}{\propto}}(t)y'(t)\right)^{\frac{1}{\propto}} R(t)\,Q(t)y^{\propto}(\theta(t))dt.$$

Taking (3.2.5) and the monotonicity of $u^{\frac{1}{\alpha}}(s)$y (t) into account, we get there at

$$\begin{aligned} u(s) \geq\ & R(s)u^{\frac{1}{\alpha}}(s)y'(s) + R(s)u^{\frac{1}{\propto}}(s)y'(s) \\ & + \frac{1}{\propto}\int_{s_1}^{s}\left(u^{\frac{1}{\propto}}(t)y'(t)\right)^{1-\alpha} R(t)\,R^{\alpha}(\theta(t)Q(t)(u(\theta(t))\left(y'(\theta(t))\right)^{\alpha} ds \end{aligned}$$

$$\geq u^{\frac{1}{\propto}}(s)y'(s)(R(s) + \frac{1}{\propto}\int_{s_1}^{s} R(t)R^{\propto}(\theta(t))Q(t)dt) \qquad (3.2.8)$$

Thus, we conclude that

$$y(\theta(s)) \geq u^{\frac{1}{\alpha}}(\theta(s))y'(\theta(s))R(\theta(s)). \qquad (3.2.9)$$

Using (3.2.9) in (3.2.5), by good value of (3.1.1), one can see that $z(s) = r(s)(y'(s))^{\alpha}$ is a positive solution of the first order delay differential inequality

$$z'(s) + Q(s)R^{\propto}(\theta(s))z(\theta(s)) \leq 0 \qquad (3.2.10)$$

In view of the connected delay differential equation

$$z'(s) + Q(s)R^{\propto}(\theta(s))z(\theta(s)) = 0 \qquad (3.2.11)$$

as well as a positive solution. However, it is well-known that form (3.2.1) or form (3.2.10) ensures oscillation of (3.2.11). This in turn around means that (3.1.1) cannot have positive solutions.

The hence proof.

Corollary 3.1

Let form (2.1.2) hold. If

$$\lim_{s\to\infty} \sup \int_{\theta(s)}^{s} q(t)R^{\alpha}(\theta(t))dt > 1,$$ (3.2.12) θ is non decreasing

(or)

$$lim_{s\to\infty} inf \int_{\theta(s)}^{s} q(t)R^{\alpha}(\theta(t))dt > \frac{1}{e}, \qquad (3.2.13)$$

then (2.1.1) is oscillatory.

Example 3.1

For s ≥ 1, consider the second-order unbiased differential equation

$$\left(\left(y'(s)\right)^{\propto}\right)' + \frac{q_0}{s^{\propto+1}} x^{\propto}(\lambda s) = 0 \qquad (3.2.14)$$

where y(s)= x(s)+ p_0x(τ (s)), α is a proportion of odd positive integers, $p0 \in [0, 1)$, τ (s) ≤ s,q_0 > 0, and λ ∈ (0, 1). By Theorem 3.1, (3.2,14) is oscillatory if

$$\rho = (1 - p_0)^{\alpha} q_0 \lambda^{\alpha} \frac{(\alpha+(1-p_0)^{\alpha} q_0\lambda^{\alpha})^{\alpha}}{\alpha^{\alpha}} In\frac{1}{\lambda} > \frac{1}{e} \qquad (3.2.15)$$

$For\ a\ particular\ case\ of\ ,equation$

$$\left(\left(x'(t)\right)^{\frac{1}{3}}\right)' + \frac{q_0}{t^{\frac{4}{3}}} x^{\frac{1}{3}}(0.9t) = 0 \qquad (3.2.16)$$

oscillation of all solution is assured by condition

$$q0>1.92916. \qquad (3.2.17)$$

To the best of our understanding, the identified related condition for (3.2.16) based on evaluation with a first-order delay differential equation gives

q_0 > 3.61643, which is a significantly weaker consequence.

