Real Time Traffic Control to Optimize Waiting Time of Vehicles at A Road Intersection

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Abstract: This paper presents a real time non-linear quadratic programming model to minimize the aggregate delay time of vehicles at each lane by minimizing number of vehicles at each lane at the signalized intersection. One of the most efficient and easily getting factors of traffic signal control is the number of vehicles in a queue at the signalized intersection. The initial number of vehicles at each lane at the intersection is counted by camera which is the most accurate method. The model is developed to minimize the number of vehicles from cycle to cycle. This proposed model includes inter green signal time which is one of the key factors compared to other existing models proposed in the past research. This model also incorporates restrictions for upper and lower bounds for green signal time and cycle time allocation which leads to accurate and appropriate allocation for green signal time. The proposed model is solved by the method of sequential quadratic programming coded in MATLAB environment.

Keywords—aggregate delay time, nonlinear sequential quadratic programming, optimization, real time, traffic signal

I INTRODUCTION

In any country traffic congestion wastes a huge amount of the national income for fuel and traffic-related environmental and socio economic problems. Traffic intersections contribute a lot to this traffic congestion. Signalized intersections are crucial points in the highway systems. The past researches [1] reveal that the real time traffic control strategy gives a better traffic control system than the fixed-time (off-line) traffic signal control strategy. Real time traffic signal control optimization is complex due to its nature. Delay [2] and the number of vehicles waiting are the important measures of effectiveness for signalized intersections. This research considers the oversaturation level at road intersections, where the intersections considered have four signals for a particular minimum cycle time, and the green time is allocated for all four signals.

Our objective of this research is to formulate a mathematical model to minimize aggregate delay [3] time of vehicles and the total number of vehicles which are waiting in the lanes of a road intersection due to red signal by allocating sufficient amount of green time for each signal and cycle time. To maintain the feasibility, the upper bound for cycle time and the upper bound of number of vehicles waiting are adjusted according to the incoming flow rate and outgoing flow rate of vehicles. The real time data is calculated by using cameras [4] installed in every lane in the road intersection. This model is solved by sequential quadratic programming algorithm coded in MATLAB environment [5].

II METHODOLOGY

In this research we consider a signalized isolated intersection with four lanes namely Lane \( j \), \( j = 1,2,3,4 \). We divide this intersection into four stages: in Stage 1, the green signal will be on for Lane 1, where the vehicles which are waiting at the Lane 1 can move into other three lanes through the intersection as shown in the figure below:
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Fig. 1. Green signal for Lane 1

Also, when the green signal is on for another lane the vehicles waiting in that lane will proceed to other lanes in a similar manner as described above in the Fig. 1.

2.1 Formulation of the Model

2.1.1 Notations
Lane $j$: $j^{th}$ lane at the road intersection, $j = 1, 2, 3, 4$,
$N_j(k)$: Number of vehicles in Lane $j$ at cycle $k$,
$I_g$: inter green time,
$W_j$: Weighting parameter of Lane $j, j = 1, 2, 3, 4$,
$t_j(k)$: Allocated green time for the signal for Lane $j, j = 1, 2, 3, 4$ at cycle $k$,
$s_j(k)$: Outgoing flow rate of vehicles for the Lane $j, j = 1, 2, 3, 4$ at cycle $k$,
$(t_j(k))_{min}$: Minimum green time for the signal for Lane $j, j = 1, 2, 3, 4$, at cycle $k$,
$(t_j(k))_{max}$: Maximum green time for the signal for Lane $j, j = 1, 2, 3, 4$, at cycle $k$,
$C(k)$: Cycle time at cycle $k$,
$C_{T_{min}}$: Minimum cycle time,
$C_{T_{max}}$: Maximum cycle time,
$D_j(k + 1)$: Aggregate delay time of vehicles in cycle $k$ at Lane $j, j = 1, 2, 3, 4$.

2.1.2 Formulation
In Stage 1, the aggregate delay time of vehicles for $(k+1)^{th}$ cycle (aggregate delay time of vehicles at the end of $k^{th}$ cycle) is calculated by the aggregate delay time of vehicles in the Lane 1 during green signal is on and after the green signal is off which is represented by the shaded area of Stage 1 in Fig.2.

