Method of Fracture Surface Matching Based on Mathematical Statistics

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ABSTRACT: Fracture surface matching is an important part of point cloud registration. In this paper, a method of fracture surface matching based on mathematical statistics is proposed. We reconstruct a coordinate system of the fractured surface points, and analyze the characteristics of the point cloud in the new coordinate system, using the theory of mathematical statistics. The general distribution of the points is determined. The method can realize the matching relation among some point cloud.

Keywords: fracture surface, mathematical statistics, matching

I. INTRODUCTION

Reverse engineering is the process of digitizing existing models or products, obtaining 3D point cloud data, and reconstructing point cloud data into CAD model. On the basis of reverse engineering development, the application of reverse engineering technology to cultural relics protection is also an inevitable trend of technological development. Fracture surface matching is an important part of point cloud registration research, and it has important research significance in the fields of cultural relics repairing, medicine, and industrial design and so on. Among them, fracture surface matching technology based on the computer graphic image, pattern recognition, machine identification and other technology is also particularly important. This technology can not only save the preservation of cultural relics, but also can be applied to the restoration of cultural relics work, speeding up the restoration of cultural relics, shortening the artificial high repair cycle, reducing the artifacts of hand pieces difficult and cost and avoiding the secondary damage to cultural relics, which has a high research value and practical significance.

The general fracture surface matching includes the following steps (fig. 1). The focus of the fracture surface matching method is how to obtain good surface segmentation results and to extract the characteristic information that can represent the fracture surface. The results of these two parts are the most important research of fracture surface matching.

![Diagram of fracture surface matching steps](https://via.placeholder.com/150)

Fig.1 the steps of fracture surface matching

Good surface segmentation algorithm can reduce the difficulty of matching, and improving the correctness of matching. Surface segmentation algorithm mainly includes watershed method, regional growth method and so on. Mangan[10] proposed an improved version of the watershed surface segmentation algorithm

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based on a given threshold to prevent over-segmentation. Vieria\cite{2} proposed a regional growth segmentation algorithm that uses curvature to extract sharp feature boundaries. The algorithm based on the contour line of the fracture surface includes: Cohen\cite{3} uses the method of linear search to find the best matching region of the volume space curve, and the distance error between the two curves is used to judge whether it is the standard. Shin\cite{4} got the matching information by calculating and comparing the contour line length, concavity. Hunholm\cite{5} calculated the curvature and torsion of the sampling points on the contour line to compose the characteristic string, to get the matching results from the longest common substring. The methods of extracting the spatial surface characteristics of the fracture surface includes: Altantsetseg\cite{6} used the Fourier transform method to extract the curve on the fracture surface as the characteristic for the cross-section matching; Ge Baozhen\cite{7} used the curvature graph as the feature description function of 3D point cloud data, and the registration of 3D point cloud data is realized by using curvature graph. Li Jijunnan\cite{8} proposed an optimized matching algorithm based on feature space surface integral calculation, integral invariant of the surface volume formed with curved convex constrained clusters and complementary cluster to get the initial matching pairs, and realize the fracture surface matching by solve the optimization problem.

The existing fracture surface matching algorithm requires that the fragmentation is not damaged. The computational complexity of the algorithm is very large, the calculation process is complex, and the feature extraction did not calculate the complete fracture surface data, but only considered part of the point cloud data to extract the features. The extracted feature cannot express the fracture surface information completely. Considering the number of debris in reality is large, and the existing algorithm having a high computational requirement for computer, this paper proposes a method of fracture surface matching method based on mathematical statistics. The method is simple, effective, having low requirements on computer configuration, and it is based on the complete fracture point cloud data to obtain the matching information. The fracture surface feature can be expressed completely.

