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Commutativity of Prime Near Γ-rings with Nonzero Reverse σ-derivations and Derivations

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Abstract

Let M be a near Γ-ring, and let σ be an automorphism on M. Let d be a reverse σ-derivation on M. In this study, we investigate the commutativity of a prime near Γ-rings M employing certain conditions on non-zero reverse σ-derivations d and non-zero derivations on M.

Keywords: Γ-rings, prime near Γ-rings, non-zero reverse σ-derivations, non-zero derivations, commutativity of prime near Γ-rings.

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I. Introduction

Let M and Γ be additive abelian groups. M is said to be a Γ-ring if there exists a mapping $M \times \Gamma \times M \to M$ (sending (x, α, y) into $x\alpha y$) such that
(a) $(x + y)\alpha z = x\alpha z + y\alpha z$,
$x(\alpha + \beta)y = x\alpha y + x\beta y$,
$x\alpha(y + z) = x\alpha y + x\alpha z$,
(b) $(x\alpha y)\beta z = x\alpha(y\beta z)$,
for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.

A Γ-ring M (not necessarily abelian under addition) is called a right (resp. left) near Γ-ring satisfying the right distribution law over addition (resp. left distribution law over addition). A right (resp. left) near Γ-ring M is said to be a zero-symmetric right (resp. left) near Γ-ring if $0\alpha x = 0$ (resp. $x\alpha 0 = 0$), for all $x \in M$ and $\alpha \in \Gamma$. A near Γ-ring M is said to be prime if $x\alpha M\beta y = 0$ implies either $x = 0$ or $y = 0$, for all $x, y \in M$ and $\alpha, \beta \in \Gamma$. A near Γ-ring M is said to be 2-torsion free if $x + x = 0$ for all $x \in M$ implies $x = 0$. The center of M is denoted by Z and is defined by $Z = \{x \in M : x\alpha y = y\alpha x \text{ for all } y \in M \text{ and } \alpha \in \Gamma\}$. The commutator $x\alpha y - y\alpha x$ is denoted by $[x, y]_\alpha$. In our paper, we use the condition $x\alpha y\beta z = x\beta y\alpha z$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$ whereabouts we need and we denote it by $(*)$. With the help of $(*)$, the useful notations $[x, y\beta z]_\alpha$ and $[x\alpha y, z]_\beta$ are given by $[x, y\beta z]_\alpha = y\alpha[x, z]_\beta + [x, y]_\alpha\beta z$ and $[x\alpha y, z]_\beta = x\alpha[y, z]_\beta + [x, z]_\alpha\beta y$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$. An additive mapping $d : M \to M$ is said to be a derivation on M if $d(x\alpha y) = x\alpha d(y) + d(x)\alpha y$ for all $x, y \in M$ and $\alpha, \beta \in \Gamma$. The derivation d is said to be commuting on M if $[d(x), x]_\alpha = 0$ for all $x \in M$ and $\alpha \in \Gamma$.

Y.Ceven [10] studied on Jordan left derivations on completely prime Γ-rings. He investigated the nonzero Jordan left derivation on a completely prime Γ-ring that can make the Γ-ring commutative by an assumption. He proved that every Jordan left derivation on a completely prime Γ-ring is a left derivation on it by the same assumption. In this paper, he created an example of Jordan left derivation on Γ-rings.

Mustafa Asci and Sahin Ceran [7] investigated a nonzero left derivation d on a prime Γ-ring M that makes M commutative with the conditions $d(U) \subseteq U$ and $d^2(U) \subseteq Z$, with an ideal U of M and the center Z of M. They also studied the commutativity of M using the nonzero left derivation d_1 and right derivation d_2 on M such that $d_2(U) \subseteq U$ and $d_1 d_2(U) \subseteq Z$.

A derivation and a Jordan derivation on Γ-rings are defined due to Sapanci and Nakajima [8]. They proved that a Jordan derivation on a certain type of completely prime Γ-ring is a derivation. They also set examples of a derivation and a Jordan derivation of Γ-rings.

