Application of thermal error in machine tools based on Dynamic Bayesian Network

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Abstract: In recent years, the growing interest toward complex manufacturing on machine tools and the machining accuracy have solicited new efforts in the area of modeling and analysis of machine tools machining errors. Therefore, the mathematical model study on the relationship between temperature field and thermal error is the core content, which can improve the precision of parts processing and the thermal stability, also predict and compensate machining errors of CNC machine tools. It is critical to obtain the thermal errors of a precision machine tools in real-time. In this paper, based on Dynamic Bayesian Network (DBN), a pioneering modeling method applied in thermal error research is presented. The dependence of thermal error and temperature field is clearly described by graph theory, and the fuzzy classification method is proposed to reduce the computational complexity, then forming a new method for thermal error modeling of machine tools.

Keywords: machine tools; thermal error; DBN; fuzzy classification; modeling;

I. Introduction

Artificial intelligence techniques have been used for many years now in the field of thermal error modeling in machine tools. The thermal model forms one of the most critical elements in machine tools. In the prediction of thermal error, most research has focused on the principle of regression [1,2]. Chen [3] used a three-layer Artificial Neural Network (ANN) with a supervised back-propagation training algorithm and the ‘sigmoid’ activation function to map the calibrated thermal errors to the temperature measurements. Zhejiang University in 2002 with the improved BP neural network on three-dimensional noncontact measurement system is analyzed in thermal error modeling [4]. Hong Yang and Jun Ni [5] proposed a fuzzy model adaptation method is used to update in the thermal error model under different processing conditions. The traditional thermal error model cannot handle abnormal situations, and the structure and the choice of the type too dependent on experience. Research on the application of the Bayesian Network in thermal error modeling of CNC machine tools on the thermal error has opened up a new way, in order to better solve the complexity and defect of thermal error modeling [6]. R. Ramesh [7] applied Hybrid Bayesian network to solve thermal error measurement and modeling problems in machine tools.

A brief explanation of the theory of Bayesian Network is presented as following:

The Bayesian Network (BN) is a graphical model consisting of nodes representing causes and effects in real-world situations and a set of edges each of which connects two nodes, it has been used in medical diagnostic system, software testing, web intelligent navigation, power system fault diagnosis, etc. A Bayesian network for a set of variables \( x = \{x^1, x^2, \ldots, x^N\} \), the joint probability distribution for \( x \) is given by

\[
p(x^1, x^2, \ldots, x^N) = \prod_{i=1}^{N} p(x^i | p_a(x^i))
\]

(1)

Where \( x^i \) denotes the variable and \( p_a(x^i) \) denotes the parents of node \( x^i \). From the chain rule of probability,

\[
p(x^1, x^2, \ldots, x^N) = \prod_{i=1}^{N} p(x^i | x^{i-1})
\]

(2)

BN is an effective tool for complex systems uncertainty reasoning and data analysis, but the thermal error of machine tool research with differences between the working conditions are different, thus requiring thermal error model must have a strong ability to learn, and according to the current working update the status of the model results. BN in dealing with real-time tracking data still has some limitations, not good with the experimental data.
reflect updates to the model. This paper seeks to address the issue of developing a thermal error model that account for a variety of operating conditions while yet generating accurate prediction. So it will propose fuzzy classification and Dynamic Bayesian Network to build a thermal machine error model, a better solution of dynamic thermal error modeling problems.

II. Fuzzy Dynamic Bayesian Network

Dynamic Bayesian Network (DBN) is a type of BN that can model time-series data to capture the fact that time flows forward, which coupled with the time constraints on the properties of the original network structure. It has the advantage both of model-based and data-based methods. And with the introduction of time factor, the data on the state of the formation of different time, reflects the development change rule represented by the variable, and its intuitive, high precision and adaptive for thermal error modeling of machine tools.

The actual processing for precision CNC machine tools, the thermal error generated by many factors and there are many uncertain factors, also the relations among these factors are perplexing, so the requirements of thermal error prediction model should not only be a strong inclusive of many complex factors, but also the structure of flexible function. In order to account for the particularity of thermal error, we present the DBN thermal error model herein is therefore very essential.

