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A Revisit To Forchheimer Equation Applied In Porous Media Flow

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Abstract: A brief reference to various non-linear forms of relation between hydraulic gradient and velocity of flow through porous media is presented, followed by the justification of the use of Forchheimer equation. In order to study the nature of coefficients of this equation, an experimental programme was carried out under steady state conditions, using a specially designed permeameter. Eight sizes of coarse material and three sizes of glass spheres are used as media with water as the fluid medium. Equations for linear and non-linear parameters of Forchheimer equation are proposed in terms of easily measurable media properties. These equations are presented in the form of graphs as quick reckoners.

Keywords: Porous media, Darcy, Forchheimer equation, tortuosity, porosity.

I. Introduction

Darcy's linear law adequately and reliably describes porous media flow at low Reynolds numbers. At higher Reynolds numbers, where a non-linear relationship between regime and resistance is anticipated, its applicability leads to inaccurate predictions. A lot of, but scattered, literature that exists indicates a little agreement among the results of different works, especially pertaining to post- Laminar flow.

Forchheimer (1901), based on experiments on a sand model for well flow, proposed a quadratic form of equation covering linear and non-linear flow regimes, as

$$i = av + bv^2 \qquad (1)$$

In this equation, 'v' is bulk velocity and '**a**' and '**b**' are linear and non-linear parameters which depend upon fluid and medium properties. Term 'av' represents the rate of energy loss in the laminar regime and 'bv^2' corresponds to post laminar regime. Equation (1) was later refined by Forchheimer himself, (Scheidegger (1960)) by adding a third term to consider transitional conditions of flow, as

$$i = av + bv^2 + c_1 v^3 \qquad (2)$$

The two term Forchheimer equation was modified also as (Rose (1951))

$$i = av + bv^2 + c_2 v^{1.5} \qquad (3)$$

Equation(1) was generalized to contain a time variant term (Polubarinova-Kochina (1952)), as

$$i = av + bv^2 + c_3 \frac{dv}{dt} \qquad (4)$$

where c_1,c_2 and c_3 are coefficients.

In addition to a quadratic form, other non-linear forms in vogue are:

Missbach (Scheidegger,1960) employed an empirical power law as

$$i = c_4 v^m \qquad (5)$$

In Eq. (5), c_4 is a coefficient determined by the properties of the fluid and porous medium, and m is an exponent having a value between 1.0 and 2.0, depending on the nature of the regime. Though 'c_4' and '**m**' are not strictly constants and depend upon the Reynolds number of flow, the available data indicate that over a given range of Re, they were considered to be constants. This form of equation is too simple to describe a wide range of seepage flow.

Willkins (1955) while investigating the permeability properties of a wide range of crushed rocks and glass balls, proposed an equation of the form

$$v_v = c_5 \mu^{\alpha} r^{\beta} i^n \qquad (6)$$

where v_v = Seepage velocity, r = Hydraulic radius = ratio of void ratio to specific surface, α and β = constants, c_5 = coefficient, n = 1/j = Constant,

μ =Dynamic viscosity of the fluid

Equation (6) is an improved form of Eq. (5) as majority of the media properties are given separate identity. Parkin (1963) used this form of equation to study the hydraulic and stability problems of inbuilt spillway dams. Widely used forms of hydraulic gradient-velocity of flow relationships are summarized in Table 1. A glance at majority of the past work indicates that Forchheimer equation (Eq. 1) is most widely used.

The scope of the present study is to express the Forchheimer coefficients in terms of easily measurable media properties. Hence, an experimental programme was carried-out using a specially designed permeameter.

II. Experimental Set up

Figure 1 shows the details of the permeameter used. It consists of a vertical circular G.I column of 6.2 m high and 15.25 cm in diameter. A cursory look at earlier investigations (Dudgeon (1966), Subramanya and Madav (1978), Pradeep Kumar and Venkatraman (1995), reveals that a single length of travel with one set of head loss readings measured from two manometers, one at the beginning and the other at the end of the section, was used for computing hydraulic gradient. The permeameter used in the present study is specially designed to have three different lengths of travel (1000 mm, 3007 mm, and 5032 mm) with three sets of piezometric tapping points enabling to compute three numbers of hydraulic gradients, average of which is used in the analysis of experimental data. This ensures a more reliable value of hydraulic gradient, as it reduces (rather, avoids) error due to non-uniformity in packing of the porous medium, if any. Proper care is exercised to file away all the burrs and projections at the tapping points. One set of ends of polythene tubes are connected to the permeameter through short copper tubes and the other ends are connected to a manometer board. Manometer heads are measured to an accuracy of ± 0.5 mm.

