Harvesting Energy Density Performance Of Cantilevered Piezoelectric Transducers

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Abstract: To estimate the harvesting performance of piezoelectric transducers under finite volume, the models of unimorph cantilevers are established and then the analyses of influences of external and internal factors on harvesting energy density is proposed. The results show that the more heavy the tip mass is, the better output performance it would turn out at the first natural frequency, which is contrary to output capacity at the second natural frequency. Moreover, the harvesting voltage density is proportional to the acceleration of the base excitation. It is found that output voltage increases along with the growth of load resistance and there exists an optimal load resistance which leads to the most desirable output power density. Besides external factors, internal factors such as thickness ratio and material properties of substrate can be of great significance to output performance. The findings exhibit that an optimum thickness ratio corresponding to the most desirable output voltage density would emerge either aluminum or copper substrate involved in energy harvesting systems and further studies apparently show that copper substrate generates higher performance in comparison with aluminum substrate.

Keywords: piezoelectric transducers, unimorph cantilever, electric energy density, external and internal factors

I. Introduction

Harvesting ambient vibrational energy to power the wireless sensors devices had been proved to be feasible in the past few decades. The energy conversion mechanisms that can be used to convert ambient vibrations into electricity energy are electromagnetic [1-3], electrostatic [4, 5], and piezoelectric [6-8] conversion mechanisms, respectively. Due to higher electromechanical coupling performance, greater voltage output, needing no external power supply and being compatible with MEMS technology, the self-powered technology based on piezoelectric transduction has attracted a great deal of attentions as exhibited in some published literatures [9-12] that directly focusing on piezoelectric energy harvesting.

Many researchers adopt the cantilevered piezoelectric energy transducers to generate electrical power from piezoelectric materials. The reason is that the cantilever is of simple structures and can be manufactured and fabricated relatively easily. In terms of this type of energy transducers, one or two layer so f piezoelectric material are usually attached to a metal substrate layer, and we named the unimorph or bimorph cantilevers. Energy transducers normally operate in the 31 mode, in which the bending stress developed in the long it udinal direction (1-direction), produces a potential difference across the thickness direction (3-direction) of the piezoelectric layer. Normally, the natural frequency of an energy transducer has to be tuned around the external in centives frequency to operate at resonance state and maximize the energy output. Consequently, a tip mass is normally attached to the free end of the cantilevered piezoelectric energy transducers for the purpose of reducing the natural frequency of energy harvesting systems.

Researchers have obtained highly significant achievements in recent years, yet certain aspects haven’t received particular attentions so far. As to the generation performance of piezoelectric harvesters, it was mainly
characterized via index such as output voltage, yet few evaluation methods based on voltage density performance had been proposed. However, the miniaturization of the self-powered system is also of great importance simultaneously [10, 13]. Kang L H [14] made investigation on performance of piezoelectric unimorph with mechanically prestressed substrate (PUMPS) evaluated via actuation displacement and force, and the findings suggest that PUMPS is of better performance than conventional ones. However, due to large dimension and low integration level caused by extra curved space under piezoelectric unimorph, PUMPS is not desirable for MEMS technology. In other words, it is an extremely important method for better compatibility with MEMS technology to obtain maximum electric energy in smallest volume, and generation performance in unit volume of piezoelectric energy harvester which are considered as harvesting electric density parameters. In this paper, the harvesting electric density performance is brought forward as energy output assessment index on the basis of analysis on external and internal factors. Meanwhile, this paper provides certain theoretical references for energy harvesting technology using piezoelectric materials with cantilever configuration.

II. Modeling Of Coupled Piezoelectric Transducers

In this article, unimorph forms of transducers are taken into account, as shown in figure 1. The unimorph consists of two layers: PZT and substrate layer, a tip mass is attached to the free end of the beam and base excitation is applied to the fix end of the piezoelectric cantilever. Meanwhile, an external load resistance $R_{\text{load}} = 12 \, \text{k}\Omega$ is presented to form an electric circuit.