On the other hand, for equation

$$\left((x'(s))^{\frac{1}{3}}\right)' + \frac{1}{6}\left(\frac{5}{18}\right)^{\frac{1}{6}} s^{-\frac{4}{3}} x^{\frac{1}{3}}(0.9s) = 0 \qquad (3.2.18)$$

form (3.2.17) fails to hold and x(s) = $s^{\frac{1}{2}}$ is a non oscillatory solution of (3.2.18).

clearly, if

$$\int_{\theta(t)}^{s} Q(t)R^{\alpha}(\theta(t))dt \le \frac{1}{e}, \qquad (3.2.19)$$

then Theorem 3.1 cannot be functional to (2.1.1). However, if (3.2.19) holds and z(t) is a positive solution of (3.2.10), then it is likely to get hold of sharper lower bounds of the ratio z(θ(s))/z(s). This will permit us to refine classical Riccati transformation techniques which are generally used in learn of oscillation of second-order differential equations. Zhang and Zhou obtain such bounds for the first order delay differential equation (3.2.11) by employing

$$\{ f_n(\rho)\}_{n=0}^{\infty}$$
$$f_0(\rho) = 1$$
$$f_n(\rho) = e^{\rho fn(\rho)}, \qquad (3.2.20)$$

n=0,1,2,...,

where ρ is a positive constant satisfying

$$\int_{\theta(s)}^{s} Q(t)R^{\alpha}(\theta(t))dt \ge \rho , s \ge s_1 \ge s_0 \qquad (3.2.21)$$

They show that, for ρ ∈ (0, 1/e], the sequence is increasing and bounded above and

$$\lim_{t\to\infty} f_n(\rho) = f(\rho) \in [1, e]$$

where f (ρ) is a real root of the equation

$$f(\rho) = e^{\rho f(\rho)} \qquad (3.2.22)$$

Their consequence plays the necessary role when prove the following lemma.

Lemma 3.3

Let us assume that θ is strictly increasing, situation (3.2.21) holds for some ρ > 0, and (2.1.1) has a positive solution x(s) on [s_0, ∞). Then, for every n ≥ 0,

z(s) = u(s)(y'(s)$^{\alpha}$ satisfies

$$\frac{z(\theta(s))}{z(s)} \ge f_n(\rho) \qquad (3.2.23)$$

for s great sufficient, where $f_n(\rho)$ is defined by (3.2.20).

Proof

Assume that (2.1.1) has a non oscillatory solution x(s) on [s_0, ∞). with no loss of generalization, suppose that there exists a $s_1 \ge s_0$ such that x(s)> 0, x(τ (s)) > 0, and x(θ (s)) > 0 for s ≥ s_1. As in the proof of Theorem 3, assume that z(s) = u(s)(y'(s))$^{\alpha}$ is a positive solution of the first order delay differential inequality (3.2.10). arranged in a similar method as in the proof of Lemma 3.1, we see that estimation (3.2.23) holds.

In the follows, we use the Riccati substitution technique to get hold of new oscillation criteria for (2.1.1), which are particular effective in the case when Theorem 3.1 fail to apply.

Theorem 3.2

Let us assume that $\theta \in C([s_0, \infty), R)$, $\theta'(s) > 0$, and situation (3.2.21) holds for some $\rho > 0$. If there exists a function $\phi \in C([t_0, \infty), (0, \infty))$ such that, for some sufficiently large $T \geq s_1$ and for some $n \geq 0$,

$$\limsup_{t\to\infty} \int_T^s \left(\varphi(t)Q(t) - \frac{\left(\varphi'(t)\right)^{\alpha+1} u\left(\theta(t)\right)}{(\alpha+1)^{\alpha+1} f_n(\rho)\varphi^{\alpha}(t)\left(\theta'(t)\right)^{\alpha}}\right)dt = \infty \tag{3.2.24}$$

where $f_n(\rho)$ is defined by (3.2.20) and $\phi'(s) = \max\{0, \phi'(s)\}$, then (2.1.1) is oscillatory.

Proof

Assume that (2.1.1) has a non oscillatory solution x(s) on $[s_0, \infty)$. with no loss of generality, assume that there exists a $s_1 \geq s_0$ such that $x(s) > 0$, $x(\tau(s)) > 0$, and

$X(\theta(s)) > 0$ for $s \geq s_1$.