II. Fracture surface segmentation and recognition

In this paper, a new method based on integral invariant partition is proposed to segment the fragment point cloud, which uses the multi-scale volume integral invariant to find the feature segmentation among the surfaces of the fragment, then uses a specific strategy to generate closed surface segmentation line, in order to achieve the surface segmentation (Fig. 2). The surface segmentation method has good robustness and stability. After the segmentation of the surface, the fracture surface and the original surface are identified according to the roughness of the surface. The roughness of the surface can be represented by the average roughness of the surface:

\[
E = \frac{1}{N(p)} \sum_{i=1}^{k} \frac{1}{k} \bigg[ \sum_{i=1}^{k} \left\| n_p - n_{q_i} \right\|^2 \bigg]
\]

\[
E_{k,r}(p_i) = \frac{1}{N(r_i)} \sum_{j=1}^{k} \left[ \sum_{j=1}^{k} \left\| p - q_j \right\|^2 \right]
\]

In which, \( p \) is the set of points on the fracture surface, which is the \( k \) nearest neighbor point of the \( q \) point, and \( N \) is the surface normal vector. The larger the average roughness of the surface, the more rough the surface, the greater probability that the surface is the fracture surface. When the roughness \( E \) of the surface is larger than the threshold value \( E' \), it is judged that the surface is a fracture surface; when \( E \) is smaller than the threshold value \( E' \), it is judged that the surface is an original surface. The size of the threshold value \( E' \) depends on the specific situation of the fragments.

![Fig.2](image_url) the original surfaces and fracture surface
III. FRACTURE SURFACE MATCHING

After the fracture surface segmentation is completed, we re-established the coordinate system of the fracture surfaces by calculating the normal vector of each point of the fracture surface and the normal vector sum of all points to get a normal vector sum, and using the normal vector as the direction of the coordinate axis, and then project all the points to the axis to get all points. The matching information is obtained by comparing the projection length of the points on the coordinate axes.

Define the normal vector of a point on the fracture surface as $\mathbf{v}_i$; define the normal vector sum of all points as $\mathbf{V}$, which will be the axis of the fracture surface. Define all projection points under new coordinate system as the new set $\{P_i\}$, in which $V = \sum v_i$.

Calculate and analyze the statistical information of each point set, such as the projection value distribution. When the distribution of the two sets of points is similar, the two sets of points are matched to the corresponding set of points, and the corresponding two fragments are matched. As showed in the following picture. This paper improves the ICPIF (Interactive Closest Points using Invariant Feature) algorithm proposed by Sharp, and aligns the fragments based on the ICPIF algorithm. The weighted characteristic distance equation is defined as follows:

$$d(p, q) = d_e(p, q) + \alpha^2 d_d(p, q)$$

$$A(x) = \left[ \frac{1}{N} \sum_{i=1}^{N} (V^n(x) - \frac{1}{2})^2 \right]^{\frac{1}{2}}$$

$$d_e(p, q) = \|A(p) - A(q)\|$$

Fig. 3: The direction of the new coordinate system

Fig. 4: The distribution of two sets of points

, in which $d_e(p, q)$ represents the Euclidean distance of $p$ and $q$. $N$ is the number of radius scales, $6 < N < 8$. The ICP algorithm based on Euclidean space invariants can greatly improve the correct rate of corresponding clicks and reduce the possibility of the algorithm falling into local extremum.
IV. EXPERIMENTAL RESULTS AND CONCLUSION
In this paper, the experimental data includes a number of point cloud fragmentation data, which are the original debris cloud model. The experimental results are shown in the following picture. The matching result is good.

![Matching results of fracture surfaces](image)

FIG.5 the matching results of fracture surfaces

Most of the current surface matching algorithms are based on the feature extraction from the points of the fault surface, or the contour of the fracture surface. When a fragment is damaged or incomplete, the extracted feature or contour does not fully represent the fracture surface information. The proposed algorithm in the paper not only improves the calculation speed, and can represent the fracture surface feature information, and still has good matching effect in the situation of the incomplete debris.

REFERENCES