![Unimorph piezoelectric energy transducer with a tip mass.](image)

For a uniform beam that undergoes undamped free vibration, the governing equation of motion can be expressed as

$$\frac{\partial^2 M(x,t)}{\partial x^2} + m \frac{\partial^2 w(x,t)}{\partial t^2} = f_0(x,t)$$

(1)

Where $M(x,t)$ is the internal bending moment, $m$ is the mass per unit length of the beam and $f_0(x,t)$ is the external force per unit length that applied to the beam.

The displacement of the beam can be obtained as [15]

$$W(x,t) = w_b(x,t) + w_{\text{rel}}(x,t)$$

(2)

Where $w_b(x,t)$ and $w_{\text{rel}}(x,t)$ are the base displacement and the transverse displacement of the beam relative to its base. Based on the proportional damping assumption, the vibration response relative to the base of the bimorph can be represented as an absolutely and uniformly convergent series of the eigenfunctions as

$$w_{\text{rel}}(x,t) = \sum_{r=1}^{\infty} \varphi_r(x) \eta_r(t)$$

(3)

Where $\varphi_r(x)$ is the mass normalized eigenfunction of the $r$-th vibration mode, $\eta_r(t)$ is the modal mechanical response expressions. The eigenfunction representing $r$-th mode shape corresponding to the undamped free vibration can be described as

$$\varphi_r(x) = A_r [\cos \beta_r x - \cosh \beta_r x + \zeta_r (\sin \beta_r x - \sinh \beta_r x)]$$

(4)
Then, $\varsigma_i$ is obtained from

$$\varsigma_i = \frac{m(\sin \beta_i L - \sinh \beta_i L) + \beta_i M_i (\cos \beta_i L - \cosh \beta_i L)}{m(\cos \beta_i L + \cosh \beta_i L) - \beta_i M_i (\sin \beta_i L - \sinh \beta_i L)}$$ \hspace{1cm} (5)

Where $A_r$ is the constant of modal amplitude, $M_t$ is the weight of tip mass and $L$ is the length of the beam. And the undamped natural frequency of the $r$-th vibration mode can be expressed as

$$\omega_r = \beta_r^2 \sqrt{\frac{Y_l}{m}}$$ \hspace{1cm} (6)

And the bending stiffness term $Y_l$ can be obtained from

$$Y_l = \frac{b}{12} (Y_s h_s^3 + 8c_{11} h_p^3 + 12c_{11} h_p^2 h_t + 6c_{11} h_p h_t^2)$$ \hspace{1cm} (7)

Here, $h_s$, $h_p$ are the thickness of substrate and piezoelectric layer, respectively. And $b$ is the width of the beam. The constitutive equation which relates the electrical and mechanical term for energy harvesting system is

$$S_i^p = S_{11}^p T_i^p + d_{31} E_3$$
$$D_1 = d_{31} T_i^p + e_{33} E_3$$ \hspace{1cm} (8)

And $D_3, d_{31}, T_1, S_1, s_{11}^E, e_{33}$ and $E_3$ are the electrical displacement, the piezoelectric strain constant, the longitudinal stress, the elastic compliance at constant electric field, the longitudinal strain, the permittivity and the component of applied electric field, respectively.

There are two kinds of longitudinal stress which can be written as the following expressions from equation (8).

$$T_{1i}^\sigma = Y S_i^\sigma$$ \hspace{1cm} (9)
$$T_{1i}^\rho = Y_p (S_i^p - d_{31} E_3)$$ \hspace{1cm} (10)

Here, $Y$ is the Young’s modulus and $Y_p=1/s_{11}^E$. 1 and 3 directions are coincident with $x$ and $y$ directions, respectively (where 1 is the axial strain direction and 3 is the polarization direction).