A. K. Halder and A. C. Paul [1] investigated the existence of a non-zero Jordan left derivation of a 2-torsion free prime Γ-ring into a faithful ΓM-module X that makes M commutative. They also proved that M is commutative if $d : M \to M$ is a derivation.

K. K. Dey and A. C. Paul [5] studied σ-derivations d on Prime Gamma-Near-Rings with automorphism σ on Prime Gamma-Near-Rings. In this study, they proved that if d is a σ-derivation such that σd = dσ with d2 $d^2 = 0$, then $d = 0$. They also investigated composition $\sigma\tau$-derivations of two derivations σ and τ on Prime Gamma-Near-Rings.

A. M. Ibraheem [3] investigated the commutativity of prime Γ-near- rings M with the help of generalized Γ-derivations F and G satisfying certain conditions.

In this study, we develop the results of A. M. Ibraheem [2] and the commutativity part of [4] in Γ-ring version. We prove the commutativity of a prime near Γ-ring M with a reverse σ-derivation $d : M \to M$ satisfying certain properties. We also prove the commutativity of a two-torsion free prime near Γ-ring in presence of a non-zero derivation with some conditions.

II. Initial Results

To prove our main results we need the following definition and lemmas:

Definition 2.1 *If M is a near Γ-ring and σ is an automorphism on M then an additive mapping $d : M \to M$ is said to be a reverse σ-derivation whenever $d(x\alpha y) = d(y)\alpha x + \sigma(y)\alpha d(x)$, for all $x, y \in M$ and $\alpha \in \Gamma$.*

Lemma 1 *For any near Γ-ring M and any arbitrary automorphism $d : M \to M$, $d(x\alpha y) = \sigma(y)\alpha d(x) + d(y)\alpha x$ if and only if $d(x\alpha y) = d(y)\alpha x + \sigma(y)\alpha d(x)$, for all $x, y \in M$ and $\alpha \in \Gamma$. Therefore d is a reverse σ-derivation on M if and only if $d(x\alpha y) = d(y)\alpha x + \sigma(y)\alpha d(x)$, for all $x, y \in M$ and $\alpha \in \Gamma$.*

Proof

Suppose $d(x\alpha y) = \sigma(y)\alpha d(x) + d(y)\alpha x$, for all $x, y \in M$ and $\alpha \in \Gamma$.
Since $(x + x)\alpha y = x\alpha y + x\alpha y$ for all $x, y \in M$ and $\alpha \in \Gamma$,

$d(x + x)\alpha y$

$$= \sigma(y)\alpha d(x + x) + d(y)\alpha(x + x)$$
$$= \sigma(y)\alpha d(x) + \sigma(y)\alpha d(x) + d(y)\alpha x + d(y)\alpha x \ldots \quad (1)$$

and

$$d(x\alpha y + x\alpha y)$$
$$= d(x\alpha y) + d(x\alpha y)$$
$$= \sigma(y)\alpha d(x) + d(y)\alpha x + \sigma(y)\alpha d(x) + d(y)\alpha x \ldots \quad (2)$$

for all $x, y \in M$ and $\alpha \in \Gamma$.
Equations (1) and (2) yield
$d(x\alpha y) = \sigma(y)\alpha d(x) + d(y)\alpha x = d(y)\alpha x + \sigma(y)\alpha d(x)$, for all $x, y \in M$ and $\alpha \in \Gamma$.
The reverse part is similar.

Lemma 2 *If d is a non-zero reverse σ-derivation on a prime near $\Gamma - ring$ M and $d(M) \subset d(Z)$ then M is a commutative near Γ-ring.*