In Stage 2, the aggregate delay time of vehicles for $(k+1)^{th}$ cycle (aggregate delay time of vehicles at the end of $k^{th}$ cycle) is calculated by the aggregate delay time of vehicles in the Lane 2 before green signal is on, during green signal is on and after the green signal is off which is represented by the shaded area of Stage 2 in Fig.2. Similarly for the other two stages, the total delay time for $(k+1)^{th}$ cycle is illustrated in the figure below.
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![Diagram showing the number of vehicles and delay of four stages](image)

Aggregate delay time for vehicles at each stage at the end of cycle $k$ is calculated from the area of the shaded region in the Fig.2 as given below:

$$D_1(k + 1) = \frac{1}{2} 2N_1(k)C(k) + f_1(k)C(k)^2 - s_1(k)(t_1(k))^2 - 2s_1(k)t_1(k)(4I_g + t_2(k) + t_3(k) + t_4(k))$$  \hspace{1cm} (1)

$$D_2(k + 1) = \frac{1}{2} 2N_2(k)C(k) + f_2(k)C(k)^2 - s_2(k)(t_2(k))^2 - 2s_2(k)t_2(k)(3I_g + t_3(k) + t_4(k))$$  \hspace{1cm} (2)

$$D_3(k + 1) = \frac{1}{2} 2N_3(k)C(k) + f_3(k)C(k)^2 - s_3(k)(t_3(k))^2 - 2s_3(k)t_3(k)(2I_g + t_4(k))$$  \hspace{1cm} (3)

$$D_4(k + 1) = \frac{1}{2} 2N_4(k)C(k) + f_4(k)C(k)^2 - s_4(k)(t_4(k))^2 - 2s_4(k)t_4(k)I_g$$  \hspace{1cm} (4)

The number of vehicles in four lanes at the end of the cycle $k$ is given by the following four equations. First equation represents number of vehicles in Lane 1 at the end of cycle $k$ (or beginning of cycle $k+1$) which is equivalent to the total of the number of vehicles waiting at the beginning of cycle $k$ (from camera readings), incoming number of vehicles into the Lane 1 during green signal time, inter signal green time and red signal time and excluding outgoing number of vehicles during green signal time for the Lane 1. Similarly, the number of vehicles in Lane 2, Lane 3 and Lane 4, at the end of cycle $k$ (for cycle $k+1$) are given by the remaining three equations respectively:

$$N_1(k + 1) = N_1(k) + (t_1(k) + I_g)f_1(k) - (t_1(k))s_1(k) + (t_2(k) + I_g)f_1(k) + (t_3(k) + I_g)f_1(k) + (t_4(k) + I_g)f_1(k)$$  \hspace{1cm} (5)

$$N_2(k + 1) = N_2(k) + (t_2(k) + I_g)f_2(k) - (t_2(k))s_2(k) + (t_1(k) + I_g)f_2(k) + (t_3(k) + I_g)f_2(k) + (t_4(k) + I_g)f_2(k)$$  \hspace{1cm} (6)

$$N_3(k + 1) = N_3(k) + (t_3(k) + I_g)f_3(k) - (t_3(k))s_3(k) + (t_1(k) + I_g)f_3(k) + (t_2(k) + I_g)f_3(k) + (t_4(k) + I_g)f_3(k)$$  \hspace{1cm} (7)

$$N_4(k + 1) = N_4(k) + (t_4(k) + I_g)f_4(k) - (t_4(k))s_4(k) + (t_1(k) + I_g)f_4(k) + (t_2(k) + I_g)f_4(k) + (t_3(k) + I_g)f_4(k)$$  \hspace{1cm} (8)
Lower and upper bounds of green signal time:
\[(t_j(k))_{\text{min}} \leq t_j(k) \leq (t_j(k))_{\text{max}}, \ j = 1,2,3,4\]  

(9)

Cycle time of cycle \(k\) is given by the total green signal time and total inter green signal time:
\[(t_1(k) + Ig) + (t_2(k) + Ig) + (t_3(k) + Ig) + (t_4(k) + Ig) = C(k)\]  

(10)

Lower and upper bounds of cycle time:
\[CT_{\text{min}} \leq C(k) \leq CT_{\text{max}}\]  

(11)

When twice the number of outgoing vehicles during green signal time in a lane at a given a cycle is less than the number of vehicles in that lane at the end of that cycle, it is defined as oversaturation condition. Each of the following four inequalities represents the oversaturation condition for each of the four lanes during cycle \(k\):
\[2s_1(k) t_1(k) \leq N_1(k + 1)\]  

(12)
\[2s_2(k) t_2(k) \leq N_2(k + 1)\]  

(13)
\[2s_3(k) t_3(k) \leq N_3(k + 1)\]  