The distribution of longitudinal bending moment can be obtained as

$$M(x, t) = \int_{a-h_b}^a T_1^b b z dz + \int_{a}^{a+h_b} T_1^p b z dz$$ \hspace{1cm} (11)

And $a$ is the distance between neutral layer and the position of the bottom of the piezoelectric layer. The distribution of longitudinal strain can be expressed as

$$S_i(x, y, t) = -y \frac{\partial^2 w_{rel}(x, t)}{\partial x^2}$$ \hspace{1cm} (12)

The electric charge $q(t)$ generated in the piezoelectric layer and collected by the electrodes can be obtained by integrating the electric displacement over the electrode area as [16]

$$q(t) = \int_A \mathbf{D} \cdot \mathbf{n} dA$$ \hspace{1cm} (13)

Where $\mathbf{D}$ is the vector of electric displacements and $\mathbf{n}$ is the unit normal vector. Then, the output voltage generated by the piezoelectric materials can be given by

$$v(t) = R_{load} i(t) = R_{load} \frac{dq(t)}{dt}$$ \hspace{1cm} (14)
Where \( i(t) \) is current of the electric circuit. Furthermore, the output voltage density can be obtained as

\[
\rho_v(t) = \frac{v(f)}{V_s + V_p + V_t}
\]  

(15)

Here, \( V_s, V_p \) and \( V_t \) represent the volume of substrate materials, piezoelectric materials and tip mass, respectively.

PZT-5H and copper are used to produce the beam and the physical attributes of original piezoelectric cantilever are shown in table 1.

### Table 1. Physical attribute parameters of piezoelectric cantilever.

<table>
<thead>
<tr>
<th>Related parameters</th>
<th>Piezoelectric ceramic</th>
<th>Copper substrate</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y(GPa) )</td>
<td>60.61</td>
<td>110</td>
</tr>
<tr>
<td>( d_{31}(C/N^\circ) )</td>
<td>-3.2x10^{-12}</td>
<td>-</td>
</tr>
<tr>
<td>( \varepsilon_{33}(F/m^\circ) )</td>
<td>3.02x10^{-8}</td>
<td>-</td>
</tr>
<tr>
<td>( L_r(mm) )</td>
<td>70.0</td>
<td>70.0</td>
</tr>
<tr>
<td>( h(mm) )</td>
<td>15.0</td>
<td>15.0</td>
</tr>
<tr>
<td>( h_s(h_p)(mm) )</td>
<td>2.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

### III. Voltage Density Response Of External Factors

The effect of external factors, such as tip mass, acceleration of base excitation and load resistance are of highly significance to maximize voltage output. Accordingly, the analysis of influences of external factors on output performance is highly significant for achieving more desirable electricity energy. Additionally, the output performance of energy transducers is evaluated with voltage density (including the volume of tip mass), as mentioned in equation (15).

#### 3.1 Frequency response of transducers with or without a tip mass

Figure 2 exhibits the effect of tip mass on harvesting voltage density for piezoelectric cantilevers. The simulation process is carried out using five different tip mass: 0, 1.6g, 3.2g, 4.8g, 6.4g and 8.0g. It can be found that, on one hand, the voltage density generated at the first mode increases along with the growing of the weight of the tip mass. Meanwhile, the first natural frequency decreases as the weight of the proof mass increased. On the other hand, it is contrary to the first mode that the voltage density produced at the second mode increases with the decrease of the weight of the tip mass except energy transducers without tip mass which exhibit a relatively high output density performance. Moreover, it can be found that the voltage density of energy transducers with 8.0g tip mass increase by 30.2% compared with no tip mass ones at the first natural frequency. On the contrary, the output density of energy transducers with no tip mass increase by 17.2% compared with 8.0g tip mass ones at the second natural frequency.