2.1 Dynamic Model

Although the pattern of heat transfer in machine tools is very complex, these are plausible cause and effect relationships that might exist between the various elements of the machine when subjected to varying operating conditions. Therefore, in order to carry out the research of complex system and model should be simplified [8].

1. The assumption of conditional probability in a finite time change process for all t is smooth. 2. To solve the problems of thermal error in this study, assuming an environment with Markov nature, that every state heat distortion temperature and sampling points are related to the time of sampling points on temperature and thermal deformation states. Bayesian network structure inference process, making inferences and decisions based on prior probabilities. 3. Assuming that the conditional probability of adjacent time process is steady, which \( p(x_{t+1} | x_t) \) has nothing to do with the time \( t \), you can easily get the transition probability \( p(x_{t+1} | x_t) \) in different time.

Based on the above assumptions, the establishment of DBN should be considered \((B_1, B_{22})\): the priori net \( B_1 \), the transfer net \( B_{12} \), the limited time in practical application stage 1, 2, ..., then we can get:

\[
p(\{x_1, x_2, \ldots, x_T\}) = p_{B_1}(x_1) \prod_{t=1}^{T} p_{B_{12}}(x_{t+1} | x_t)
\]

(3)

If we use \( p(x_t | x_{t-1}) \) to represent the probability of the current state occurs when any previous time state is variable. \( x_t \) represents the value of the variable \( i \) in time \( t \), \( pa(x_t) \) denotes the parents of node \( x_t \). So, joint probability distribution of DBN can be calculated for any node as the following shown:

\[
p(\{x_1^{T}, x_2^{T}, \ldots, x_T^{T}\}) = \prod_{i=1}^{N} p_{B_1}(x_1^i) \prod_{t=1}^{T} p_{B_{12}}(x_t^i | pa(x_t^i))
\]

(4)

The research of thermal error on precision machine tools showed us the parameters of sampling point temperature and thermal deformation data had discrete features. In the experiment, the decision variable is always discrete, and certain temperature observation is continuous, to solve such problems in pure discrete Dynamic Bayesian Network have some difficulties. In this regard, this paper choose the calculation method of discrete fuzzy DBN that fuzzy classification techniques combined with the discrete DBN [9] to solve these problems. It will not only make full use of the advantages of discrete DBN which has relatively faster inference speed compared with continuous DBN, but also can achieve qualitative reasoning under continuous observation without loss of information provided.

2.2 Fuzzy Classification

In the fuzzy theory, an element \( x \) is the degree of \( \mu_A(X) \) belongs to the set \( A \), this degree means membership. The element \( x \) must also exit in certain membership with other sets, when \( 0 \leq \mu_A(A) < 1 \). Fuzzy set \( A \) is defined in \( x \) and its affiliated with \( A \) and \( \mu_A(X) \) as the following:
First, the fuzzy classification process is divided the sample space into several subsets, which is a subset of fuzzy sets. Fuzzy sets is the corresponding discrete DBN node status. Then defines membership function according to actual condition, that membership of fuzzy classification is the characteristics of the response variable, not a specific numerical size. Discrete BN with \( N \) hidden points and \( M \) observation points, along with the time development can get the discrete DBN with \( T \) time slices, if the observed value is only one combination of state, so this observation distribution of hidden variables are:

\[
p\left( x_1, x_2, \ldots, x_N, y_1, y_2, \ldots, y_M \right) = \prod_{i,j} p\left( y_j^i \mid pa\left( y_j^i \right) \right) \prod_{i,k} p\left( x_k^i \mid pa\left( x_k^i \right) \right)
\]

\[
= \sum_{x_1, x_2, \ldots, x_N} \sum_{y_1, y_2, \ldots, y_M} \left[ \prod_{i,j} p\left( y_j^i \mid pa\left( y_j^i \right) \right) \prod_{i,k} p\left( x_k^i \mid pa\left( x_k^i \right) \right) \right] \times \prod_{j} p\left( Y_j^i = y_j^i \right)
\]

(\( t = 1, \ldots, T; \ j = 1, \ldots, M; k = 1, \ldots, N \))