(14)
\[2s_4(k) t_4(k) \leq N_4(k + 1)\]  

(15)

The incoming number of vehicles into the lane during a cycle time is less than or equal to the outgoing number of vehicles from the lane during green signal time for that lane is also considered as a condition. Each of the following four inequalities represents that condition for each of the respective four lanes during cycle \(k\):
\[f_1(k) C(k) \leq s_1(k) t_1(k)\]  

(16)
\[f_2(k) C(k) \leq s_2(k) t_2(k)\]  

(17)
\[f_3(k) C(k) \leq s_3(k) t_3(k)\]  

(18)
\[f_4(k) C(k) \leq s_4(k) t_4(k)\]  

(19)

2.2 Flow Chart of the Model and the Method of Solution

Fig.3. Flow chart for the model and the method of solution
III OPTIMIZE GREEN TIME

Objective is to minimize the total number of vehicles waiting at the intersection subject to the oversaturation condition, the delay, additional condition and some constraints related to the traffic signal control problem, which are described above, are combined into the model formulation and is illustrated below to calculate the duration of the green signal time on at the beginning of the cycle k:

Minimize $Z = \sum_{j=1}^{4} W_{j} N_{j}(k + 1)$

Subject to

$N_{1}(k + 1) = N_{1}(k) + (t_{1}(k) + 1g) \times f_{1}(k) - (t_{1}(k)) \times s_{1}(k) + (t_{2}(k) + 1g) \times f_{1}(k) + (t_{3}(k) + 1g) \times f_{1}(k)$,

$N_{2}(k + 1) = N_{2}(k) + (t_{2}(k) + 1g) \times f_{2}(k) - (t_{4}(k)) \times s_{2}(k) + (t_{1}(k) + 1g) \times f_{2}(k) + (t_{3}(k) + 1g) \times f_{2}(k)$,

$N_{3}(k + 1) = N_{3}(k) + (t_{3}(k) + 1g) \times f_{3}(k) - (t_{4}(k)) \times s_{3}(k) + (t_{1}(k) + 1g) \times f_{3}(k) + (t_{2}(k) + 1g) \times f_{3}(k)$,

$N_{4}(k + 1) = N_{4}(k) + (t_{4}(k) + 1g) \times f_{4}(k) - (t_{4}(k)) \times s_{4}(k) + (t_{1}(k) + 1g) \times f_{4}(k) + (t_{2}(k) + 1g) \times f_{4}(k)$,

$2s_{1}(k) t_{1}(k) \leq N_{1}(k + 1)$,

$2s_{2}(k) t_{2}(k) \leq N_{2}(k + 1)$,

$2s_{3}(k) t_{3}(k) \leq N_{3}(k + 1)$,

$2s_{4}(k) t_{4}(k) \leq N_{4}(k + 1)$,

$f_{1}(k) C(k) \leq s_{1}(k) t_{1}(k)$,

$f_{2}(k) C(k) \leq s_{2}(k) t_{2}(k)$,

$f_{3}(k) C(k) \leq s_{3}(k) t_{3}(k)$,

$f_{4}(k) C(k) \leq s_{4}(k) t_{4}(k)$,

$D_{1}(k + 1) = \frac{1}{2} \left[ N_{1}(k) C(k) + f_{1}(k) C(k)^{2} - s_{1}(k) t_{1}(k) \left(41g + t_{2}(k) + t_{3}(k) + t_{4}(k)\right) \right] \geq 0$,

$D_{2}(k + 1) = \frac{1}{2} \left[ N_{2}(k) C(k) + f_{2}(k) C(k)^{2} - s_{2}(k) t_{2}(k) \left(31g + t_{3}(k) + t_{4}(k)\right) \right] \geq 0$,

$D_{3}(k + 1) = \frac{1}{2} \left[ N_{3}(k) C(k) + f_{3}(k) C(k)^{2} - s_{3}(k) t_{3}(k) \left(21g + t_{4}(k)\right) \right] \geq 0$,

$D_{4}(k + 1) = \frac{1}{2} \left[ N_{4}(k) C(k) + f_{4}(k) C(k)^{2} - s_{4}(k) t_{4}(k) \left(1g\right) \right] \geq 0$,

$(t_{1}(k) + 1g) + (t_{2}(k) + 1g) + (t_{3}(k) + 1g) + (t_{4}(k) + 1g) = C(k)$,

$(t_{j}(k))_{\text{min}} \leq t_{j}(k) \leq (t_{j}(k))_{\text{max}}$, $j = 1, 2, 3, 4$,

$CT_{\text{min}} \leq C(k) \leq CT_{\text{max}}$.

