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Generalized Pre-Closed Set in Topological Space

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Abstract
In this paper was study a generalized pre-closed sets in a topological space. We will provide the relationship between generalized closed sets and generalized pre-closed sets. Furthers we discuss sg-submaximal space, disconnected space by using various kinds of generalized closed sets.

Keywords:
Generalized closed set, Tgs-space, submaximal, extremely disconnected, Generalized pre-closed sets.

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I. Introduction

In 1970, N.Levine [13] initiated the study of so-called generalized closed sets. By definition, a subset A of X is said to be pre-closed if cl(int A)⊆A, the pre-closure of A is denoted by PclA, is the smallest set X containing A. Complement of pre-closet set are called pre-open. By definition, a subset A of a topological space X is cl(A)⊆U whenever A⊆U and U is open. Moreover A is called generalized open or g-open if X/A is g is closed. In [17] Makietal introduce the concept of pg-closed set and gp-closed sets in analogous manner.

Definition 1

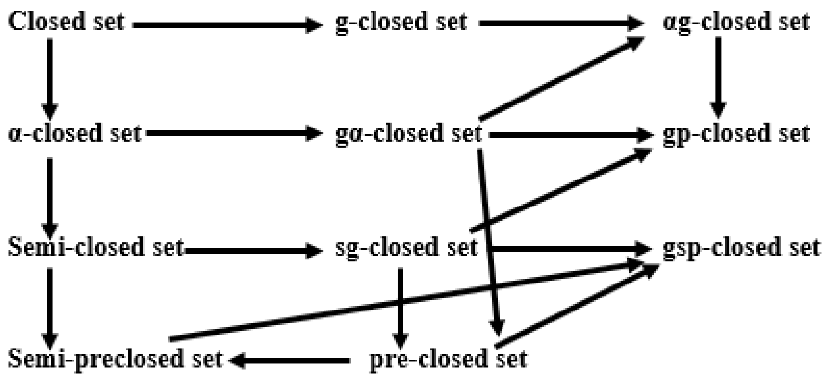
Let X be a topological space subset A of X is called

1) Pre-generalized closed (briefly, pg-closed) [17], if PclA⊆U, whenever A⊆U and is pre-open.
2) Generalized pre-closed (briefly,gp-closed) [17], if PclA⊆U, whenever A⊆U and U is open.
3) Semi-generalized closed (briefly, sg-closed) [2], if SclA⊆U whenever A⊆U and U is semi-open.
4) Generalized semi-closed (briefly, gs-closed) [2], if SclA⊆U whenever A⊆U and U is open.
5) Generalized α-closed (briefly, gα-closed) [15], if α-clA⊆U whenever A⊆U and U is α-open or equivalently, if A is g-closed w.r.t α (X).
6) α -generalized closed (briefly, αg-closed) [14], if A⊆U and U is open.
7) Generalized semi-preclosed (briefly, gs-closed) [6], if SPclA⊆U whenever A⊆U and U is open.

By definition (1) A space X has been called pre $T_{1/2}$ – space [17] if every pg-closed sets of X is pre-closed iff every singleton set of X is either pre-closed set or pre-open. Every pg-closed set is pre-closed or every space is pre $T_{1/2}$. However in any topological space, a singleton is either open or pre-closed.

So, let us recall some basic concept is A subset A of a topological space X is called α-open (resp. Semi-open, semi-preopen, if A⊆int (cl(intA)) (resp), A⊆cl(intA).The α-closed (resp, semi-closed, semi-preclosed) if X/A is α-open (resp, semi-open, semi-preopen) or equivalently if cl(int(clA)⊆A(resp, int(clA)⊆A, the α-closure (resp, semi-closure, semi-preclosure of A⊆X is the smallest α-closed (resp, semi-closed, semi-preclosed) set contain in A. it is well-known that α-clA=AUcl(int(cl(A)))SclA=AU int(clA) and SPclA=AUint(cl(int(A)).

In [7] Dontchev summarized the fundamental relationship between various type of generalized closed sets in following diagram.



[Diag.1]

Converse of some implications in the above diagram. Donrchev [7] two questions asking for the class of space in which every semi-preclosed subset is sg-closed, and for the class of space in which every pre-closed subset is gα-closed. These two questions have been considered and answered by Cao, Ganster and Reilly in [5]. Other possible converse of implications were investigated in [6].

