Analyzing the Main Factors Affecting the Reactive Power of Induction Motors Used in the Production of Metallic Concentrates

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Abstract: The dependence of the values of the reactive power required for the consumption of an induction motor used in the production of metallic concentrates on the peculiarities of the technological process is substantiated. A mathematical model for the reactive power consumed by the induction motor is developed, taking into account the changes in the mains voltage, frequency, loading, and power factor. The impact of different factors on the consumed power of the synchronous motor is investigated and estimated. Critical points have been revealed at which the consumption reactive power of the induction motor remains unchanged. The developed model allows to comprehensively estimate the consumed reactive power of the induction motor in case of changing the power and technological factors. The results obtained can be used to improve the power consumption regimes of both metallic concentrates and other productions.

Keywords: frequency, induction motor, loading, mains voltage, model, power factor, reactive power.

I. INTRODUCTION

Power investigations for industrial enterprises show that the enterprises producing metallic concentrates are the most power-intensive ones characterized by a considerable concentration of electric power. The structure of electric load used in the technological process for obtaining metallic concentrates is complex. Electromechanical systems with both powerful synchronous and induction electric drives are used in them. This allows to use the produced power resources more effectively, thus favoring the improvement of the power consumption regimes.

In particular, the powerful synchronous motors of the ore-grinding mill electric drive used in the production of metallic concentrates can be a source of reactive power for the ore crushers serving as reactive power consumers in the technological process of grinding, classifiers, and the electric drive induction motors of pulp pumps.

It is known that in the production of metallic concentrates, the operation modes of electric equipment are conditioned by the topology of the technological scheme, the type and number of devices used in the scheme, the unstable character of process implementation, and the qualitative requirements set to the developed concentrate [1-3]. Therefore, the determination and estimation of reactive powers generated by the synchronous motor and consumed by the induction motor used in the production process should be carried out taking their operating modes into account. It is evident that the values of the reactive powers generated by the synchronous motor and consumed by the induction motor used in the production of metallic concentrates are conditioned by the peculiarities of the technological process. At the same time, it should be mentioned that induction motors use about 40% of the consumed reactive power [4]. That is why, the operation modes of induction motors are often the main factors affecting the consumption of the total reactive power and the magnitude of power factor in the industrial enterprise, which undergo special standardization depending on the level of the power mains voltage [5].

Taking into account what was said above, as well as given the investigation and analyses of the main factors influencing the reactive power generated by the electric drive synchronous motor used in the production of metallic concentrates [6], we have to consider also the consumption power of induction motors used in the mentioned production.

The goal of the work is to investigate the main factors, affecting the reactive power of induction motors used in the production of metallic concentrates.

II. THE STATEMENT OF THE PROBLEM AND THE SUBSTANTIATION OF THE METHOD

In the works devoted to the investigation of the reactive power of consumption of an induction motor, the impact of the load on the reactive power is considered [7-11], which does not allow to comprehensively...
investigate the value of the reactive power of consumption of induction motors used in the production process at different operating modes of the motor. Based on this, the following problems are solved in the present work:
- A mathematical model for complex estimation of the consumed reactive power in different operation modes of a synchronous motor is developed;
- The impact of different factors on the consumption power of the synchronous motor is investigated and estimated.

On the whole, reactive power of consumption of induction motors consists of two components – the magnetization reactive power \(Q_o\), which is consumed in the no-load operation of the motor to create a magnetic flux and the leakage fields \(Q_P\) [12]:

\[
Q = Q_o + Q_P = 3U I_2 \sin \phi' + 3U I_0,
\]

where \(U\) is the supply mains voltage; \(I_2\) - the given current of the rotor; \(\sin \phi'\) - the phase shift of the rotor’s given current in relation to the mains voltage; \(I_0\) - the no-load current of the induction motor.

To determine the magnetization and dispersion components of the reactive power of consumption of the induction motors, it becomes necessary to consider the main factors influencing the operating modes of the motor. Below is introduced the algorithm for developing the model of the reactive power consumed by the induction motor, taking into account the mains voltage and frequency, loading, and changes of the power factor.