A horizontal perforated pipe provided at the delivery end of inlet pipe enabled water (fluid medium used in the present study) to fall in the form of shower instead of a thick jet of water. Further, a 3.5 mm thick aluminum screen with 85% perforations placed at the entry of permeameter facilitated relatively turbulent-free entry of water into the permeameter. A similar screen at the exit of permeameter allowed the retention of media in the permeameter.

Eight sizes of coarse granular media and three sizes of glass spheres are used as media in the present study. Media properties are listed in Table 2.

III. Experimental Procedure

Permeameter is filled with medium under gravity, ensuring even packing by varying height of fall. Water is allowed to flow for about 5 to 6 hrs to avoid further reorientation during experimentation. All the entrapped air is sucked by a suitable system. Once the flow attained steady state conditions, discharge and corresponding manometer readings are noted. Temperature of the outflow is recorded for every run. Using these data, velocity and hydraulic gradients of flow are computed.

IV. Analysis And Results

Assumptions made during the course of analysis are

(i) Flow is steady and one dimensional,
(ii) Porosity of the medium is uniform,
(iii) Material used in the media is homogeneous and inert and,
(iv) Single-phase flow of fluid through unconsolidated rigid packing.

In order to make use of findings of any experimental investigation, it is first of all necessary to check the reliability of trend of the results by comparing it with that of past. In the present study, limits of Reynolds number for segregation of data into two broad regimes i.e. laminar regime (Re < 10), where linear relationship holds good and post laminar (Re $\geq$ 10) are adopted. relationship is applicable. Reynolds number for all the experimental data for various sizes of coarse granular media and glass spheres used in the present study are computed and they are then segregated into two groups, $R_e < 10$ and $R_e \geq 10$. The first group of data is used to ascertain the reliability in the Laminar zone. That is, the trend of results of present experimental study is compared with that of Darcy. After ascertaining, the later set of data is used for further analysis on the applicability of Forchheimer Equation.

V. Reliability of Present Experimentation

Figure 2 depicts the variation of ‘i' with ‘v’ for the experimental data pertaining to 3.25 mm, 4.73 mm, 10.00 mm, and 13.1 mm size of media packed to 48.9 %, 47.0%, 48.34% and 43.05% porosity with Re<10. (To avoid possible confusion from clustering of data of all sizes, data of only the four sizes are presented in Fig.2.). In order to facilitate comparison of trend of present work with that of past, nature of variation of i with v for Darcy's original experimental data is reproduced in Fig. 3. It may be noted that the i-v data, for different sizes are found to fall along separate straight lines, with different positive slopes, the slope of each line being a measure of hydraulic conductivity, as is represented in the original work of Darcy. Thus, hydraulic conductivity values for all the sizes are determined. For example, for 3.25 mm size, it is 12.704 cm/s, for 4.73 mm size, it is 18.25 cm/s, for 10.00 mm size, it is 61.95 cm/s and for 13.10 mm size, it is 87.43 cm/s.

Comparing trends of variation of i with v of both the works, it may be noted that

(i) Similar trends are observed in both cases.
(ii) Linear relationship between v and i is found to be valid for Re < 10.
(iii) This indicates that as the velocity of flow increases, correspondingly, there is an increase in the value ohydraulic gradient, i. This is in conformity with Hagen-Poiseuille equation for head loss through pipe flow and earlier findings (Dudgeon (1966), Subramanya and Madav (1978), Pradeep Kumar and Venkatraman (1995)).
(iv) Further, as size of media increases, there is an obvious increase in hydraulic conductivity of media. This may be due to decrease in specific surface as the size of the medium increases (Scheiddegger, 1960). Therefore, smaller is the size of the medium, larger is the extent of contact of the fluid with the solid matrix, resulting in increase of the resistance to flow.
(v) Finally, as the trends of variation of i with v for the data of the present study agreed well with that of Darcy's data, reliability of experimentation is ascertained.

VI. Examining Applicability of Forchheimer Equation

Equation (1) may be rewritten as,

$$\frac{i}{v} = a + bv \tag{7}$$

When a plot is made between i/v on y-axis and v on x-axis, then the data must lie along a straight line, with linear parameter '**a** 'equal to y-intercept (i/v intercept) and non-linear parameter '**b'** is equal to slope of (i/v vs v) line. However, the next question is whether such an approach can be applied to complete experimental data without checking the limits of applicability of such an approach.