II. Definition

T_{gs}-spaces and Pre-closed sets (generalized) It is obvious that in any topological space X, every Sg-closed subset of X is gs-closed. We start with establishing relation between various generalized pre-closed set. In [16] the class of T_{gs}-space was introduced where a space X is called T_{gs} if every gs-closed subset of X is sg-closed. The following result T_{gs}-space has been obtained in [5] and [6].

Lemma 1

For a space X the following are equivalent.

1) X is a T_{gs}-Space
2) Every generalized Semi-Preclosed subset of X is semi-preclosed (ie) X is Semi-Pre $T_{1/2}$ [6].
3) Every singleton is either preopen or closed [5].
4) Every αg-closed subset of X is gα-closed [6].

In our next result we offer additional characterization of T_{gs}-space thereby answering several possible questions in Diag [1].

Corollary 1.1

Every $T_{1/2}$ space is T_{gs}

A space X is called Semi-T_1[19] if each singleton is semi-closed, it is called semi-$T_{1/2}$ [3] if every singleton is either semi-closed or semi-open. Let S(X) be the semi-regulation of a space X. The closure of a subset A of X w.r.t S(X) will be denoted by δ-clA. A subset A of X is called δ-generalized closed if δ-clA⊆U hen A⊆U and U is open in X.

Theorem 2.1

For any space X is

1) Every gsp-closed subset of X is gp-closed
2) Every semi-pre-closed subset of X is gp-closed
3) The space X is extremely disconnected

Proof : 1→2 is obvious. Therefore we have to show that 2→3 and 3→1.

1) 2→3. Let A be regular open subset of X. then A is semi-preclosed. By hypothesis A is gp-closed and so PclA⊆A→ A=cl(int A). Therefore A is closed and hence X is extremely disconnected.

2) 3→1. Let A be a gsp-closed subset of X, and let U⊆X be open A⊆U. If B=SPclA, then by assumption, A⊆B⊆U. Since B is semi-preclosed. By theorem [5] we have that B is pre-closed. We have PclA⊆B⊆U (ie) A is gp-closed.

Corollary 2.1

For a space X the following are equivalent

1) Every gsp-closed subset of X is pre-closed
2) X is T_{gs} and extremely disconnected

III. Dontchev's questions

In recent paper [7] Dontchev posed the following two open question concerning generalized closed sets.

Question 3.1

1) Every semi-preclosed t is sg-closed.
2) Every pre-closed set is gα-closed.

In order to answer these question we need some preparation. Let S be a subset of a space X. A resolution of S is a pair $<E_1,E_2>$ of disjoint dense subset of S. The subset is said to be resolvable if it possesses a resolution, otherwise S is called irresolvable. In addition, S is called strongly, S is called strongly irresolvable, if every open subspace of S is irresolvable, observe that of $<E_1, E_2>$ is a resolution of S then E_1 and E_2 are condense in X, (ie) have empty interior.

Note: Every submaximal space is hereditarily irresolvable. Every space of X has a unique decomposition X=FUG, where F is closed and resolvable and G is open and hereditarily irresolvable [11, 18].

In this paper, the representation X=FUG, where F and G are as in (Note) will be called Hewitt decomposition of X.

Theorem 3.1

For a space X the following are

1) Every θp-closed set is αg-closed
2) Every pre-closed set is αg-closed
3) Every pre-closed set is gα-closed

Proof: from 1→2 and 3→2 are obvious and 2→3 follows by question(1).

2→1. Let A be gp-closed and A⊆U where U is open. If B=PclA then B⊆U, by B is αg-closed and so α-clA⊆α-clB⊆U (ie) A is αg-closed.

Corollary 3.1

For any space X the following are

1) Every gp-closed set is gα-closed
2) X is T_{gs} and every gp-closed set is αg-closed

Proof: 1→2 we show that X is T_{gs}. Let xϵX and suppose that {x} is now where dense and not closed. Then X\{x} is α-open and gp-closed and so α-cl(X\{x}\{x}). Thus X\{x} is α-closed and {x} is open, a contradiction. This prove that X is T_{gs}.

2→1 follows from Lemma1.

