International Journal of Research in Engineering and Science (IJRES)
ISSN (Online): 2320-9364, ISSN (Print): 2320-9356
www.ijres.org Volume 8 Issue 11 ǁ 2020 ǁ PP. 48-55

Decision of optimum number of lobes for a stair climber with conjugate wheels

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***Abstract**: The paper puts forth detailed optimization procedure for number of lobes of an innovative 'Low Cost'design of staircase climbing platform. Total new concept of specially designed wheels is put forth. The saidconfigurationuses 'Lobbed' driving wheels withthe shape of each lobe,conjugate with the profile of steps. There was an open choice to select number of lobes. Hence it was necessary to decide optimum number of lobes for the proposed design. Universal method of 'Design of Experiments' is used for the said optimization and the results are presented in last section of this paper.*

***Keywords:** staircase climbing platform; lobbed wheels; conjugate profile; design of experiments*

Date of Submission: 28-10-2020 Date of acceptance: 09-11-2020

LIST OF ABBRIVATIONS
θ = angle of slope of the staircase
ϕ = wheel rotation angle when contact is with Going
r_0 = minimum radius of Logarithmic Spiral
ϕ_0 = angle of wheel radius w.r.t min. radius position
H = step height
D = step depth
F = ratio (D / H) of the step
n = number of lobes of the wheel
Hyp= hypoteneus of the right angled triangle bounded by Rise ang Go of thestep

I. INTRODUCTION

Many old buildingsfrom densely populated areas of society are without lifts. People with restricted mobility find it difficult to climb stairs. In order to tackle their difficulties, a new platform with conjugate wheels was proposed [1] earlier. The platform is in the form of chassis. One can change the body that is built on it, to get 'Stair Climbing Wheelchair' or 'Stair Climbing Trolley'. The wheels of that platform are completely conjugate with the staircase for which they are designed. DOE technique is used to find optimum number of lobes suitable for this application.

II. CONJUGATE PROFILE GENERATION PROCESS

'Rack Shift' method is used for generation of so called conjugate wheel. Combination of steps is treated as 'Rack' that has triangular profile. 'Wheel Blank' of appropriate dimension in relation with staircase rack is shown in Fig 1

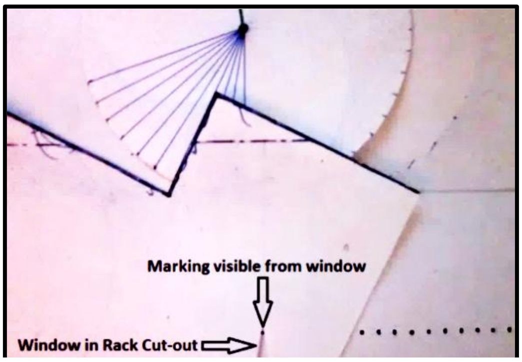


Figure 1: Wheel blank in relation with staircase rack

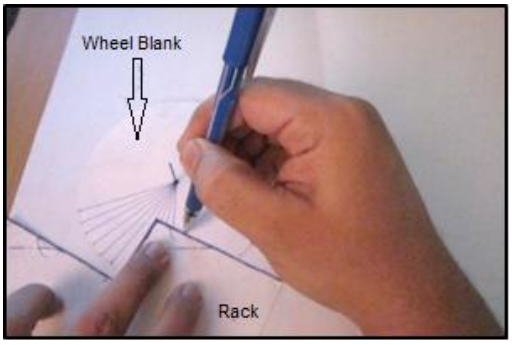


Figure 2: Transfer of rack profile on wheel blank

Rack profile is manually transferred on the wheel blank as shown in figure 2. The procedure is gradually repeated by shifting rack ahead and by rotating the blank till successive graduation. After completing one rotation of the blank we get an output as shown in Fig 3. By increasing blank diameter we can even get four lobes as shown in Fig 4.