The cycle time is equal to the sum of the total green signal time and total inter green time. Each green time value has a minimum value $(t_{j}(k))_{\text{min}}$ and a maximum value $(t_{j}(k))_{\text{max}}$ which are fixed for a cycle.

In order to maintain the feasibility, the sum of the total minimum green signal time values and inter green signal time values is assumed to be greater than or equal to $CT_{\text{min}}$ and also, the sum of the total maximum green signal time values and inter green signal time values is assumed to be less than or equal to $CT_{\text{max}}$.

The objective function consists of waiting parameters $W_{j}, j = 1, 2, 3, 4$ assigned to each lane. The default value of $W_{j} = 1, 2, 3, 4$ is assigned to 1. The objective function can be optimized by selecting different waiting parameters $W_{j}$ according to different criteria: lane priority, emergency vehicle passing etc.

To optimize green time for each signal we apply sequential quadratic programming algorithm implemented in the MATLAB optimization toolbox for the above nonlinear programming problem.

IV RESULTS AND DISCUSSION

The method is tested to a hypothetical data set:

Distance between upstream camera and downstream camera installed in any lane is 125 m. If we assume that the average length of a small vehicles is approximately 5 m, then the maximum number of vehicles in a lane within 125 m is 25.

Incoming flow rates of vehicles for the lanes are fixed over the cycles given by $f_{1} = 0.05$ vehicles/sec., $f_{2} = 0.05$ vehicles/sec., $f_{3} = 0.05$ vehicles/sec., $f_{4} = 0.1$ vehicles/sec.

Outgoing flow rates of vehicles for the lanes are fixed over the cycles given by $s_{1} = 0.5$ vehicles/sec., $s_{2} = 0.5$ vehicles/sec., $s_{3} = 0.5$ vehicles/sec., $s_{4} = 0.5$ vehicles/sec.
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Inter green signal time is given by $I_g = 1$ sec.

Weighting parameters of the lanes are given by $W_j = 1$ for $j = 1, 2, 3, 4$.

Simulation results and corresponding graphs are given in the following Table 1, Fig.4 and Fig.5 for scenario 1:

From camera readings: $N_1(1) = 20, N_2(1) = 23, N_3(1) = 21, N_4(1) = 25$

<table>
<thead>
<tr>
<th>Cycle $k$</th>
<th>Number of vehicles in lanes at the beginning of the cycle $N_j(k), j = 1, 2, 3, 4$</th>
<th>Green signal time (sec.) $t_j(k), j = 1, 2, 3, 4$</th>
<th>Cycle time (sec.) $C(k)$</th>
<th>Aggregate delay time at the end of the cycle (sec.) $D_j(k+1), j = 1, 2, 3, 4$</th>
<th>Number of vehicles in lanes at the end of the cycle $N_j(k+1), j = 1, 2, 3, 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20 23 21 25</td>
<td>16 21 17 22</td>
<td>80</td>
<td>1184, 1609, 1564, 2188</td>
<td>16 21 17 22</td>
</tr>
<tr>
<td>2</td>
<td>16 21 17 22</td>
<td>13 19 14 19</td>
<td>69</td>
<td>817, 1255, 1096, 1656</td>
<td>13 18 13 19</td>
</tr>
<tr>
<td>3</td>
<td>13 18 13 19</td>
<td>11 16 11 17</td>
<td>59</td>
<td>560, 924, 719, 1214</td>
<td>10 16 10 16</td>
</tr>
<tr>
<td>4</td>
<td>10 16 10 16</td>
<td>08 14 08 14</td>
<td>48</td>
<td>362, 659, 458, 827</td>
<td>08 14 08 14</td>
</tr>
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<td>07 12 07 12</td>
</tr>
<tr>
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<td>07 12 07 12</td>
<td>06 10 06 10</td>
<td>36</td>
<td>185, 377, 239, 467</td>
<td>06 11 06 11</td>
</tr>
<tr>
<td>7</td>
<td>06 11 06 11</td>
<td>05 10 05 10</td>
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<td>05 09 05 09</td>
</tr>
<tr>
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<td>04 08 04 08</td>
<td>28</td>
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<td>04 08 04 08</td>
</tr>
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<td>04 07 04 07</td>
<td>26</td>
<td>73, 181, 99, 226</td>
<td>03 07 03 07</td>
</tr>
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<td>10</td>
<td>03 07 03 07</td>
<td>03 06 03 06</td>
<td>22</td>
<td>47, 133, 64, 166</td>
<td>03 06 03 06</td>
</tr>
<tr>
<td>11</td>
<td>03 06 03 06</td>
<td>03 05 03 05</td>
<td>20</td>
<td>42, 106, 57, 131</td>
<td>03 06 03 06</td>
</tr>
<tr>
<td>12</td>
<td>03 06 03 06</td>
<td>03 05 03 05</td>
<td>20</td>
<td>42, 106, 57, 131</td>
<td>03 06 03 06</td>
</tr>
</tbody>
</table>