Proof
Suppose $d(x) \in Z$ for all $x \in M$. Then

$$d(x)\alpha z = z\alpha d(x) \ldots \quad (3),$$

for all $z \in Z$, $x \in M$ and $\alpha \in \Gamma$.
We replace x by $x\beta y$ in equation (3) and then use the action of d to get

$$(d(y)\beta x + \sigma(y)\beta d(x))\alpha z = z\alpha(d(y)\beta x + \sigma(y)\beta d(x)),$$

which with $(*)$ yields

$$\sigma(y)\alpha d(x)\beta z - z\beta\sigma(y)\alpha d(x)$$
$$= -d(y)\alpha x\beta z + z\beta d(y)\alpha x$$
$$= -d(y)\alpha x\beta z + d(y)\alpha z\beta x \ldots \quad (4),$$

for all $x, y \in M$, $z \in Z$ and $\alpha, \beta \in \Gamma$.
Replacing $\sigma(y)$ by $d(x)$ in equation (4) and then using (3), we have

$$d(y)\alpha(-x\beta z + z\beta x) = d(y)\alpha[z, x]_\beta = 0 \quad (5),$$

for all $x, y \in M$, $z \in Z$ and $\alpha, \beta \in \Gamma$.
We write $z\alpha y$ for z in equation (5) and then use equation (5) and $(*)$ in the obtained result to get

$$d(y)\alpha z\beta[y, x]_\alpha = 0,$$

for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.
Since M is prime and d is non-zero,

$$[y, x]_\alpha = 0,$$

for all $x, y \in M$ and $\alpha \in \Gamma$. Therefore M is a commutative prime near Γ-ring.

Lemma 3 *If d is a non-zero reverse σ-derivation on a prime near Γ-ring M with center Z then $d(Z) \subset Z$.*

Proof
Let $x \in M$, $z \in Z$ and $\alpha \in \Gamma$. Then we have

$$d(x\alpha z) = d(z\alpha x).$$

Now by using Lemma 1 and then replacing $\sigma(z)$ by Z in the obtained result, we have

$$d(x\alpha z) = z\alpha d(x) + d(z)\alpha x \ldots \qquad (6),$$

for all $x, z \in M$ and $\alpha \in \Gamma$.
Also,

$$d(z\alpha x) = d(x)\alpha z + \sigma(x)\alpha d(z) \ldots \qquad (7),$$

for all $x, z \in M$ and $\alpha \in \Gamma$.
Equations (6) and (7) give

$$d(z)\alpha x = \sigma(x)\alpha d(z),$$

for all $x, z \in M$ and $\alpha \in \Gamma$.
Since σ is an automorphism on M, $\sigma(x) = x$ yielding $d(z)\alpha x = x\alpha d(z)$ for all $x, z \in M$ and $\alpha \in \Gamma$. Therefore $d(z) \in Z$ and so $d(Z) \subset Z$.

Lemma 4 *If d is non-zero reverse σ-derivation on a prime near Γ-ring M and $x\alpha d(M) = 0$ or $d(M)\alpha x = 0$ for all $x \in M$ and $\alpha \in \Gamma$ then $x = 0$.*

Proof Suppose

$$x\alpha d(v) = 0 \ldots \qquad (8),$$

for all $v \in M$ and $\alpha \in \Gamma$.
Replacing v by $u\beta v$ in equation (8), and by definition of d, we have

$$x\alpha d(v)\beta u + x\alpha\sigma(v)\beta d(u) = 0 \ldots \qquad (9),$$

for all $x, u, v \in M$ and $\alpha, \beta \in \Gamma$.
Since σ is an automorphism on M, $\sigma(v) = v$ and we have by equations (8) and (9)

$$x\alpha v\beta d(u) = 0,$$

for all $x, u, v \in M$ and $\alpha, \beta \in \Gamma$.
This implies that

$$x\alpha M\beta d(u) = 0,$$

for all $x, u \in M$ and $\alpha, \beta \in \Gamma$.
Since M is prime, $x = 0$.
Similarly, if $d(M)\alpha x = 0$ for all $x \in M$ and $\alpha \in \Gamma$, it can be shown that $x = 0$.