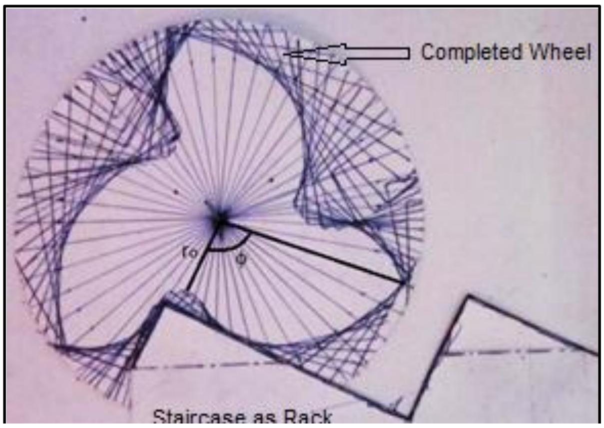


Figure 3: Completed profile of the proposed wheel with three lobes

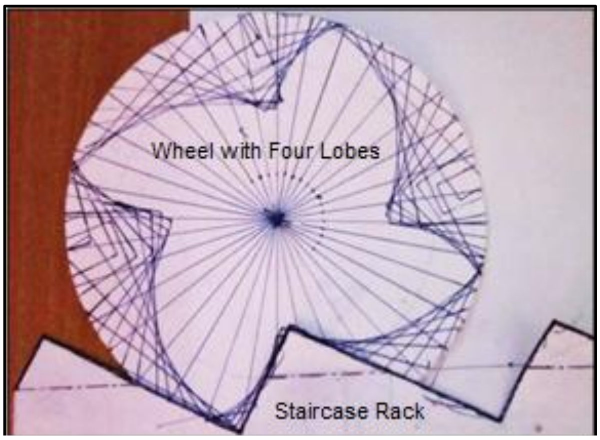


Figure 4: Completed profile of the proposed wheel with four lobes

Any number of lobes is possible, the minimum number being two. Optimum number of lobes can be decided based on theDOE analysis of the candidate wheels.
While climbing, the longer curve will maintain contact with ‘Going’ (i.e. horizontal part of step) while smaller curve is supposed to touch ‘Riser’ (i.e vertical surface) but vertical crawling is impossible hence that smaller curve can be neglected and the wheel can be relieved in that section.

III. MATHEMATICALLY DERIVED EQUATION OF THE PROFILE

From the conjugate wheel generation process it can be seen that the generated curve issuitable forspecific staircase. If thedimensions of the steps and H:D ratio is varied, the shape of the wheel changes.Hence it was must toderivemathematical equation of the curve.

By using first principals, the most generalized equation of 'conjugate profile' is obtained as follows[1]:

$$r = r_0 . e^{\tan\theta.(\emptyset)} \text{……………………..(1)}$$

Wherer_0 and ϕare 'Minimum Radius' and 'Angle of rotation of the wheel during ascend' respectively. These values are based on required number of lobes for the conjugate wheel and step proportions as well as dimensions.

Systematic procedure is followed to find out these two parameters for the generalized staircase. After rigorous mathematical analysis [1] the minimum radius is found to be as follows:

$$r_0 = \frac{D * H}{Hyp\ (e^{\tan\theta.(\emptyset)} - 1)} \text{…..................(2)}$$

The angle of rotation of the wheel during which it maintains contact with the horizontal surface (∅)is also found out systematically. The equation derived is as follows:

$$\emptyset = \frac{2\pi .\ F^2}{n(F^2 + 1)} \text{………..…. (3a) (Angle in Radians)}$$

If the angles are considered in degrees then the equation (19) will take the form:

$$\emptyset = \frac{360 .\ F^2}{n\ (F^2 + 1)} \text{…………..…. (3b) (Angle in Degrees)}$$

Equations (2) and (3) can to be used to find values of $\mathbf{r_0}$ and **ϕ** for the wheel that has **'n'** lobes and which is conjugate to a staircase with step height **'H'** as well as depth to height ratio of steps **'F'.**

3. USE OF DESIGN OF EXPERIMENTS TECHNIQUE FOR OPTIMIZATION

Formula for calculating the total torque required to drive the platform is also derived **[1]** and the same is as follows:

$$T = (W \sin\theta)\ (r_{max}) \text{………………(4)}$$

By substituting (2) and (3) in (1), we get,

$$rmax = \left(\frac{D * H}{Hyp\ \left(e^{\tan\theta.\left(\frac{2\pi .\ F^2}{n(F^2 + 1)}\right)} - 1\right)} \right) . e^{\tan\theta.\left(\frac{2\pi .\ F^2}{n(F^2 + 1)}\right)} \text{………….....(5)}$$