Defined the Riccati function by

$$w(s) = \varphi(s)u(s)\left(\frac{y'(s)}{y(\theta(s))}\right)^{\alpha}, s \geq s_1 \tag{3.2.25}$$

Then $w(s) > 0$ for $s \geq s_1$.Differentiating(3.2.25), arrive at

$$w'(s) = \frac{\varphi'(s)}{\varphi(s)}w(t) + \varphi(s)\frac{u(s)(y'(s)^{\alpha})'}{y^{\alpha}(\theta(s))} - \alpha\varphi(s)\theta'(s)u(s)\left(\frac{y'(s)}{y(\theta(s))}\right)^{\alpha}\left(\frac{y'(\theta(s))}{y(\theta(s))}\right) \tag{3.2.26}$$

It follows from Lemma 3.4 that there exists a $T \geq s_1$ great sufficient such that

$$\left(\frac{y'(\theta(s))}{y'(s)}\right) \geq \left(\frac{f_n(\rho)u(s)}{u(\theta(s))}\right)^{\frac{1}{\alpha}}, s \geq T \tag{3.2.27}$$

By virtue of (3.2.3) and (3.2.27), applications of (3.2.25) and (3.2.26) yield

$$w'(s) = \frac{\varphi'(s)}{\varphi(s)}w(t) - \varphi(s)Q(t) - \frac{\alpha f_n^{\frac{1}{\alpha}}(\rho)u'(s)}{(\varphi(s)u(\theta(s)))^{\frac{1}{\alpha}}} w^{\alpha+\frac{1}{\alpha}}(s) \tag{3.2.28}$$

let

$$A = \frac{\varphi'(s)}{\varphi(s)} \text{ and } B = \frac{\alpha f_n^{\frac{1}{\alpha}}(\rho)u'(s)}{(\varphi(s)u(\theta(s)))^{\frac{1}{\alpha}}}$$

In (3.1.2), it follows now from Lemma 3.2 and (3.2.28) that

$$w'(s) \leq -\varphi(s)Q(s) + \frac{\left(\varphi'(s)\right)^{\alpha+1}u(\theta(s))}{(\alpha+1)^{\alpha+1} f_n(\rho)\varphi^{\alpha}(s)\left(\theta'(s)\right)^{\alpha}} \tag{3.2.29}$$

$$\int_T^s \left(\varphi(t)Q(t) - \frac{\left(\varphi'(t)\right)^{\alpha+1}u(\theta(t))}{(\alpha+1)^{\alpha+1} f_n(\rho)\varphi^{\alpha}(t)(\theta'(t))^{\alpha}}\right)dt \leq w(T)$$

which contradicts form (3.2.24).

Hence the proof.

Remark 3.1

Theorem 3.2 is new because of the constant $f_n(\rho)$ (for some $n \geq 0$) appearing in (3.2.24). So far, all consequences obtained in a similar method have been formulated for $n = 0$. Thus, for any given $n > 0$, our consequence fundamentally improves the previous ones.

Letting $\phi(s) = R^{\alpha}(\theta(t))$ in (3.2.24), Theorem 3.2 yields the following result.

Corollary 2

Let form (2.1.1) hold and assume that $\theta \in C([s_0, \infty), R)$, $\theta'(t) > 0$, and condition (3.2.21) holds for some $\rho > 0$. If, for some sufficiently large $T \geq t_1$ and for some $n \geq 0$,

$$\limsup_{s\to\infty} \int_T^s (R^{\alpha}(\theta(t))Q(t) - \left(\left(\frac{\alpha}{\alpha+1}\right)^{\alpha+1} \frac{\theta'(t)}{f_n(\rho)R(\theta(t))u^{\frac{1}{\alpha}}(\theta(t))}\right) ds = \infty \tag{3.2.30}$$

where $f_n(\rho)$ is defined by (3.2.20), then (2.1.1) is oscillatory.