A glance at the work reported by Subramanya and Madav (1978) infer that the values of **a** and **b** were determined simply by extrapolating i/v- v lines in the reverse direction to intersect i/v axis. While the intercept on i/v axis is taken as linear parameter (**a**) and slope of the line is non-linear parameter (**b**) of Forchheimer Equation. Thus, values of **a** and **b** for all the sizes of the media used by them are determined. Then equations were proposed relating **a** with size and **b** with size, without prescribing limits of applicability.

Figure 4 presents variation of i/v with v for complete range of data covering both $R_e < 10$ and $R_e \geq 10$ for 3.25mm,4.73mm,10.00mm,and 13.10 mm size coarse grains. Stunningly, the trend of variation of i/v and v lines is not a continuous rising straight line. At lower velocities, the trend is found to lie along a curve, with a steep negative slope and is found to raise with the increase in velocity. Therefore, simple extension of i/v-v line in reverse direction to intercept i/v axis and determination of **a** and **b** from such an analysis is incorrect. The work reported by Murali et al., (2004) (Figure 5) also supports the inference drawn. Therefore, it is obvious that the equations purporting to relate linear and non-linear parameters with size without prescribing limits of applicability are of limited use.

Since Darcy's law is very much valid for Laminar regime ($R_e < 10$), an emphasis is laid on the post laminar regime in the present study.

Reynolds Number for all the experimental data were computed and those data having $R_e \geq 10$ are segregated for further analysis. Figure 6 presents variation of i/v with v for $R_e \geq 10$ for 3.25 mm, 4.73 mm,10.00 mm, 11.64 mm, 13.10 mm, 20.10 mm, 28.90 mm and 39.50 mm packed to 48.9%. 47.00%, 48.34%, 44.03%, 43.05%, 45.88%, 48.73% and 48.26% porosity respectively. Similarly, experimental data ($R_e \geq 10$) of Nasser (1970) and Niranjan (1973) on gravel are also shown in Fig.6. Relationship between i/v and v for glass spheres of 15.41 mm, 18.03 mm and 28.37 mm size (packed to 41.81%, 41.94% and 42.50% respectively) used in the present study, is depicted in Fig. 7. Similarly, experimental data of Niranjan of 13.0 mm, 17.0 mm, 20.0 mm and 25.0 mm sizes for $R_e \geq 10$ are also shown in Fig.7.

For any sample, irrespective of size and shape for $R_e \geq 10$,

- it is seen that i/v, which is a measure of energy loss in the medium, increases as velocity of flow is increases, which is in agreement with earlier findings.
- There is a systematic and regular orientation of i/v Vs v lines of all sizes. The i/v – v line of small size medium is found to have a steeper slope and higher i/v value. Large size medium has a flatter slope and smaller i/v values for a given rate of flow. This infers the rate of increase of total resistance is higher for small size samples as compared with that of large size.

The reason for this kind of response from solid matrix may be at very low velocities, viscous forces, which cause skin friction, will be predominant when compared to effect caused due to size of the wake. The skin friction, which is proportional to the extent of contact with the surface of the body (specific surface) in addition to surface roughness, appears to cause higher value of resistance at lower velocities. As the velocity increases, effect of viscous forces gradually diminishes resulting in reduction of skin friction. A continuous decrease in resistance is found up to a certain value of velocity after which the effect of turbulence predominates. In a situation, where turbulence overweighs laminar conditions, resistance gradually becomes more dependent on relative roughness of the solid media.

Further, a glance at the systematic orientation of i/v –v lines of all the available data, indicates that there is a relation between i/v and v. Using linear regression, values of **a** and **b** are computed for the all the data and are summarized in Table. 3. For the present study, value of **a** for coarse media varies from 0.0016 to 0.0837 sec/cm and for glass spheres it varies from 0.0025 to 0.0049 sec/cm and value of **b** for coarse media lies between 0.0015 and 0.0459 $(sec/cm)^2$ and for glass spheres it is between 0.0024 and 0.0064 $(sec/cm)^2$. It may be observed that both **a** and **b** decrease with increase in size of the media. These observations are in agreement with those noted by Nasser and Niranjan and there is a good dependence between them. Both **a** and **b** are plotted as the ordinates and the size, d of the particle along the abscissa, as shown in Fig. 8. It may be observed that points representing **a** and **b** with size of crushed rock or glass spheres are found to lie along respective smooth curves.