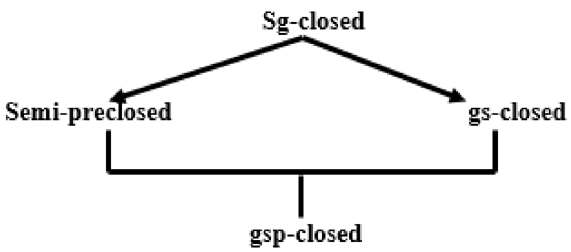
In concluding this section we provide an example of a space where every gp-closed set is αg-closed but which fails to be T_{gs}, hence must have a gp-closed subset which is not gα-closed.

Example 3.2

Let X be the set of natural number with ϕ, X and sets of the form {1,2,…n}, nϵN, as open sets. Since {1}⊆U for every open set U,X is strongly irresolvable and so, by question (1) every gp-closed set is αg-closed. If m>1, then cl{m} = {m, m+1,…}. Hence {m} is nowhere dense but not closed, so X is not T_{gs}.

IV. gp-closed sets and sg-closed sets

In this section we shall consider the relationship between gp-closed sets and sg-closed set (resp, gs-closed set).First observe that every sg-closed set is obviously gs-closed. The relationship between sg-closed sets (gs-closed) sets and other generalized pre-closed set can be illustrated in the diagram.



[Diag.2]

In general the notions of gp-closed sets and sg-closed (gs-closed) sets are independent each other (ie) X is said to be sg-submaximal [5] if every dense subset is sg-open.

Theorem 4.1

For a space X the following equivalent

1) Every gs-closed set of X is gp-closed
2) Every sg-closed subset of X is gp-closed
3) Every semi-closed subset of X is gp-closed

4) Every space X is extremely disconnected

Proof: 1→2→3 are obvious. We shall show 3→4→1

3→4. Let A be a regular open subset of X. then A is semi-closed. By assumption, A is gp-closed and A⊆A, SoPclA=clA⊆A is closed and thus X is extremely disconnected.

4→ 1. Let A be gs-closed with A⊆U are U is open. Then SclA=AU int (clA)⊆U. By int (clA)is closed and so clearly PclA=AUcl(intA)⊆U (ie) A is gp-closed.

Theorem 4.2

For a space X the followings are

1) Every gp-closed set is gs-closed
2) Every pre-closed set is gs-closed
3) X is sg-submaximal

Proof: 1→2→3 is Theorem (4.5 in [7])

2→1. Let A be gp-closed with A⊆U where U is open. If B=PclA then B is preclosed and B⊆U. Therefore B is gs-cloesd and so SclA⊆SclB⊆U. (ie) A is gs-closed. The proof of the result is similar to that of Theorem 4.2, thus is omitted.

Theorem 4.3

For space X the following are

1) Every gsp-closed set of X is gs-closed
2) Every semi-preclosed set of X is gs-closed

Preposition 4.1

If every gp-closed subset of a space X is sg-closed, then X is T_{gs}.

Proof : Suppose that {x} is nowhere dense but not closed. Then X \{x} is semi-open and gp-closed. By assumption, X{x} is sg-closed and thus semi-closed. So {x} is semi-open, contradicting the fact that {x} is nowhere dense.

Corollary 4.1

A space X is gg-submaximal iff every preclosed subset of X is gs-closed

Proof: The necessity is trivial by (Every pre-closed subset of X is sg-closed). For the sufficiency, suppose that every pre-closed subset is gs-closed. Let X=FUG be the Hewitt decomposition of X, and let $<E_1, E_2>$ be a resolution of int F.

We first claim that every open set V⊆int F is regular open. V⊆E_1 is co-dense and contained in V. since co-dense sets are preclosed, by assumption, they are gs-closed. Thus int(cl(V⊆E_1))⊆V. on the other hand, E_1 is dense in int F, hence we have int (cl(V⊆E_1))=int(clV). It follows V=int(clV).

Let x∊int F and V=int F∩(X\cl{x}). Suppose that {x} is nowhere dense. Then X\cl{x} is dense and int (clV)=int (cl(int F)) =int F, Our claim int F = V. Hence F⊆X\{x}. acontradiction. {x} is pre-open. We have proved that int F⊆X_2 (ie) X_1⊆clG. By the theorem X is sg-submaximal.

Remarks 4.1

One way ask whether every sg-submaximal space has to be T_{gs}. This is, however not the case. The space is our example 3.2 is not T_{gs} and has the property that every pre-closed set is gα-closed and thus gs-closed. Hencely theorem 4.2 it is sg-submaximal.

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