From (4) and (5) we get,

$$T = (W \sin\theta) \left(\frac{D * H}{Hyp\ \left(e^{\tan\theta.\left(\frac{2\pi .\ F^2}{n(F^2 + 1)}\right)} - 1\right)} \right) . e^{\tan\theta.\left(\frac{2\pi .\ F^2}{n(F^2 + 1)}\right)} \text{…………(6)}$$

From the configuration it is seen that one lobe is used to climb one step. If number of lobes on the wheel are more, we need to rotate the wheel slower in order to maintain same rate of ascend. Hence as the number of lobes increases, rotational speed (ω) decreases proportionately. Power required to drive the platform is the actual point of concern and the same is given by the following equation [1].

$$P = T.\omega \text{…………....... (7)}$$

Substituting (6) in (7), we get

$$P = (W \sin\theta) \left(\frac{D * H}{Hyp\ \left(e^{\tan\theta.\left(\frac{2\pi .\ F^2}{n(F^2 + 1)}\right)} - 1\right)} \right) . e^{\tan\theta.\left(\frac{2\pi .\ F^2}{n(F^2 + 1)}\right)} \times \omega \text{ …...(8)}$$

In the above formula, weight (W) is constant while rotational speed (ω) is to be adjusted according to the number of lobes (n). Terms (Hyp) (θ) and (F) depend on (D) and (H) so the effective variables are (D), (H) and (n).

Power required to drive the platform can be calculated by using formula (8) and it can be treated as a response for these variables. It is necessary to choose or select optimum values of these variables.

Standard ranges of (D) and (H) vary according to countries. Standards from USA [2], UK [3], India [4] and Australia [5] are studied in depth.

First three standards specify same maximum value for riser as 190mm and same minimum value of going as 250mm. Building Code of Australia (BCA) specifies the upper and lower limit for 2(H) + (D) as 735 and 470 respectively but the ranges of (H) and (D) are slightly wider.

After clubbing all these standards, we decide following ranges for 'Rising' and 'Going':

Table1: Standard dimensions of staircases

Riser (H)		Going (D)		2(H) + (D)	
Maximum	Minimum	Maximum	Minimum	Maximum	Minimum
190	115	355	240	735	470

Intermediate values of these variables are decided exactly midway and the number of lobes is varied as 3, 4 and 5. Thus it is a 3 level 3 factor design.

An Excel program is generated fo3^3=27 calculations of torque for different combinations of (H) (D) and (n)

	A	B	C	D	E	F	G	H	I
1			Min	Med	Max				
2	W in N	1200	1200	1200	1200	1200	1200	1200	1200
3	RPM	10							
4	Going	D	240	297.5	355		240	240	240
5	Rise	H	115	152.5	190		115	115	115
6	No	N	3	4	5		3	4	5
7	Factor	F					2.086956522	2.086956522	2.086956522
8									
9	D SQ						57600	57600	57600
10	H SQ						13225	13225	13225
11	SUM						70825	70825	70825
12	SQRT	S					266.1296676	266.1296676	266.1296676
13									
14	Phi deg	ϕ d					97.59265796	73.19449347	58.55559478
15	Phi rad	ϕ c					1.70399879	1.277999092	1.022399274
16									
17	Delta deg	δ d					120	90	72
18	Alfa deg	α d					22.40734204	16.80550653	13.44440522
19									
20	tan θ						0.479166667	0.479166667	0.479166667
21	(tan θ).ϕ						0.81649942	0.612374565	0.489899652
22	e^tan θ. ϕ						2.262565666	1.844806816	1.632152428
23	coeff of rmin						1.262565666	0.844806816	0.632152428
24	DH/S						103.7088433	103.7088433	103.7088433
25	rmin						82.14134605	122.7604245	164.0567032
26	rmax						185.8501893	226.4692678	267.7655465
27									
28	TORQUE						96371.54083	117434.329	138848.2755