Hence, the tip mass has the ability to tune the natural frequency of the beam. So, in order to obtain better voltage density, increasing the weight of tip mass properly when excitation frequency is around the first natural frequency or reducing the weight when it comes to the second natural frequency.
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3.2 Acceleration response

The acceleration of external base excitation is also of great influence on piezoelectric energy transducers. The process is carried out using 8.0g tip mass at the first natural frequency (240 Hz). Here, as demonstrated in figure 3, the acceleration is set from 0.25g to 2.0g, where g is the gravitational acceleration ($g=9.81 \text{ m/s}^2$). It can be seen clearly that the output voltage density increases in proportion with the growing of the acceleration of external base excitation. Furthermore, the proportion between the output voltage density and the acceleration is found to be $0.81\times10^7 \text{ V/(m}^3\text{g)}$, which shows that higher excitation acceleration can lead to more desirable output voltage density.

Figure 3. Acceleration response of output voltage density using 8.0g tip mass at 240Hz.

3.3 Load resistance response

The electric circuit of energy harvesting system consists of structure of piezoelectric beam and external load resistance. Figure 4 presents the relationships among the load resistance (from 10Ω to 1000kΩ), output voltage density and normalized output power density (output power density/8) using 8.0g tip mass at its first natural frequency 240Hz. For the purpose of revealing the change law of output voltage/power density along with different load resistance, logarithmic x-coordinate is used to exhibit clearly. It is evident that output voltage density increases rapidly when load resistance is relatively small, it increases slowly when load resistance is relatively large. When load resistance is more than about 40kΩ, the output voltage density curve is almost a straight line. On the contrary, there exists an peak of output power density with various external load resistance which found to be 3.16kΩ.
IV. Voltage Density Response Of Internal Factors

Besides external factors, internal factors such as thickness ratio of piezoelectric layer and substrate layer ($\alpha = h_s/h_p$), material properties of substrate have distinct influences on output voltage density. Here, as shown in figure 5, two kinds of most commonly used metal materials, aluminum and copper are taken into consideration of substrate material. It can be found that there always exists an optimal thickness ratio corresponding to the most desirable output voltage density either aluminum or copper substrate involved in energy harvesting systems. In addition, the first natural frequency increases along with the growing of thickness ratio and the reason is that more metal components (bigger thickness ratio) increase the equivalent stiffness of the systems. In terms of output voltage density at the first natural frequency of various thickness ratio, the performance increases firstly then decreases with growing thickness ratio. Moreover, from a global view, cantilevered piezoelectric energy transducers using copper substrate generated more output voltage density than the ones with aluminum substrate at the first natural frequency.

![Figure 5](image)

(a) Comparison of output voltage density with different thickness ratio, (a) aluminum substrate, (b) copper substrate.

V. Conclusions

The models of unimorph cantilevers are established to evaluate the harvesting performance of piezoelectric transducers under finite volume, and then the analyses of influences of external and internal factors on harvesting energy density is proposed. The results show that the more heavy the tip mass is, the better output performance it would be at the first natural frequency, which is contrary to output capacity at the second natural frequency. Moreover, the harvesting voltage density is proportional to the acceleration of the base excitation, and the proportion between the output voltage density and the acceleration is found to be $0.81 \times 10^7 V/(m^3.g)$, which shows that higher excitation acceleration can lead to more desirable output voltage density. In terms of external load resistance of the electric circuit, it is found that output voltage density increases along with the growth of load resistance and there exists an optimal load resistance which leads to the most desirable output power density.
Besides external factors, internal factors such as thickness ratio and material properties of substrate can be of highly importance to output density performance. The findings reveal that an optimum thickness ratio corresponding to the most desirable output voltage density would emerge either aluminum or copper substrate involved in energy harvesting systems and further studies apparently show that copper substrate generates higher performance in comparison with aluminum substrate.

Acknowledgments
This work was supported by Shanghai University of Engineering Science Innovation Fund for Graduate Students (No.15KY0610). The authors are all indebted to the generous support.

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