### III. THE ALGORITHM FOR DEVELOPING THE MATHEMATICAL MODEL

Taking into account the fact that the operation mode of the electric drive induction motors used in the ore-grinding process is mainly conditioned by the moment of resistance generated by the mechanisms used in the technological process, which can periodically change depending on the qualitative characteristics of the processed ore, it becomes necessary to consider the electromagnetic moment of the induction motor while determining its reactive power of consumption. For practical calculations, it is expedient to introduce the electromagnetic moment of the induction motor by relative values [13]:

\[
\frac{P}{P_{max}} = \frac{M}{M_{max}} = \frac{2}{s + \frac{s_k}{s} + \frac{2R_1 s_k c_1 R_2}{s}},
\]

where \(P\) is the electromagnetic power transmitted to the rotor from the stator of the induction motor; \(P_{max}\) – the maximum power transmitted to the rotor in case of the critical slip; \(M\) – the rotation electromagnetic moment of the motor; \(M_{max}\) - the maximum value of the rotation electromagnetic moment; \(s_k\) - the critical slip; \(R_1\) and \(R_2\) - the resistances of the stator and rotor respectively:

\[c_1 = 1 + \frac{x_k}{x_o},\]

where \(x_k\) - is the inductive impedance of the stator winding; \(x_o\) - the inductive impedance of the stator magnetization.

For practical calculations, the resistance of the stator motor is usually neglected. In this case, the formula (2) will have a simpler form:

\[
\frac{P}{P_{max}} = \frac{M}{M_{max}} = \frac{2}{s + \frac{s_k}{s}}.
\]

Let us designate:

\[
\frac{P_{max}}{P_H} = \frac{M_{max}}{M_H} = b_H.
\]

In this case, the rotation electromagnetic moment of the motor with relative units will have the following form:

\[
\frac{M}{M_H} = m = \frac{2b_H}{s + \frac{s_k}{s}}.
\]

Since the dispersion inductive impedances of the stator and rotor windings are relative to frequency [13], the multiplicity coefficient of the maximum moment in relation to the nominal moment will change inversely to the square of frequency.
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\[ M_{\text{max}} = M_H b_H \frac{f_H^2}{f^2}, \quad (6) \]

and directly proportional to the square of the voltage change:

\[ M_{\text{max}} = M_H b_H \frac{U_H^2}{U^2}. \quad (7) \]

Let us designate \( k_f = f / f_H \) and \( k_U = U / U_H \). Then we can write:

\[ \frac{M_{\text{max}}}{M_H} \approx b_H \frac{k_u}{k_f^2}. \quad (8) \]

If the motor load is different from the nominal load, i.e. \( m \neq 1 \), the multiplicity of the maximum moment in relation to the moment will be \( b_H / m \).

Based on the Claus formula and equations (4) and (5), we will obtain the expression of the moment’s relative value, depending on the parameters changes of the supplying mains [12,13]:

\[ m = \frac{M}{M_H} = \frac{2b_H k_u^2}{k_f s_k + k_f^2 s_k}. \quad (9) \]

By solving equation (2) in relation to the slip, and assuming that the motor operates in the nominal mode, we will obtain:

\[ s = s_k \left( \frac{b_H}{m} + \frac{b_H^2}{m^2} - 1 \right) = \frac{s_k b_H}{m + \sqrt{b_H^2 + b_H^2}}. \quad (10) \]

At nominal load and nominal voltage, when \( m = 1 \), the equation (10) will have the following form:

\[ s_H = \frac{s_k b_H}{b_H + \sqrt{b_H^2 - 1}}, \quad (11) \]

where \( s_{\text{th}} \) is the critical slip in the nominal mode.

One of the main parameters, characterizing the operation mode of an induction motor and its power quality is also the current of its stator. It can be introduced by the geometric sum of its active and reactive current components [13]:

\[ I_1 = \sqrt{\left( I_0 \sin \phi_o + I_2 \sin \phi \right)^2 + \left( I_0 \cos \phi_o + I_2 \cos \phi \right)^2}, \quad (12) \]

where \( I_{1a} = I_1 \cos \phi_H = I_0 \cos \phi_o + I_2 \cos \phi \) is the active component of the current; \( I_{1p} = I_1 \sin \phi_H = I_0 \sin \phi_o + I_2 \sin \phi \) - reactive component of the current; \( \cos \phi_o \) - the phase shift of the given current of the rotor in the no-load operation mode in relation to the mains voltage.

Neglecting the losses of the no-load operation, i.e. when \( \sin \phi_o = 1, \cos \phi_o = 0 \), we will obtain [13]:

\[ I_1 = \sqrt{\left( I_0 + I_2 \sin \phi \right)^2 + \left( I_2 \cos \phi \right)^2}. \quad (13) \]

To determine the stator current of the motor, it is necessary to determine the regularity of the change of the motor current and the phase shift angle \( \phi \) of the mains voltage [12]:

\[ \tan \phi = \frac{1}{s_k} \frac{b_H}{m + \sqrt{\frac{b_H^2}{m^2}} - 1}. \quad (14) \]