Equations for **a** and **b** in terms of d are obtained by regression technique as

$$a = \frac{0.024}{d^{1.4}} \text{ (Crushed rock)} \tag{9a}$$

$$b = \frac{0.018}{d^{1..5}} \text{ (Crushed rock)} \tag{9b}$$

$$a = \frac{0.03}{d^{1.5}} \text{ (Glass spheres)} \tag{10a}$$

and

$$b = \frac{0.013}{d^{1.5}} \text{ (Glass spheres)} \tag{10b}$$

Therefore, using Eqs. [(9a) and (9b)] or [(10a) and (10b)] for a known size of the medium packed in parallel flow configuration as used in the range of sizes of media used in the present study, either for coarse media or round particles, for the given rate of flow corresponding head loss can be determined, as **a** and **b** are measures of energy loss.

VII. Conclusions

1. Whole range of data is divided into two groups, with one category of data having $R_e < 10$ and the other set has $R_e \geq 10$. Trend of present experimentation is ascertained.
2. Limitations of equations relating Linear parameter and non- Linear parameter of Forchheimer equation with size have been brought out.
3. Expressions relating for Linear and non- Linear parameters for ($R_e \geq 10$) with size of the medium have been obtained. These equations are depicted in the form of charts, which can be used readily and easily to evaluate the resistance offered to flow by the media.

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Notation

a = Darcy parameter
$(a)_{cr}$ = Darcy parameter for crushed rock
$(a)_{gs}$ = Darcy parameter for glass spheres
b = non-Darcy parameter
$(b)_{cr}$ = non-Darcy parameter for crushed rock
$(b)_{gs}$ = non-Darcy parameter for glass spheres
c_1, c_2, c_3, c_4, c_5 = coefficients
C_r = regression coefficient
D = diameter of the circular permeameter
d = volume diameter of a particle, size of the particle
d_p = diameter of pore
i = hydraulic gradient
j = an exponent
k = Darcy’s coefficient of permeability
m = an exponent
n = constant
r = hydraulic mean radius
R_e = Reynolds number
v = bulk or superficial or bulk velocity flow
α = constant
β = constant
μ = dynamic viscosity of the fluid
υ = kinematic viscosity of the fluid
ρ = density of the fluids

Table 1. Different Forms of Non-linear Equations

Sl. No.	Equation	Proposed by	Remarks
1.	$i = av + bv^2$	Forchheimer (1901)	Empirical
2.	$i = av + bv^2 + c_1 v^3$	Forchheimer(Scheidegger,1960)	Empirical
3	$i = av + bv^2 + c_2 v^{1.5}$	Forchheimer (Rose,1951)	Empirical
4	$i = c_4 v^m$	Missbach (Scheidegger,1960)	Empirical
5	$i = av + bv^2 + c_3 \frac{dv}{dt}$	Polubarinova-Kochina (1952)	Empirical
6	$v_v = c_5 \mu^\alpha r^\beta i^n$	Wilkins (1955)	Semi-empirical

Table 2. Characteristics of media used in the study

Sl. No.	Description of the media	Volume Diameter (mm)	Porosity (%)
1	Crushed rock	3.25	48.90
2	Crushed rock	4.73	47.00
3	Crushed rock	10.00	48.34
4	Crushed rock	11.64	44.03
5	Crushed rock	13.10	43.05
6	Crushed rock	20.10	45.88
7	Crushed rock	28.90	48.73
8	Crushed rock	39.50	48.26
9	Glass spheres	15.41	41.81
10	Glass spheres	18.03	41.94
11	Glass spheres	28.37	42.50

Table 3 Values of a and b

Size and shape of the medium	Porosity %	**a** (Sec/cm)	**b** $(Sec/cm)^2$	$\mathbf{C}_r$
Present study				
3.25 mm Cr.Rock	48.90	0.0837	0.0459	0.93
4.73 ,,	47.00	0.0437	0.0619	0.95
10.00 ,,	48.34	0.0101	0.0178	0.93
11.64 ,,	44.03	0.0159	0.0203	0.94
13.10 ,,	43.05	0.0880	0.0133	0.97
20.10 ,,	45.88	0.0154	0.0057	0.95
28.90 ,,	48.73	0.0112	0.0031	0.98
39.50 ,,	48.26	0.0016	0.0015	0.97
Nasser's data				
6.9 mm Cr. Rock	47.20	0.009	0.0336	0.93
16.8 ,,	44.50	0.003	0.0133	0.95
43.6 ,,	50.00	0.002	0.0023	0.92
Niranjan's data				
3.18 mm Cr. Rock	42.00	0.288	0.098	0.95
6.36 ,,	43.50	0.100	0.067	0.86
11.15 ,,	43.00	0.016	0.026	0.93
17.50 ,,	46.50	0.010	0.010	0.94
23.80 ,,	44.70	0.005	0.005	0.91
33.30 ,,	50.00	0.008	0.002	0.95
Present study				
15.41 mm Gl. Sp.	41.81	0.0049	0.0064	0.94
18.03 ,,	41.94	0.0041	0.0052	0.95
28.37 ,,.	42.50	0.0025	0.0024	0.93
Niranjan's data				
13.0 mm Gl. Sp.	35.50	0.0323	0.0081	0.93
17.0 ,,	36.10	0.0293	0.0068	0.95
20.0 ,,	36.70	0.0279	0.0053	0.94
25.0 ,,	38.10	0.0198	0.0035	0.95