3 Commutativity of Prime Near Γ-rings With Non-zero Reverse σ-derivations

Theorem 1 *Let d be a non-zero reverse σ-derivation on a prime near Γ-ring M with center Z satisfying $[x, d(x)]_\alpha = 0$ for all $x \in M$ and $\alpha \in \Gamma$. Then M is commutative.*

Proof
Let $[x, d(x)]_\alpha = 0$ for all $x \in M$ and $\alpha \in \Gamma$. We replace $d(x)$ by $y\beta d(x)$ in $[x, d(x)]_\alpha = 0$ and then use $(*)$ and the given condition in the obtained result to get

$$[x, y]_\alpha \beta d(x) = 0 \ldots \qquad (10),$$

for all $x, y \in M$ and $\alpha, \beta \in \Gamma$. Putting $z\alpha y$ for y in equation (10) and then using $(*)$ and (10) in the obtained result, we get

$$[x, z]_\alpha \alpha y \beta d(x) = 0,$$

for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.
This gives

$$[x, z]_\alpha \alpha M \beta d(x) = 0,$$

for all $x, z \in M$ and $\alpha, \beta \in \Gamma$.
Since M is prime and d is non-zero, it follows that $[x, z]_\alpha = 0$ and that $x \in Z$ for any fixed $x \in M$. Thus by Lemma 3, $d(x) \in Z$ and so $d(M) \subset Z$. Hence by Lemma 2, M is commutative.

Theorem 2 *If d is a non-zero reverse σ-derivation on a prime near Γ-ring M with center Z such that $[d(y), d(x)]_\alpha = 0$ for all $x, y \in M$ and $\alpha \in \Gamma$, then M is commutative.*

Proof
Let

$$[d(y), d(x)]_\alpha = 0 \ldots \qquad (11),$$

for all $x, y \in M$ and $\alpha \in \Gamma$.
Replacing y by $y\beta x$ in (11) and then using action of d, $(*)$, and (11) in the obtained result, we have

$$d(x)\alpha[y, d(x)]_\beta + [\sigma(x), d(x)]_\alpha \beta d(y) = 0 \ldots \qquad (12),$$

for all $x, y \in M$ and $\alpha, \beta \in \Gamma$.
For $z \in Z$, we replace $z\alpha y$ for y in (12) and use $(*)$ to get

$$d(x)\alpha z\alpha[y,d(x)]_\beta + d(x)\alpha[z,d(x)]_\alpha\beta y + [\sigma(x),d(x)]_\alpha\alpha d(y)\beta z$$
$$+[\sigma(x),d(x)]_\alpha\alpha\sigma(y)\beta d(z) = 0\ldots \qquad (13),$$

for all $x,y,z \in M$ and $\alpha,\beta \in \Gamma$.
Since σ is an automorphism on M, $\sigma(x) = x$ and $\sigma(y) = y$ in (3) and then (12) i (13) gives that

$$[x,d(x)]_\alpha\alpha y\beta d(z) = 0,$$

for all $x,y,z \in M$ and $\alpha,\beta \in \Gamma$.
This gives that (13) gives that

$$[x,d(x)]_\alpha\alpha M\beta d(z) = 0,$$

for all $x,z \in M$ and $\alpha,\beta \in \Gamma$.
Since M is prime and d is non-zero, $[x,d(x)]_\alpha = 0$ for all $x \in M$ and $\alpha \in \Gamma$.
Thus by Theorem 1, M is commutative.

Theorem 3 *If d is a non-zero reverse σ-derivation on a prime near Γ-ring M with center Z satisfying $[x,d(y)]_\alpha \in Z$ for all $x,y \in M$ and $\alpha, \in \Gamma$, then M is commutative.*

Proof
Suppose that $[x,d(y)]_\alpha \in Z$ for all $x,y \in M$ and $\alpha, \in \Gamma$. Then for any $u \in M$ and $\beta \in \Gamma$, we have

$$[[x,d(y)]_\alpha,u]_\beta = 0\ldots \qquad (14)$$

Replacing x by $x\alpha d(y)$ in (14) and then using $*$ and (14) in the obtained result, we have

$$[x,d(y)]_\alpha\alpha[d(y),u]_\beta = 0\ldots \qquad (15),$$

for all $x,y,u \in M$ and $\alpha,\beta \in \Gamma$.
Putting $u\alpha x$ for x in (15) and then using $(*)$ and (15) in the gained result, we get