Using the known trigonometric ratio, we determine:

\[ \sin \phi = \frac{1}{\sqrt{\frac{2b_H}{m} \left( \frac{b_H}{m} + \sqrt{\frac{b_H^2}{m^2} - 1} \right)}}. \quad (15) \]
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\[
\cos \varphi = \sqrt{\frac{b_H + m \sqrt{b_H^2 - 1}}{2b_H^2}}. \tag{16}
\]

When the values of frequency and voltage are deviated against their nominal values, equations (4), (5), and (6) will respectively have the following forms:

\[
\tan \varphi = \frac{1}{\sqrt{\frac{b_H k_u^2}{mk_f^2} + \left(\frac{b_H k_u^2}{mk_f^2}\right)^2 - 1}}. \tag{17}
\]

\[
\sin \varphi = \frac{1}{\sqrt{\frac{2b_H k_u^2}{mk_f^2} \left(\frac{b_H k_u^2}{mk_f^2}\right)^2 - 1}}. \tag{18}
\]

\[
\cos \varphi = \frac{b_H k_u^2 + mk_f^2 \left(\frac{b_H k_u^2}{mk_f^2}\right)^2 - 1}{2b_H k_u^2}. \tag{19}
\]

Now, let us consider the change in the given current of the rotor, depending on the load in case of the nominal values of frequency and voltage \( k_u = k_f = 1 \)

\[
I'_2 = \frac{U_H}{Z_{2l}} = \frac{U_H \sqrt{1 + s_k^2}}{z_{2k} \sqrt{1 + \left(\frac{s_k}{s}\right)^2}}. \tag{20}
\]

Taking (10) into account, we will obtain:

\[
I'_2 = \frac{U_H \sqrt{1 + s_k^2}}{z_{2k} \sqrt{1 + \left(\frac{b_H}{m}\right)^2 - 1}} = \frac{U_H \sqrt{1 + s_k^2}}{z_{2k} \sqrt{2b_H \left(\frac{b_H}{m}\right) + \left(\frac{b_H}{m}\right)^2 - 1}}. \tag{21}
\]

At nominal load \((m = 1)\):

\[
I'_{2H} = \frac{U_H \sqrt{1 + s_k^2}}{z_{2k} \sqrt{2b_H \left(\frac{b_H}{m}\right) + \left(\frac{b_H}{m}\right)^2 - 1}}. \tag{22}
\]

Dividing (21) by (22), we will have:

\[
\frac{I'_2}{I'_{2H}} = \sqrt{\frac{m b_H + \sqrt{b_H^2 - 1}}{b_H + \sqrt{b_H^2 - 1}}}. \tag{23}
\]

In case of the change in the supplying mains voltage and frequency in relation to the nominal, the equation (23) will have the following form:

\[
I'_2 = I'_{2H} \sqrt{\frac{m b_H + \sqrt{b_H^2 - 1}}{b_H + \sqrt{b_H^2 - 1}}} \cdot \left(\frac{b_H k_u^2}{mk_f^2} + \left(\frac{b_H k_u^2}{mk_f^2}\right)^2 - 1\right). \tag{24}
\]

It is known that the relation between the currents of the rotor and stator is characterized by the following expression \([13,14]\):
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\[ I_{2H} = I_{1H} \frac{\cos \phi_H}{\cos \phi}. \]  

(25)

Taking (25) and (19) into account, equation (24) can be introduced in the following way:

\[ I'_{2} = I_{1H} \cos \phi_H \frac{m\left(b_H + \sqrt{b_H^2 - 1}\right)}{b_H + \sqrt{\left(b_H^2 - 1\right)}} \cdot \frac{2b_H k_u^2}{mk_f^2} \times \frac{2b_H k_u^2}{mk_f^2} - 1. \]  

(26)

The expression of the given current of the motor rotor, when it operates in the nominal mode \( m = k_f = k_U = 1 \) will be:

\[ I'_{2} = I_{1H} \cos \phi_H \frac{m\left(b_H + \sqrt{b_H^2 - 1}\right)}{b_H + \sqrt{\left(b_H^2 - 1\right)}} \cdot \frac{2b_H k_u^2}{mk_f^2} \times \frac{2b_H k_u^2}{mk_f^2} - 1. \]  

(27)

The magnetization current of the stator winding is practically equal to the no-load current of the motor and changes directly proportional to voltage and inversely proportional to frequency, that is:

\[ I_0 = I_{OH} \frac{k_U}{k_f}. \]  

(28)

The no-load current at nominal voltage can be determined according to the nominal power coefficient and the maximum moment multiplicity. In that case, the reactive component of the stator current can be introduced in the following form:

\[ I_{1H} \sin \phi_H = I_{OH} + I_{2H} \sin \phi'. \]  

(29)