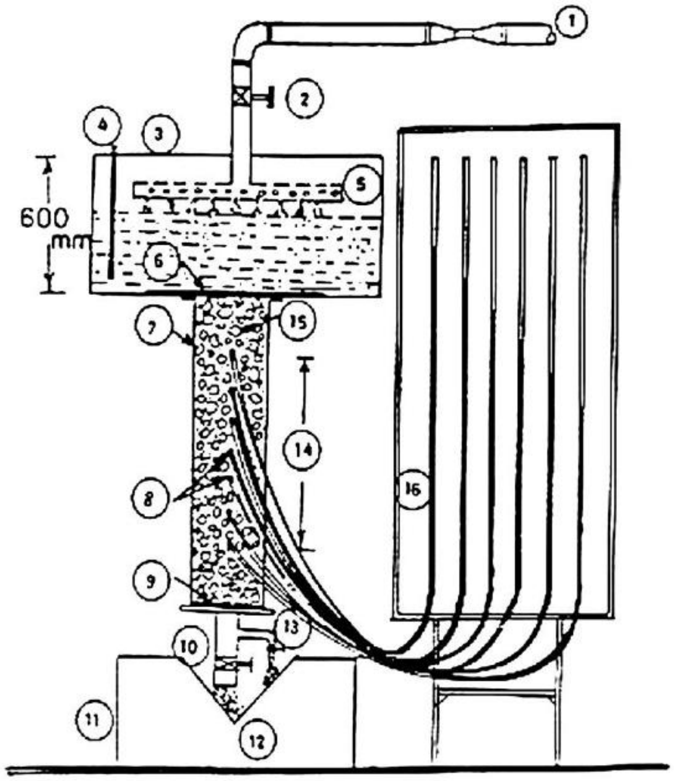

Fig.1 Permeameter

1. 75 mm dia delivery pipe
2. 75 mm dia inlet valve
3. Header tank
4. Static head gauge
5. Perforated pipe
6. Pertorated screen
7. Galvanised iron Pipe (15.5 cm ∅)
8. Piezometric tappings
9. Perforated retaining Screen
10. 75 mm outlet valve
11. Measuring tank
12. 90^0 V-Notch
13. By pass valve
14. Test section
15. Media
16. Manometer board