$$[u,d(y)]_\alpha\alpha x\beta[d(y),u]_\alpha = 0$$

for all $x,y,u \in M$ and $\alpha,\beta \in \Gamma$. This implies that

$$[u,d(y)]_\alpha\alpha M\beta[d(y),u]_\alpha = 0$$

for all $y,u \in M$ and $\alpha,\beta \in \Gamma$.
Since M is prime, either

$$[u,d(y)]_\alpha = 0\ldots \qquad (16)$$

for all $y,u \in M$ and $\alpha \in \Gamma$,
or

$$[d(y),u]_\alpha = 0\ldots \qquad (17)$$

for all $y, u \in M$ and $\alpha \in \Gamma$.
Now, replacing $d(y)$ by $v\alpha d(y)$ in equations (16) and (17) and using equations (16) and (17) in the obtained results, we have $[u, v]_\alpha \alpha d(y) = 0$ or $[v, u]_\alpha \alpha d(y) = 0$ for all $y, u, v \in M$ and $\alpha \in \Gamma$.
Then by Lemma 4, we have $[u, v]_\alpha = 0$ and $[v, u]_\alpha = 0$ for all $u, v \in M$ and $\alpha \in \Gamma$. Hence M is commutative.

Theorem 4 *If d is d is a non-zero reverse σ-derivation on a prime near Γ-ring and $x \in M$ with center Z such that $[x, d(x)]_\alpha = 0$ for all $\alpha \in \Gamma$, then $d(x) = 0$ or $x \in Z$, and so M is commutative.*

Proof
Suppose

$$[x, d(x)]_\alpha = 0 \ldots \qquad (18)$$

for all $x \in M$ and $\alpha \in \Gamma$.
Replacing $d(x)$ by $y\beta d(x)$ in equation (18) and then using $(*)$ and (18) in obtained results, we have

$$[x, y]_\alpha \beta d(x) = 0 \ldots \qquad (19)$$

for all $x, y \in M$ and $\alpha, \beta \in \Gamma$.
Putting y by $z\alpha y$ in equation (19) and then using $(*)$ and (19) in the gained result, we have

$$[x, z]_\alpha \alpha y \beta d(x) = 0,$$

for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$. This implies that $[x, z]_\alpha \alpha M \beta d(x) = 0$ for all $x, z \in M$ and $\alpha, \beta \in \Gamma$.
Since M is prime and d is non-zero, $[x, z]_\alpha = 0$ for all $x, z \in M$ and $\alpha \in \Gamma$. This implies $x \in Z$ for any fixed $x \in M$, and so by Lemma 3, $d(M) \subset Z$. Therefore by Lemma 2, M is commutative.

Theorem 5 *If d is a non-zero reverse $\sigma-$derivation on a prime near Γ-ring M such that $d([x, y]_\alpha) = [x, d(y)]_\alpha$ for all $x, y \in M$ and $\alpha \in \Gamma$, then M is commutative.*

Proof
Let

$$d([x, y]_\alpha) = [x, d(y)]_\alpha \ldots \qquad (20),$$

for all $x, y \in M$ and $\alpha \in \Gamma$.
Writing $y\beta x$ in equation (20) and then employing $(*)$ and (20) in the obtained outcome, we have

$$[x, d(x)]_\alpha \beta y + [x, \sigma(x)]_\alpha \beta d(y) = 0 \ldots \qquad (21),$$

for all $x, y \in M$ and $\alpha, \beta \in \Gamma$. Since σ is an automorphism on M, $\sigma(x) = x$ and so (21) becomes

$$[x, d(x)]_\alpha \beta y = 0 \ldots \qquad (22),$$

for all $x, y \in M$ and $\alpha, \beta \in \Gamma$.
Now, replacing y by $y\alpha d(x)$ in equation (22) and using $(*)$, we have

$$[x, d(x)]_\alpha \alpha y \beta d(x) = 0,$$

for all $x, y \in M$ and $\alpha, \beta \in \Gamma$.
This implies $[x, d(x)]_\alpha \alpha M \beta d(x) = 0$ for all $x \in M$ and $\alpha, \beta \in \Gamma$.
Since M is prime and d is non-zero,

$$[x, d(x)]_\alpha = 0,$$

for all $x \in M$ and $\alpha \in \Gamma$.
Therefor by Theorem 1 it follows that M is commutative.