In the Table 1 given above, the results of cycle 11 and cycle 12 are the same. If this process continues into more cycles, the results will be the same as the results of cycle 11. Because of oversaturation situation some vehicles are still waiting in each lane in the last cycle (cycle 11).

![Number of vehicles at each cycle in all four lanes](image)

Fig.4.Number of vehicles at each cycle in Lane 1, Lane 2, Lane 3 and Lane 4 at scenario 1
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Simulation results and corresponding graphs are given in the Table 2, Fig.6 and Fig.7 for scenario 2 below:

From camera readings: $N_1(1) = 19$, $N_2(1) = 10$, $N_3(1) = 08$, $N_4(1) = 23$

### Table 2. Cycles and the optimum feasible results for scenario 2

<table>
<thead>
<tr>
<th>Cycle $k$</th>
<th>Number of vehicles in lanes at the beginning of the cycle $N_j(k), j = 1,2,3,4$</th>
<th>Green signal time (sec.) $t_j(k), j = 1,2,3,4$</th>
<th>Cycle time (sec.) $C(k)$</th>
<th>Aggregate delay time at the end of the cycle (sec.) $D_j(k+1), j = 1,2,3,4$</th>
<th>Number of vehicles in lanes at the end of the cycle $N_j(k+1), j = 1,2,3,4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>19 10 08 23</td>
<td>10 10 07 19</td>
<td>50</td>
<td>788, 455, 377, 1175</td>
<td>17 10 07 19</td>
</tr>
<tr>
<td>2</td>
<td>17 10 07 19</td>
<td>13 10 06 16</td>
<td>49</td>
<td>617, 460, 340, 979</td>
<td>13 10 06 16</td>
</tr>
<tr>
<td>3</td>
<td>13 10 06 16</td>
<td>10 10 05 14</td>
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<td>415, 587, 258, 724</td>
<td>10 09 06 13</td>
</tr>
<tr>
<td>4</td>
<td>10 09 06 13</td>
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<td>08 09 05 11</td>
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<tr>
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<td>06 08 04 10</td>
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<td>07 08 05 09</td>
</tr>
<tr>
<td>6</td>
<td>07 08 05 09</td>
<td>06 07 04 08</td>
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<td>146, 209, 142, 283</td>
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<tr>
<td>7</td>
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<td>04 06 04 07</td>
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<td>20</td>
<td>62, 106, 57, 131</td>
<td>04 06 03 06</td>
</tr>
</tbody>
</table>

In Table 2 given above, the results of cycle 9 and cycle 10 are the same. If this process continues more cycles, the results will be same as the results of cycle 9. Because of oversaturation situation some vehicles are still waiting in each lane in the final cycle (cycle 9).
V CONCLUSION

A non-linear programming optimization model is developed to minimize the real time traffic signal control efficiency (number of vehicles and delay time) at a road intersection. A flow-chart is designed to solve the model with the aid of MATLAB optimization toolbox. Entering vehicles to the road intersection is counted using cameras installed in the road intersection which is proven to be an effective technique widely used around the world. This model incorporates inter green signal time. Our proposed model provides more practical solution than the existing developed models. The method is tested using hypothetical data set and considered two situations. Results are given in Table 1 and Table 2. It can be observed that the total number of vehicles waiting in each lane is decreasing from cycle to cycle. Subsequently, the cycle time and average aggregate delay time of vehicles are also decreasing. Finally, it can be concluded that the proposed model is delivering a promising result which can be practically implemented at road intersections consist of four lanes.

**Fig. 6.** Number of vehicles at each cycle in Lane 1, Lane 2, Lane 3 and Lane 4 at scenario 2

**Fig. 7.** Aggregate delay time at each cycle for Lane 1, Lane 2, Lane 3, Lane 4 at scenario 2
REFERENCES