Using (25) and modifying (29) at nominal voltage we will obtain:

\[ I_{OH} = I_{1H}\left(\sin \phi_H - Tg \phi_H \cos \phi_H\right). \]  

(30)

Taking the moment multiplicity into account, the equation (30) will take the following form:

\[ I_{OH} = I_{1H}\left(\sin \phi_H - \frac{\cos \phi_H}{b_H + \sqrt{b_H^2 - 1}}\right). \]  

(31)

If the motor operates in the non-nominal mode, the magnetization current will be determined by the following expression:

\[ I_{OH} = I'_{2H} \cos \phi_H \left(\sin \phi_H - \frac{\cos \phi_H}{b_H + \sqrt{b_H^2 - 1}}\right) \frac{k_u}{k_f}. \]  

(32)

Having the required parameters for analyzing the reactive power of the motor, the full reactive power of consumption of the induction motor based on its dispersion and magnetization reactive power components is determined.

Considering (18) and (24), for the dispersion component of the reactive power we obtain:

\[ Q_p = 3U I'_{2H} \cos \phi_H \left[ \frac{b_H + \sqrt{b_H^2 - 1}}{m \left(b_H + \sqrt{b_H^2 - 1}\right)} \cdot \frac{1}{\left(b_H + \sqrt{b_H^2 - 1}\right)} \right]. \]  

(33)

Considering formula (32) for the determination of the magnetization current, we will obtain the following expression for the magnetization reactive power of the motor:

\[ Q_0 = 3U I'_{2H} \frac{k_u}{k_f} \left(\sin \phi_H - \frac{\cos \phi_H}{b_H + \sqrt{b_H^2 - 1}}\right) \frac{\cos \phi'}{\cos \phi_H}. \]  

(34)

In the nominal mode, when \( k_U = 1, k_f = 1 \), the motor’s magnetization reactive power will be:
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\[ Q_{0H} = 3U_H I_{2H} \frac{k_u}{k_f} \left( \sin \varphi_H - \frac{\cos \varphi_H}{b_H + \sqrt{b_H^2-1}} \right) \frac{\cos \varphi_H}{\cos \varphi_H} . \] 

Given the expressions (19), (33), and (35), for the full reactive power of the induction motor taken from the mains, we will obtain:

\[ Q = 3U_H I_{2H} \left( m \frac{b_H + \sqrt{b_H^2-1}}{b_H + \sqrt{b_H^2-1} - 1} \right) + k_u \frac{\lg \varphi_H - 1}{b_H + \sqrt{b_H^2-1}} \left( \frac{b_H k_u^2 + m k_f^2 \left( \frac{b_H k_u^2}{m k_f^2} \right)^2 - 1}{2 b_H k_u^2} \right) . \] 

The nominal value \( Q_H \) of the induction motor reactive power is determined from equations (36), when \( m = k_u = k_f = 1 \):

\[ Q_H = 3U_H I_{2H} \left( \frac{1}{2 b_H \left( b_H + \sqrt{(b_H)^2-1} \right)} \right) + \left( \frac{\lg \varphi_H - 1}{b_H + \sqrt{(b_H)^2-1}} \right) \frac{\sqrt{b_H + \sqrt{(b_H)^2-1} - 1}}{2 b_H} . \] 

To estimate the change in the value of induction motor reactive power, depending on the supply voltage and frequency values, as well as the motor loading, its relative value is determined:

\[ K_q = \frac{Q}{Q_H} . \]

By means of the developed mathematical model, the impact of different factors on the reactive power of consumption of the synchronous motor is investigated and estimated.

**Fig.1.** The dependence of the induction motor reactive power on the supply voltage at different values of the moment multiplicity

The investigation shows that the increase in the supply voltage of the mains leads to a decrease in the reactive power of consumption of the induction motor, and the smaller is the moment multiplicity, the more abrupt is the decrease of the reactive power conditioned by the supply voltage increase (Fig. 1). At the same time, it should be mentioned that regardless of the value of the moment multiplicity, at \( K_u = 1,045 \), the consumed reactive power \( K_q \) remains unchanged.
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Regardless of the value of $\cos \phi$, at the relative value of the load $m = 0.97$, the consumed reactive power of the induction motor remains unchanged (Fig. 6).

IV. Conclusion

1. A mathematical model for complex estimation of the reactive power of the synchronous motor’s consumption at different operation modes is developed. The impact of changes in the mains voltage, frequency, loading, and power factor on the consumption power of the synchronous motor is investigated and estimated.

2. The critical values of the load, the mains supply voltage, and frequency are revealed at which the reactive power of consumption of the synchronous motor remains unchanged in case of changing other factors.

3. The obtained results can be successfully used in different productions to improve the modes of power consumption.

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