4 Commutativity of 2-torsion free Prime Near Γ-rings With Non-zero Derivations

Theorem 6 *If d is a non-zero derivation on a 2-torsion free prime near Γ-ring M such that $[d(x), y]_\alpha = [x, d(y)]_\alpha$ for all $x, y \in M$ and $\alpha \in \Gamma$, then M is commutative.*

Proof
Let

$$[d(x), y]_\alpha = [x, d(y)]_\alpha \ldots \qquad (23),$$

for all $x, y \in M$ and $\alpha \in \Gamma$.
Replacing $d(y)\beta x$ for x in equation (23),and then using $(*)$, we get

$$d(d(y)\alpha x)\beta y, -y\beta d(d(y)\alpha x) = d(y)\alpha d(x)\beta y - d(y)\alpha y \beta d(x)$$

which gives

$$d(y)\alpha d(x)\beta y + d^2(y)\alpha x \beta y - y\alpha d^2(y)\beta x - y\alpha d(y)\beta d(x) = d(y)\alpha d(x)\beta y - d(y)\alpha y \beta d(x).$$

Since $y\alpha d(y) = d(y)\alpha y$, the last equation becomes

$$d^2(y)\alpha x \beta y = y\alpha d^2(y)\beta x \ldots \qquad (24),$$

for all $x, y \in M$ and $\alpha, \beta \in \Gamma$. Putting $z\alpha x$ for x in equation (24) with $(*)$, we have

$$d^2(y)\alpha z \alpha x \beta y = y\alpha d^2(y)\alpha z \beta x = d^2(y)\alpha z \alpha y \beta x,$$

for all $x, y \in M$ and $\alpha, \beta \in \Gamma$.
yielding

$$d^2(y)\alpha M \beta [x, y]_\alpha = 0,$$

for all $x, y \in M$ and $\alpha \in \Gamma$.
Therefore by (Theorem 3.6, [9]), M is commutative.

Theorem 7 *If d is a non-zero derivation on a 2-torsion free prime near Γ-ring M such that $[d(x), y]_\alpha = [d(x), d(y)]_\alpha$ or $[x, d(y)]_\alpha = [d(x), d(y)]_\alpha$ for all $x, y \in M$ and $\alpha \in \Gamma$, then M is commutative.*

Proof
First suppose that

$$[d(x), y]_\alpha = [d(x), d(y)]_\alpha \ldots \qquad (25),$$

for all $x, y \in M$ and $\alpha \in \Gamma$.
Using $d(x)\beta y$ for y in equation (25) and employing $(*)$ with simplification, we get

$$d(x)\alpha d(d(x)\beta y) - d(d(x)\beta y)\alpha d(x) = d(x)\alpha d(x)\beta d(y) - d(x)\alpha d(y)\beta d(x)$$

which leads to

$$d(x)\alpha d(x)\beta d(y) + d(x)\alpha d^2(x)\beta y - d^2(x)\alpha y\beta d(x) - d(x)\alpha d(y)\beta d(x) = d(x)\alpha d(x)\beta d(y) - d(x)\alpha$$

which reduces to

$$d(x)\alpha d^2(x)\beta y = d^2(x)\alpha y\beta d(x) \ldots \qquad (26),$$

for all $x, y \in M$ and $\alpha, \beta \in \Gamma$.
Replacing y by $z\alpha y$ in equation (26) to get

$$d^2(x)\alpha z\beta y\alpha d(x) = d(x)\alpha d^2(x)\alpha z\beta y = d^2(x)\alpha z\beta d(x)\alpha y,$$

for all $x, y \in M$ and $\alpha, \beta \in \Gamma$,
yielding

$$d^2(x)\alpha M\beta[d(x), y]_\alpha = 0$$

for all $x, y \in M$ and $\alpha, \beta \in \Gamma$.
Since M is prime either $d^2(y) = 0$ or $[d(x), y]_\alpha = 0$.
Putting $d(y)$ for y in equation (25), we have

$$[d(x), d(y)]_\alpha = 0,$$

for all $x, y \in M$ and $\alpha \in \Gamma$.
Therefore M is commutative due to (Theorem 3.6, [9]).
The proof of the part assuming $[x, d(y)]_\alpha = [d(x), d(y)]_\alpha$ for all $x, y \in M$ and $\alpha \in \Gamma$ is straightforward.