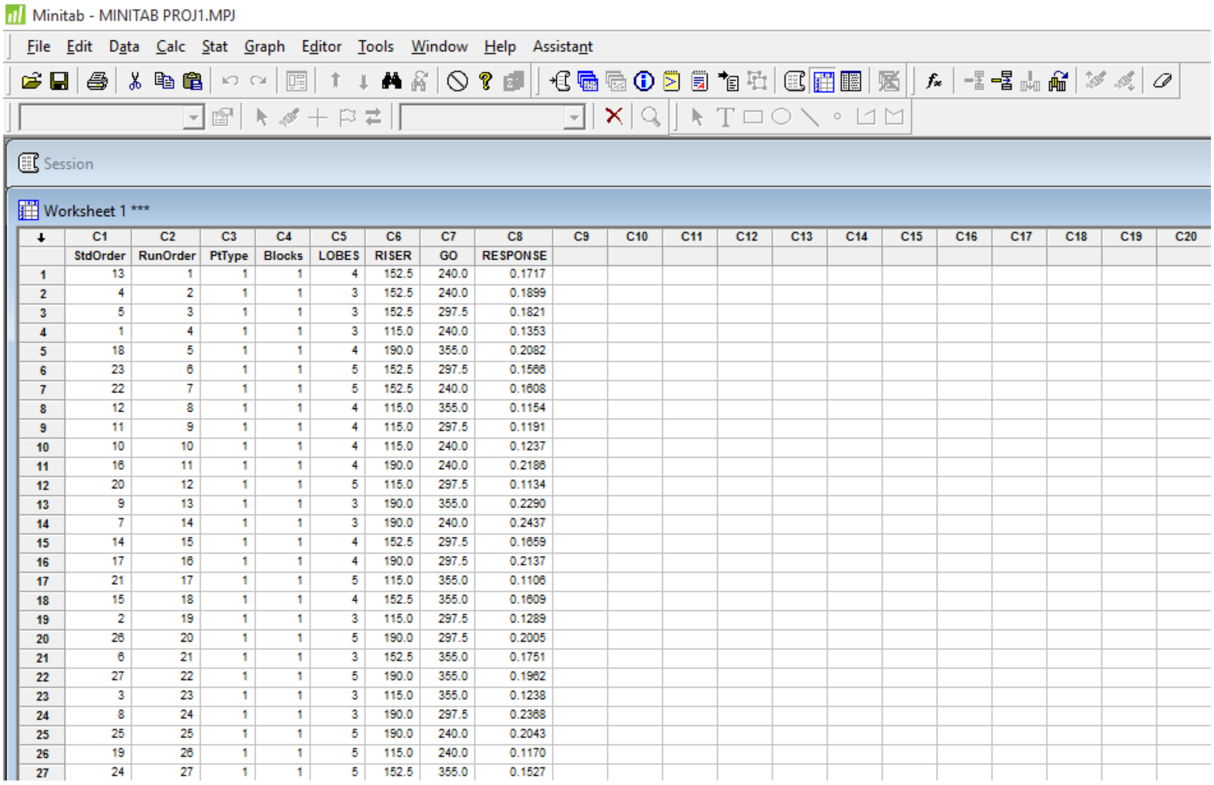
Figure 5: Spread sheet of the torque calculations

It is seen that one step is climbed by using one lobe of the wheel hence Rotational velocity (ω) can be adjusted for different numbers of lobes so as to keep velocity of ascend of the platform as constant.

Power requirement for climbing the stairs is used as the response of the three variables.Similar to torque calculation, power calculation (in kW) is also added in excel sheet and the corresponding values are stored. Full factorial optimization is completed using Design of Experiments (DOE) technique.

'MINITAB' is a statistics package developed by Pennsylvania state university for performing statistical analysis. It uses different variables along with their responses to study 'Main Effects' as well as 'Interactions' of different variables with the response.

The said software is effectively used for studying the characteristics for the above mentioned problem. Fig 6 shows the input screen of the said software for the above problem.



↓	C1 StdOrder	C2 RunOrder	C3 PtType	C4 Blocks	C5 LOBES	C6 RISER	C7 GO	C8 RESPONSE
1	13	1	1	1	4	152.5	240.0	0.1717
2	4	2	1	1	3	152.5	240.0	0.1899
3	5	3	1	1	3	152.5	297.5	0.1821
4	1	4	1	1	3	115.0	240.0	0.1353
5	18	5	1	1	4	190.0	355.0	0.2082
6	23	6	1	1	5	152.5	297.5	0.1566
7	22	7	1	1	5	152.5	240.0	0.1608
8	12	8	1	1	4	115.0	355.0	0.1154
9	11	9	1	1	4	115.0	297.5	0.1191
10	10	10	1	1	4	115.0	240.0	0.1237
11	16	11	1	1	4	190.0	240.0	0.2186
12	20	12	1	1	5	115.0	297.5	0.1134
13	9	13	1	1	3	190.0	355.0	0.2290
14	7	14	1	1	3	190.0	240.0	0.2437
15	14	15	1	1	4	152.5	297.5	0.1659
16	17	16	1	1	4	190.0	297.5	0.2137
17	21	17	1	1	5	115.0	355.0	0.1106
18	15	18	1	1	4	152.5	355.0	0.1609
19	2	19	1	1	3	115.0	297.5	0.1289
20	26	20	1	1	5	190.0	297.5	0.2005
21	6	21	1	1	3	152.5	355.0	0.1751
22	27	22	1	1	5	190.0	355.0	0.1962
23	3	23	1	1	3	115.0	355.0	0.1238
24	8	24	1	1	3	190.0	297.5	0.2368
25	25	25	1	1	5	190.0	240.0	0.2043
26	19	26	1	1	5	115.0	240.0	0.1170
27	24	27	1	1	5	152.5	355.0	0.1527