In this formula, \( x_k^i \) represents a value of \( X_k^i \), value of \( y_j^i \) for observation variable \( Y_j^i \), \( pa\left( y_j^i \right) \) denotes the parents of note \( y_j^i \). As according to get fuzzy DBN inference formula is:

\[
p\left( x_1, x_2, \ldots, x_N, y_1, y_2, \ldots, y_M \mid Y_1^i, Y_2^i, \ldots, Y_T^i \right) = \sum_{y_1, y_2, \ldots, y_M} \left[ \prod_{i,j} p\left( y_j^i \mid pa\left( y_j^i \right) \right) \prod_{i,k} p\left( x_k^i \mid pa\left( x_k^i \right) \right) \right] \times \prod_{j} p\left( Y_j^i = y_j^i \right)
\]

(\( t = 1, \ldots, T; \ j = 1, \ldots, M; k = 1, \ldots, N \))

Where \( p\left( Y_j^i = y_j^i \right) \) is the value of \( Y_j^i \) membership in continuous observation that belongs to each states.

### III. Development of DBN Model on Thermal Error

Aiming to improve the effectiveness and accuracy of abnormal situation management in complex process system, thermal error should be studied and modeled in a scientific and systematic way. This paper proposed a DBN framework for this purpose. The overall workflow is shown in Figure 1.
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3.1 Temperature Points Development

A number of studies on domestic and international machine tools show that: one of the primary factors that influence machine tool accuracy is the rise in temperature of the critical elements of the machine. The data available from the machine on a real-time basis is the temperature values as measured by the thermal sensors mounted at critical locations on the machine. Comprehensive expertise this paper establishes a model of 14 nodes [11], respectively: X, Y and Z axle nuts and rail temperatures, motor temperature, main shaft bearing temperature, after the bearing temperature, the temperature of the machine bed, spindle thermal error, then obtained Dynamic Bayesian Network model for thermal error of precision CNC shown in Figure 2.

Figure 1. DBN framework of thermal error research
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3.2 DBN Applied in Thermal Error

The above model and the discrete DBN inference algorithms, constitutes the thermal error of DBN. DBN in addition to the network structure, also need to define the parameters. For discrete DBN, the conditional probability is an expert knowledge which reacts experts views of the causal relationship among the connected nodes in the network. State transition probability between two time slices is random probability that changes over time\(^{[12]}\).

The specific calculation model can be expressed according to the conditional probability between expertise and nodes. The transition probabilities of DBN can be acquired by the priori knowledge of experts. Using fuzzy classification to divide the experimental measurements of state parameter domain, in this paper respectively divided into five states \(\{1, 2, 3, 4, 5\}\).

According to the formula (7), to calculate probabilities: \(p(d(L)|T', T^2, \cdots T^n)\) which means we can obtain conditional probability at the corresponding \(d(L)\) of different states when \(T' (i = 1, 2, \cdots, 13)\) take any state. Through the judgment of values \(p_1, p_2, p_3, p_4, p_5\) to decide the state of thermal error area, then finally predict thermal error value.

IV. Conclusions

(1) DBN structure is different from the other fitting modeling theory, it comes from the probability of data, and combined with the specific language of graph theory clearly express the dependent relationships between the various factors affecting the thermal error. And applied the fuzzy classification and membership the concept of the measurement data and other scientific probability distribution, reducing the computational complexity, concluding the probability distributions, making the numerical prediction and modeling has the high-precision thermal deformation characteristics;

(2) DBN model is widely used in the present study in the field of threat estimation and target assignment, etc. Applied it to the machine tools thermal error modeling is the current research in the field of thermal error for another attempt and application, which also provides a new effective way to study CNC machine tool thermal error.
error.

(3) Thermal error model based on DBN takes full use of the prior knowledge and sample data, and can dynamically predict thermal error according to the time of machine tools, and with the increase of sample to obtain data updated. It can reflect the change of working condition of machine tools in the process of reasoning, to make it better meet the needs of real-time compensation.

In summary, this paper use DBN structure for thermal error of machine tools to make a series of useful analysis and exploration, get the corresponding conclusions. Based on DBN structure in the study of thermal error is novelty, this paper is for further study on the experimental data and model combination.

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