Theorem 8 *If d is a non-zero derivation on a 2-torsion free prime near Γ-ring M such that $[d(x), y]_\alpha = -[d(x), d(y)]_\alpha$ or $[x, d(y)]_\alpha = -[d(x), y]_\alpha$ for all $x, y \in M$ and $\alpha \in \Gamma$, then M is commutative.*

Proof

Suppose that

$$[d(x), y]_\alpha = -[d(x), d(y)]_\alpha \dots \qquad (27),$$

for all $x, y \in M$ and $\alpha \in \Gamma$.

Replacing y by $d(x)\beta y$ in equation (27), and then using $(*)$ with simplification, we get

$$d(x)\alpha d(d(x)\beta y) - d(d(x)\beta y)\alpha d(x) = d(x)\alpha d(x)\beta d(y) - d(x)\alpha d(y)\beta d(x)$$

which gives

$$d(x)\alpha d(x)\beta d(y) + d(x)\alpha d^2(x)\beta y - d^2(x)\alpha y\beta d(x) - d(x)\alpha d(y)\beta d(x) = d(x)\alpha d(x)\beta d(y) - d(x)\alpha$$

yielding

$$d(x)\alpha d^2(x)\beta y = d^2(x)\alpha y\beta d(x) \dots \qquad (28),$$

for all $x, y \in M$ and $\alpha, \beta \in \Gamma$.

Substituting $z\alpha y$ for y in equation (28), we have

$$d^2(x)\alpha z\beta y\alpha d(x) = d(x)\alpha d^2(x)\alpha z\beta y = d^2(x)\alpha z\beta d(x)\alpha y,$$

for all $x, y \in M$ and $\alpha, \beta \in \Gamma$,

leading to

$$d^2(x)\alpha M\beta[d(x), y]_\alpha = 0$$

for all $x, y \in M$ and $\alpha, \beta \in \Gamma$.

Since M is prime either $d^2(y) = 0$ or $[d(x), y]_\alpha = 0$.

Replacing y by $d(y)$ in equation (27), we have

$$[d(x), d(y)]_\alpha = 0,$$

for all $x, y \in M$ and $\alpha \in \Gamma$.

Therefore M is commutative by (Theorem 3.6, [9]).

By the same process M is commutative if we assume that $[x, d(y)]_\alpha = -[d(x), y]_\alpha$ for all $x, y \in M$ and $\alpha \in \Gamma$.

5 Comparison

K. K. Dey and A. C. Paul [5] worked on σ-derivations and their compositions on Prime Gamma-Near-Rings while we used non-zero reverse σ-derivations on prime near Γ-rings M and non-zero derivations d on 2-torsion free prime near Γ-rings M to show the commutativity of M. We proved the commutativity of prime near Γ-rings applying non-zero reverse σ-derivations on the whole prime near Γ-ring M while A. M. Ibraheem [3] investigated the commutativity of prime Γ-near- rings M with the help of generalized Γ-derivations F and G satisfying certain conditions considering subsets of prime Γ-near-rings M.

6 Conclusion

Under certain conditions on a non-zero reverse σ-derivation d on a prime near Γ-ring M with center Z of M and with an automorphism σ on M, M is commutative. M is also commutative under non-zero derivations d on M with conditions $[d(x), y]_\alpha = [x, d(y)]_\alpha$, $[d(x), y]_\alpha = [d(x), d(y)]_\alpha$, $[x, d(y)]_\alpha = [d(x), d(y)]_\alpha$, $[d(x), y]_\alpha = -[d(x), d(y)]_\alpha$ and $[x, d(y)]_\alpha = -[d(x), y]_\alpha$ for all $x, y \in M$ and $\alpha \in \Gamma$.

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