Figure 6: Minitab input screen for above problem

Main effects plot for Response (Power) against Number of lobes (n) is as shown in figure 7 below:

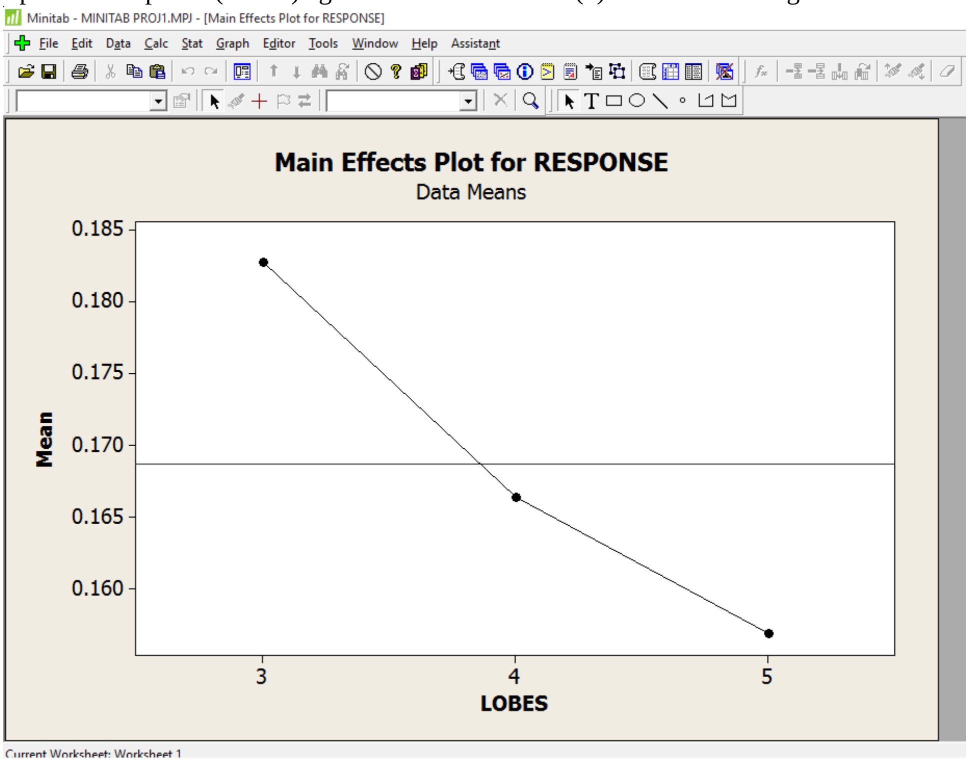


Figure 7: Main effects plot for Power against number of lobes

From the above plot it is seen that the power requirement reduces as the number of lobes is increased but the rate of reduction in power requirement also reduces as the lobes are increased.
Interactions plot for Rise (H), Go (D) and number of lobes (n) against Response (Power is as shown in Fig 8 below:

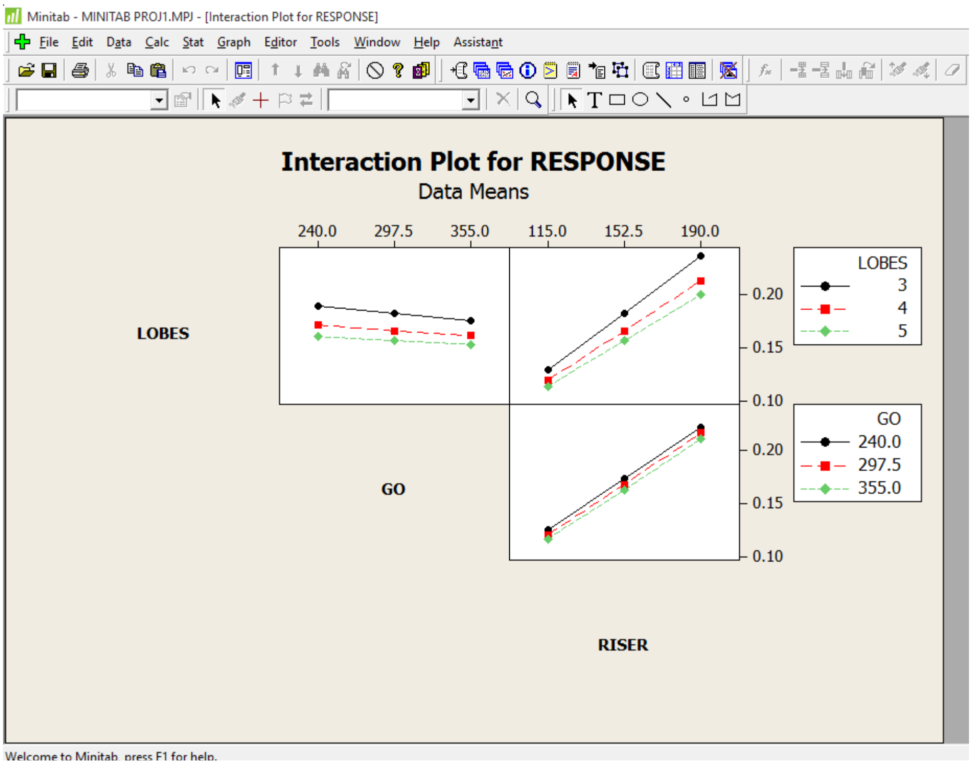


Figure 8: Interactions plot for Power against (H) (D) and (n)

It is seen that power requirement reduces with increase in the 'Go' dimension (H) for all number of lobes but the overall power requirement is minimum for n = 5

Also the power requirement increases with increase in 'Riser' i.e. step height but the overall increase in power is less if the number of lobes is more (n=5 in this case)

One way analysis of Means (ANOM) is as follows:

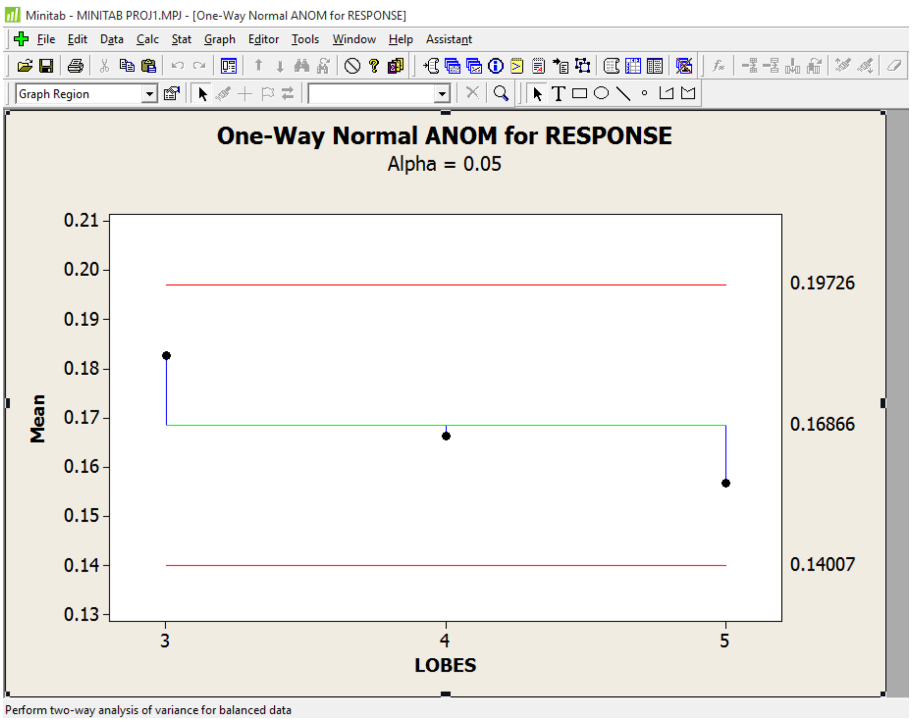


Figure 9: Interactions plot for Power against (H) (D) and (n)

With more number of lobes (n=5) the power requirement for ascend is well below its mean value.