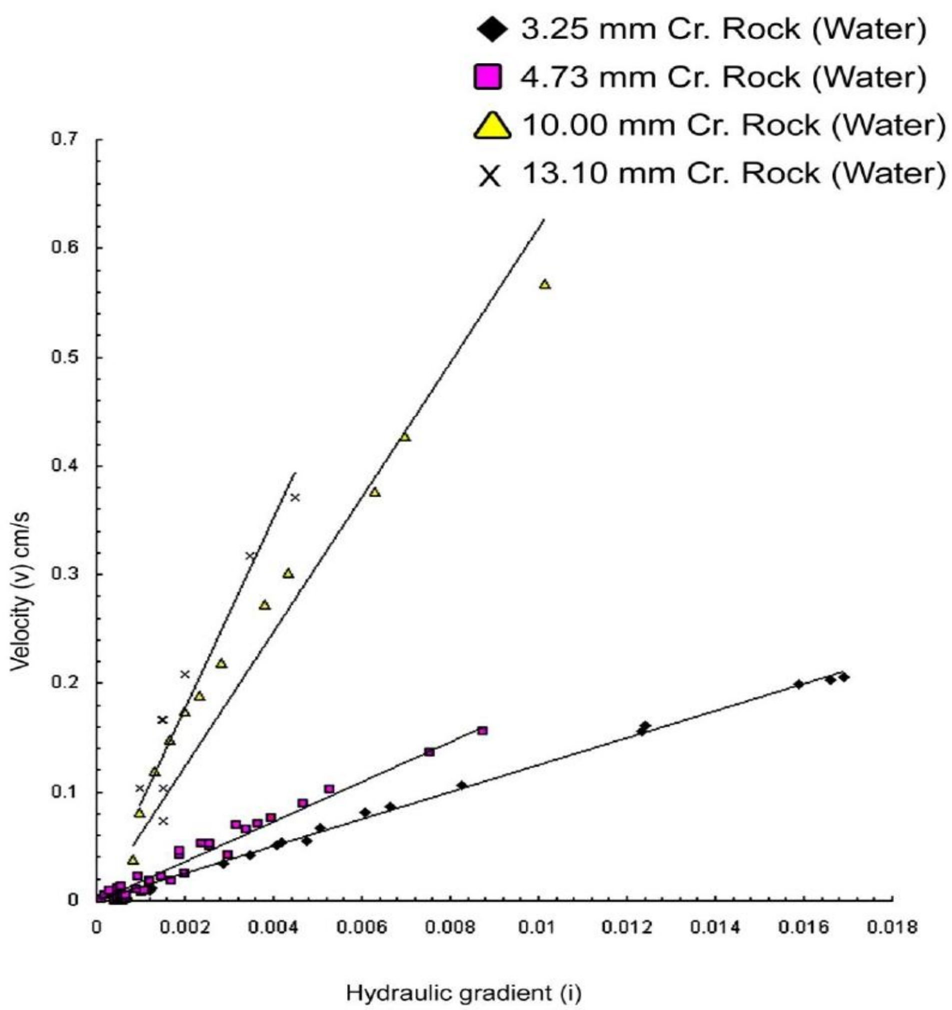

Fig. 2 Variation of Velocity (v) With Hydraulic gradient (i) for Re< 10

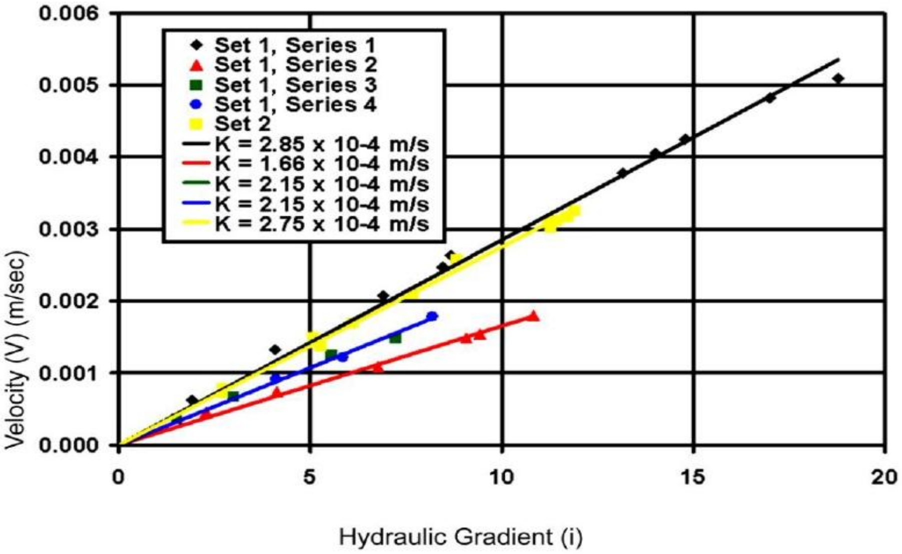

Fig. 3 Variation of Velocity (V) with Hydraulic Gradient (i)