Example 3.2

As in Example 3.1, we consider (3.2.14). If we assume that$\rho \le \frac{1}{e}$ in (3.2.15), then the Sequence $\{f_n(\rho)\}_{n=0}^{\infty}$defined by(3.2.20) has a finite limit (3.2.22),which can be expressed as

$$f(\rho) = \lim_{n\to\infty} f_n(\rho) = -\frac{W(-\rho)}{\rho}$$

where W standard denotes the principal division of the Lambert function. Then, by Corollary 3.2, (3.2.14) is oscillatory if

$$(1-p_0)^\alpha q_0 \lambda^\alpha f(\rho) > \left(\frac{\alpha}{\alpha+1}\right)^{\alpha+1} \qquad (3.2.31)$$

In order to illustrate the efficiency of the above criterion, we stress that an application of (2.1.6) yields that condition

$$(1-p_0)^\alpha q_0 \lambda^\alpha > \left(\frac{\alpha}{\alpha+1}\right)^{\alpha+1} \qquad (3.2.32)$$

ensure oscillation of (3.2.14). For a particular case of (3.2.14), equation

$$\left(\left(x'(s)\right)^3\right)' + \frac{0.3}{s^4}\, x^3(0.9s) = 0, \qquad (3.2.33)$$

situation (3.2.31) gives 3.5876 > 0.3164, which is implies that (3.2.33) is oscillatory. However, one may see that the left hand side of inequality (3.2.32) becomes 0.2187, which means that form (3.2.32) fails to hold for (3.2.33). Moreover, one can easily verify that the condition resultant from Theorem 3.1 cannot be applied to (3.2.33).The following theorem serves as an different to Theorem 3.2.

Theorem 3.3

Let form (2.1.1) be satisfied and assume that there exists a function $\psi \in C([s_0, \infty), (0, \infty))$ such that, for some sufficiently greatest $T \ge s_1$,

$$\lim_{s\to\infty} sup \int_T^s \left(\psi(t)Q(t)R(t) - \frac{(\psi'(t))^{\alpha+1}u(t)}{(\alpha+1)^{\alpha+1}\psi^\alpha(t)}\right)dt = \infty \qquad (3.2.34)$$

where $\psi'_+(s) = max\{0, \psi'(s)\}$. Then (2.1.1) is oscillatory.

Proof

Assume that (2.1.1) has a non oscillatory solution x(s) on $[s_0, \infty)$. with no loss of generality, we can suppose that there exists a $s_1 \ge s_0$ such that x(s)> 0, x(τ (s)) > 0, and x(σ (s)) > 0 for $s \ge s_1$. Defined the Riccati function by

$$w(s) = \psi(s)u(s)\left(\frac{y'(s)}{y(\theta(s))}\right)^\alpha, s \ge s1 \qquad (3.2.35)$$

Then w(s) > 0 for $s \ge s_1$ and

$$w'(s) = \frac{\psi'(s)}{\psi(s)}w(t) + \psi(s)\frac{u(s)(y'(s)^\alpha)'}{y^\alpha(\theta(s))} - \alpha\psi(s)u(s)\left(\frac{y'(s)}{y(s)}\right)^{\alpha+1} \qquad (3.2.36)$$

The proof of Theorem 3.1, we get (3.2.8),(i.e).,

$$y(s) \ge R(s)u^{\frac{1}{\alpha}}(s)y'(s)$$

or

$$\frac{y'(s)}{y(s)} \le \frac{1}{R(s)u^{\frac{1}{\alpha}}(s)}$$

Integrating the latter inequality from σ (t) to t, we obtained

$$\frac{y(\theta(s))}{y(s)} \ge \exp\left(-\int_{\theta(s)}^{s} \frac{dt}{R(t)u^{\frac{1}{\alpha}}(t)}\right) \qquad (3.2.37)$$

Combining (3.2.5)and (3.2.37),it follows that,

$$\frac{\left(u(s)(y'(s))^\alpha\right)'}{y^\alpha(s)} \le Q(s)\left(\frac{y(\theta(s))}{y(s)}\right)^\alpha$$
$$\le Q(s)\ \exp\left(-\alpha\int_{\theta(s)}^{s} \frac{dt}{R(t)u^{\frac{1}{\alpha}}(t)}\right)$$
$$\le -Q(s)R(s).$$