IV. FINDINGS AND EXPERIMENTAL VALIDATION

For checking the feasibility of projectedconfiguration, a full scalesetup is built [1]. It is shown in fig 10. Chain-sprocket drive connects the two shafts. The drive isa closed loop drive hence the petals of wheel aremaintained in phase. Alternating Current motor drives the configuration via gearbox.

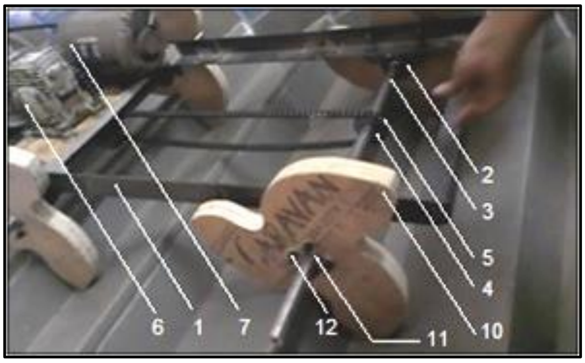


Figure 10: Prototype of proposed platform

Parts in the setup:-

1. Chassis
2. Bearing housings
3. Axles
4. Sprocket wheels
5. Mechanical Chain
6. Reduction drive
7. 1ph motor as prime mover
8. Switches (Not shown)
9. Cable (Not shown)
10. Driving wheels having generated outline
11. Hubs for mounting the drive wheels
12. Nuts and Bolts

Various other parameters are as follows:

- Platform chassis (in cm) :- 110 x 58
- Chassis Weight (in kg) :- 38
- Total Design load (in kg) :- 90
- Actuator specification (in HP) :- 0.75
- Climbing Period (in sec) :- 36
- RPM of the drive axle :- 10

It is found that the shafts are moving exactly along path parallel to the 'pitch line'.
The force being applied at the farthest point of contact is calculated by using load cells. Force value is found out to be 42 kgf ≈413 N
In the basic analysis[1] it was projected earlier that the three lobes design would be ideal one[1]. This projection was totally based on the torque requirement.
For comparing different configurations, rate of climbing must be kept constant. Hence the wheel with lesser lobes must be rotated faster than the one with more number of lobes. This can be done by changing the gearbox.
Comparison of initially used transmission for 10 RPM with the new transmission for 6 RPM is given in Table 2 below:

Table2: Comparison of transmissions in the two trials

Parameter	for Three Lobe design	For Five Lobe design
Motor RPM	1440	1440
Reduction in Pulley	1:2	1:2
Reduction in Gearbox	1:60	1:80
Reduction in Sprocket	5:6	2:3
Shaft speed in RPM	$1440 \times \frac{1}{2} \times \frac{1}{60} \times \frac{5}{6} = 10$	$1440 \times \frac{1}{2} \times \frac{1}{80} \times \frac{2}{3} = 6$

Following table shows comparative observations as well as calculations of different parameters, ultimately leading to power required to drive the setup.

Table3: Comparative observations and calculations for Three and Five lobed designs

Parameter (Unit)	Value for setup with three lobes	Value for setup with five lobes
Force (N)	413 N	414 N
Longest Torque Arm (m)	0.2 m	0.25 m
Torque (N-m)	82.6 Nm	103.5 Nm
Shaft speed (RPM & Rad/s)	10 RPM = 1.047 rad/s	06 RPM = 0.6283 rad / s
Power required for 1 wheel (W)	86.48 W	65.03 W
Total power requirement (W)	345.92 W	260.12 W

Calculated torque for the wheel with laser number of lobes is less than the one for wheel with more number of lobes. But the speed of rotation for the later is smaller than the former. Effect of all these parameters is accumulated in the power requirement.

V. CONCLUSION

Effect of increase in the torque requirement by increasing the number of lobes is less that the effect of corresponding decrease in the rotational speed.

Hence, based on the 'Power required to drive the staircase climbing platform', it was found that the minimum power requirement is for five lobed design even though the corresponding torque requirement is more.

CONFLICT OF INTEREST

No conflict of Interest for the presented work.

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