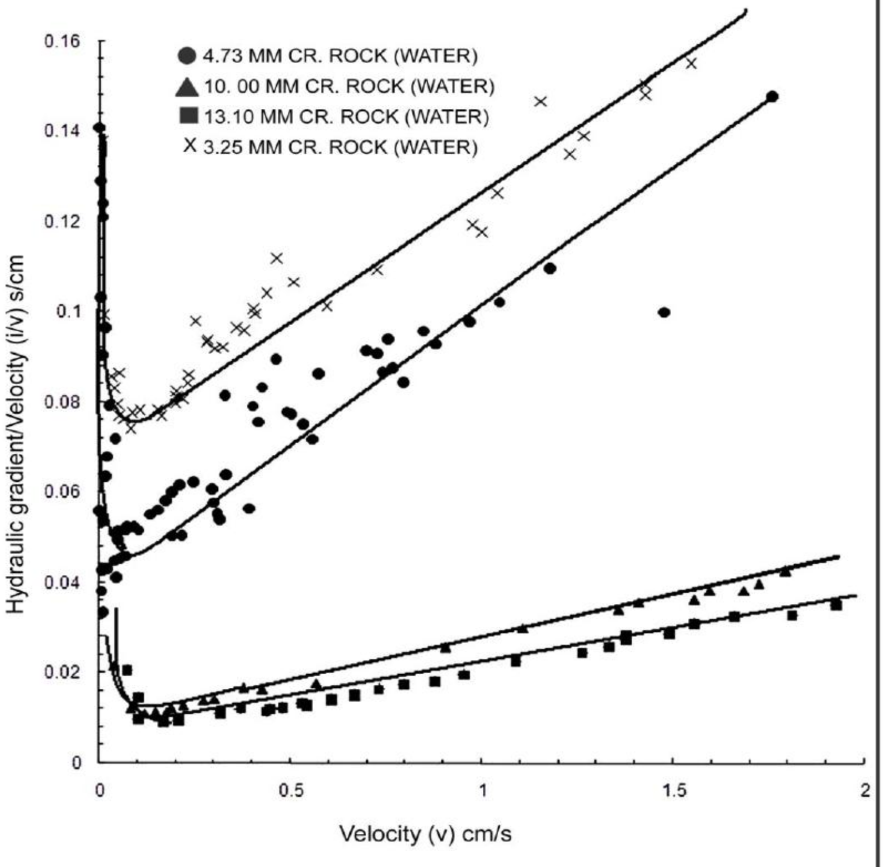

Fig. 4 Variation of Hydraulic gradient/Velocity (i/v) with Velocity (v) - All R_e

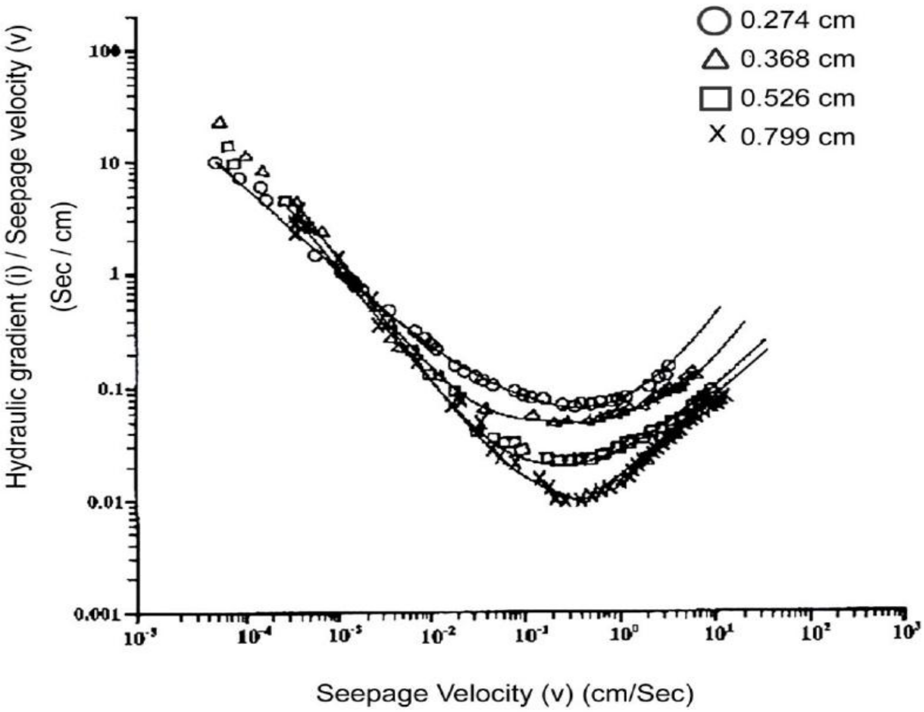

Fig. 5 Variation of Hydraulic gradient/Velocity (i/v) with Velocity (v) - Murali Data