Hence, by (3.2.35) and (3.2.36), we deduce that

$$w'(s) = \frac{\psi'_+(s)}{\psi(s)}w(t) - \psi(s)Q(s)R(s) - \frac{\alpha}{\psi(s)u(s)^{\frac{1}{\alpha}}}w^{\frac{\alpha+1}{\alpha}}(s). \qquad (3.2.38)$$

Let

$$A = \frac{\psi'_{+}(s)}{\psi(s)} \text{ and } B = \frac{\alpha}{\psi(s)u(s)^{\frac{1}{\alpha}}}$$

In (3.1.2).it follows from Lemma2 and (3.2.38) that is,

$$w'(s) \leq -\psi(s)Q(s)R(s) + \frac{\left(\psi'_{+}(s)\right)^{\alpha+1} u(t)}{(\alpha+1)^{\alpha+1}\psi^{\alpha}(s)} \qquad (3.2.39)$$

let $T \geq s_1$ be sufficiently large. Integrating (3.2.39) from T to s, we have

$$\int_T^s \left(-\psi(t)Q(t)R(t) + \frac{\left(\psi'_{+}(t)\right)^{\alpha+1} u(s)}{(\alpha+1)^{\alpha+1}\psi^{\alpha}(t)}\right) dt \leq w(T),$$

which contradict condition (3.2.34).

Hence the proof.

Example 3.3

As in the Example 3.1, we consider (3.2.14). By Theorem 3.3, (3.2.14) is oscillatory if

$$(1 - p_0)^{\alpha} q_0 \lambda^{\alpha u} > \left(\frac{\alpha}{\alpha+1}\right)^{\alpha+1} \qquad (3.2.40)$$

where $u = (\alpha/(\alpha + (1 - p_0)^{\alpha} q_0 \lambda^{\alpha}))^{\alpha}$. An function of (2.1.7) yields that (3.2.14) is oscillatory provide that It is easy to observe that u< 1, and thus our condition (3.2.40) provide a stronger consequence.

II. CONCLUSIONS

In this present dissertation, we have considered the oscillatory behavior of the second order half linear unbiased delay differential equation. As it has been illustrate throughout one more than a few examples, the results obtain improve a greatest number of the existing ones. Our method lies in establish some sharper estimate connecting a non oscillatory solution with its derivative in the folder when criteria equivalent to fails to be valid. The consequences existing in this dissertation strongly depend on the properties of first order delay differential equations. An attractive problem research is to establish different iterative techniques for testing oscillations in independently on the constant.

REFERENCE:

[1]. R. P. Agarwal, M. Bohner, and W.-T. Li, Nonoscillation and Oscillation: Theory for Functional DifferentialEquations, vol. 267 of Monographs and Textbooks in Pure and Applied Mathematics, Marcel Dekker, NewYork, NY, USA, 2004.

[2]. Bohner, M., Grace, S.R., Jadlovská, I.: Oscillation criteria for second-order neutral delay differential equations. Electron.J. Qual. Theory Differ. Equ. 2017, 60 (2017)

[3]. Baculikova, B., Džurina, J.: Oscillation theorems for second-order nonlinear neutral differential equations comput.Math appl.62(12),4472-4478(2011)

[4]. Liu, L., Bai, Y.: New oscillation criteria for second-order nonlinear neutral delay differential equations.J.comput. appl math231(2),657-663(2009)

[5]. Liu, L., Bai, Y.: Erratum to: "New oscillation criteria for second-order nonlinear neutral delay differential equations" [J. Comput. Appl. Math. 231 (2009) 657–663]. J. Comput. Appl. Math. 233(10), 2755 (2010)

[6]. Dong, J.-G.: Oscillation behavior of second order nonlinear neutral differential equations with deviating arguments.Comput. Math. Appl. 59(12), 3710–3717 (2010)

[7]. Grace, S.R., Graef, J.R., Tunç, E.: Oscillatory behavior of second order damped neutral differential equations withdistributed deviating arguments. Miskolc Math. Notes 18(2), 759–769 (2017).