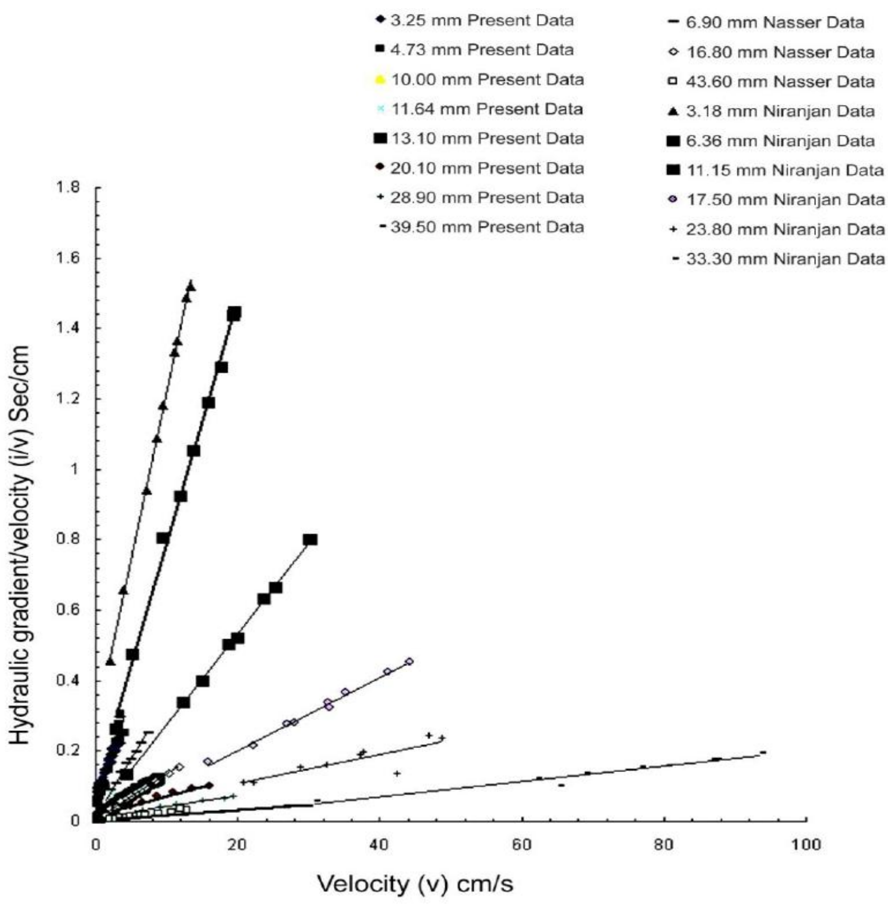

Fig. 6 Variation of Hydraulic gradient/Velocity (i/v) with Velocity (v) - Combined Data - Crushed Rock

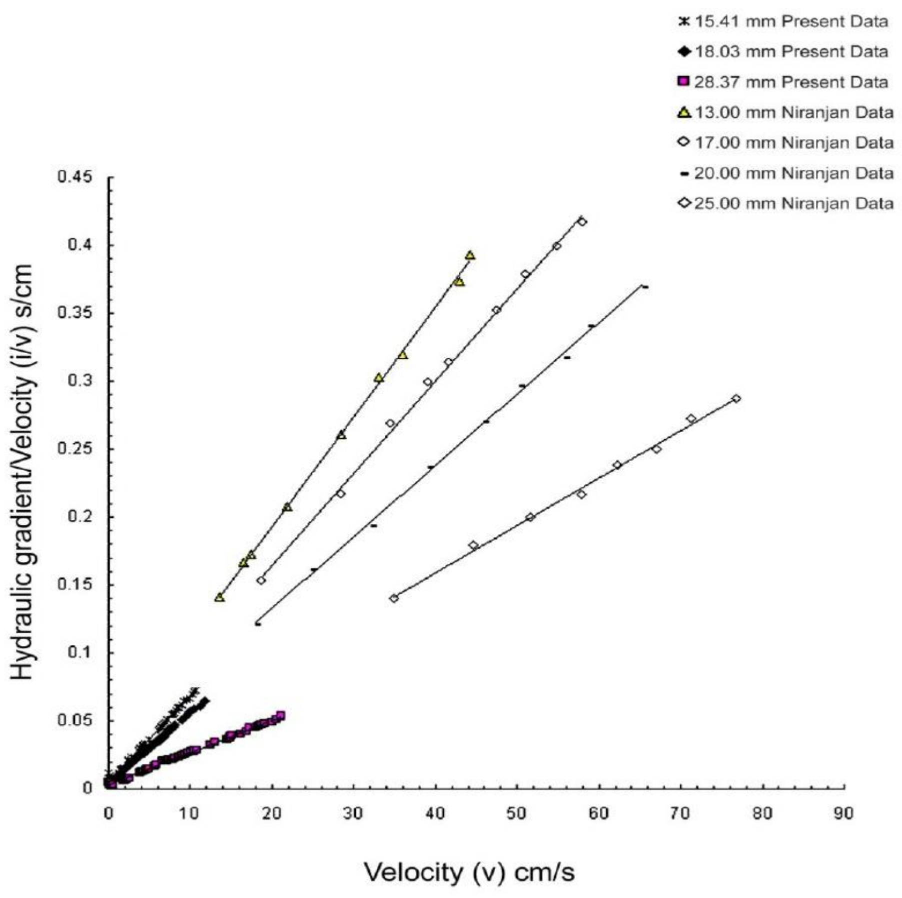

Fig. 7 Variation of Hydraulic gradient/Velocity (i/v) with Velocity (v) - Combined Data - Glass Spheres